Adaptive Sensing Schedule for Dynamic Spectrum Sharing in Time-varying Channel

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Abstract—Dynamic spectrum sharing is considered as one of the key features in the next-generation communications. In this correspondence, we investigate the dynamic tradeoff between the sensing performance and the achievable throughput, in the presence of time-varying fading (TVF) channels. We first establish a unified dynamic state-space model (DSM) to characterize the involved dynamic behaviors. On this basis, a promising dynamic sensing schedule framework is proposed, whereby the sensing duration is adaptively adjusted based on the estimated real-time TVF channel. We formulate the sensing-throughput tradeoff problem mathematically, and further show that there exists the optimal sensing duration maximizing the throughput for the secondary user (SU), which will change dynamically with channel gains. Relying on our designed recursive sensing paradigm that is able to blindly acquire varying channel gains as well as the PU states, the sensing duration can be then adjusted in line with the evolving channel gains. Numerical simulations are provided to validate our dynamic sensing schedule algorithm, which can significantly improve the SU’s throughput by reconfiguring the sensing duration according to dynamic channel conditions.

Index Terms—Sensing-throughput tradeoff, time-varying fading channel, dynamic sensing schedule, spectrum sensing, channel gain estimation.

I. INTRODUCTION

By permitting dynamic spectrum sharing, cognitive radio (CR) provides a new paradigm for opportunistic access of secondary users (SUs) to licensed bands, which are allocated originally to primary users (PUs) [1]. To share the primary band harmoniously without interfering the legal PUs, the SUs should firstly monitor licensed bands within given sensing time [2], and then opportunistically emit signals in transmission slots if none of the ongoing licensed operations are detected.

One of the fundamental importance to CR networks is the tradeoff between the sensing accuracy and the SU’s throughput. That is, in the context of shared access, an optimal sensing duration should be determined, by maximizing the achievable shared throughput under the constraint that the PUs are sufficiently protected against harmful interference. Such an optimization problem was firstly considered by [3]. According to [3], the optimal sensing duration exists indeed, and a periodical sensing-transmission frame structure was proposed. Thereafter, the trade-off over Nakagami fading channel was considered in [4], which focused on the effects from fading parameters on the achievable throughput. In general, previous works assume the knowledge of wireless channels to be fully available at SUs, which will become impractical in some application scenarios [5], e.g. mobile or dynamic environments. Lately, an estimation-sensing-throughput tradeoff was formulated in [6] to solve the optimization problem with imperfect link knowledge. Reference [7] investigated this problem in the presence of Rayleigh fading channel by using multi-antenna system to improve the sensing-throughput tradeoff. However, the way to deal with random fading channel is based on the a priori statistical probability density function (PDF). Hence, it can only characterize the instantaneous random behavior, while may fail to model the evolutions of fading channels.

In this correspondence, we focus on the effects from time-varying fading (TVF) channel to sensing-throughput tradeoff. The main contributions are summarized into three aspects. Firstly, we formulate a dynamic state-space model (DSM) which fully characterizes the spectrum sharing system in the presence of TVF channel. In our stochastic model, relying on the transfer characteristic of TVF channel, the frame period is fixed while the sensing duration changes adaptively. Secondly, the sensing-throughput tradeoff is formulated mathematically as a dynamic optimization problem under the constraint condition of predefined sensing accuracy. Based on this formulation, we prove that the optimum sensing duration is uniquely existing for identical channel gain but varies with respect to dynamic channel gain. Finally, an adaptive schedule with joint spectrum sensing algorithm is designed. Joint spectrum sensing algorithm can track the TVF channel gains and detect the PU states. Based on the real-time channel estimation, the adaptive optimal sensing duration is derived.

The paper is organized as follows. In Section II, we formulate the DSM. The traditional sensing-throughput tradeoff problem is smartly introduced in Section III. In Section IV, the adaptive sensing schedule program is formulated. Numerical results and conclusions are provided in Section V and VI.

The notations used are defined as follows. $Q(\cdot)$ denotes $Q$ function which is the tail probability of standard normal distribution. $P_{T}$, $P_{U}$, $S_{T}$, $S_{U}$ represent the sensing channel link from PU to SU, interframe link from PU to SU receiver and secondary link from SU to SU receiver, respectively. $SNR_{ps}$ denotes the signal-to-noise ratio (SNR) of $P_{T}$ link.
II. System Model

Fig. 1 shows the frame structure designed for SU with periodic spectrum sensing. Each frame consists of one sensing slot and one data transmission slot, i.e., $T_f = T_s(n) + T_i(n)$, $n = 0, 1, \cdots$. In the sensing slot, the detection result is obtained based on the sampling signals. $M(n)$ and $\tau$ denote the sampling size and period respectively, $T_s(n) = M(n)\tau$. In this work, we design a dynamic sensing schedule scheme with considering TVF channel of PU-$\text{SU}_1$ link. Specifically, the frame period is fixed while the sensing duration may change adaptively relying on the transitional behavior of fading channel. Given the sampling frequency $f_s = 1/\tau$ is generally fixed for a receiving device, then the sampling size $M(n)$ will be adjustable.

![Frame structure for SU with dynamic sensing duration](image)

**Fig. 1.** Frame structure for SU with dynamic sensing duration

The discrete time DSM is formulated as:

$$
\begin{align*}
\mathbf{x}(n) &= \Phi(\mathbf{x}(n-1)), \\
h(n) &= \Psi(h(n-1)), \\
\mathbf{y}(n) &= \Omega(\mathbf{x}(n), h(n), \mathbf{w}(n)).
\end{align*}
$$

In the state equation (1), $x$ denotes the PU states and come into two forms: active $\mathcal{H}_1$ and dormant $\mathcal{H}_0$, the complex PSK modulated signal is considered under $\mathcal{H}_1$. The evolution behavior of PU state is characterized as a two-state Markov chain. The durations of active and dormant states are assumed exponentially distributed with mean values of $\alpha_1$ and $\alpha_0$, respectively [8]. Parameters $\alpha_1$ and $\alpha_0$ can be estimated using statistical methods based on finite samples and regarded as a priori information. Then, the stationary probabilities of two states can be calculated as $p(\mathcal{H}_1) = \frac{\alpha_0}{\alpha_1 + \alpha_0}$ and $p(\mathcal{H}_0) = \frac{\alpha_1}{\alpha_1 + \alpha_0}$. The transition probability matrix (TPM) $\mathbf{P}_s$ is time-varying but can be estimated easily with established parameters $\alpha_1$ and $\alpha_0$.

$$
\mathbf{P}_s = \\
= \begin{bmatrix} p_{\mathcal{H}_0\rightarrow\mathcal{H}_0} & p_{\mathcal{H}_0\rightarrow\mathcal{H}_1} \\
p_{\mathcal{H}_1\rightarrow\mathcal{H}_0} & p_{\mathcal{H}_1\rightarrow\mathcal{H}_1} \end{bmatrix},
$$

where

$$
\begin{bmatrix} p_{\mathcal{H}_0\rightarrow\mathcal{H}_0} \\
p_{\mathcal{H}_0\rightarrow\mathcal{H}_1} \end{bmatrix} = \begin{bmatrix} \alpha_0 + \alpha_1e^{-(\alpha_0+\alpha_1)T_f} \\
\alpha_0 - \alpha_1e^{-(\alpha_0+\alpha_1)T_f} \end{bmatrix},
$$

$$
\begin{bmatrix} p_{\mathcal{H}_1\rightarrow\mathcal{H}_0} \\
p_{\mathcal{H}_1\rightarrow\mathcal{H}_1} \end{bmatrix} = \begin{bmatrix} \alpha_0 + \alpha_1e^{-(\alpha_0+\alpha_1)T_f} \\
\alpha_0 - \alpha_1e^{-(\alpha_0+\alpha_1)T_f} \end{bmatrix}.
$$

In state equation (2), $h = ae^{i\theta}$ denotes the TVF channel state of PU-$\text{SU}_1$ link, with fading amplitude $a$ and phase $\theta$. The phase $\theta$ is uniformly distributed within the domain $[0, 2\pi]$. The amplitude $a$ is assumed to be Rayleigh fading [9] and its statistical PDF with scale parameter $\sigma_a$ is:

$$
f_a(a) = \frac{a}{\sigma_a^2} \exp \left(-\frac{a^2}{2\sigma_a^2}\right), \quad a > 0.
$$

Since our study focuses on the slow-fading case, we assume the fading channel changes at a rate much slower than the PU state, i.e., the coherence time of channel $T_c \approx 1/f_D$ is greater than the frame period $T_f$, where $f_D$ denotes the maximum Doppler shift. Furthermore, the channel gain is assumed to be invariant within several successive frames, i.e., $T_c = JT_f$ and $J$ is a positive integer. To this end, we define the transition frame as the frame where the channel gain will possibly vary and whose indexes are $n = Jl$ ($l = 0, 1, \cdots$).

As far as the slow-fading channel is considered, the first-order finite-state Markov chain (FSMC) model is adopted owing to its effectiveness [9]. In the FSMC model, the fading amplitude is partitioned to $K$ non-overlapping regions, i.e., $[v_0, v_1), [v_1, v_2), \cdots, [v_{K-1}, v_K)$, and each region is represented by one feasible state $A_k, A_k \in [v_k, v_{k+1})$ with the stationary probability $\pi_k = \sum_{a} f_a(a)da, k = 0, 1, \cdots, K - 1$. A common strategy in constructing the FSMC model is the equal partition rule, i.e., let $\pi_k = 1/K$. And the representative fading state is $A_k = \sum_{a} f_a(a)da/\pi_k$. The set of representative fading states is given as $\mathcal{A} = \{A_0, A_1, \cdots, A_{K-1}\}$. The amplitude $a(n) \in \mathcal{A}$ and evolves according to the first-order Markov process at the transition frames but stays the same at the remaining frames. The transitional probabilities in the TPM of the channel evolution process $\mathbf{P}_s$ relate only to the PDF of channel amplitude and states number [9], and hence can be regarded as static and established as a priori information.

Without losing the generality, the ED-based sensing is used and the observation in (3) conditioned on two hypotheses is:

$$
\begin{align*}
\mathbf{y}(n) &= \left\{ \begin{array}{ll}
\sum_{m=1}^{M(n)} |w(n, m)|^2, & H_0, \\
\sum_{m=1}^{M(n)} a(n)x(n, m) + w(n, m), & H_1.
\end{array} \right.
\end{align*}
$$

Here, $w(n, m)$ denotes circularly symmetric complex Gaussian (CSCG) noise. In the following analysis, we make the following assumptions for the above DSM.

(AS1) The emitted signals are independent and identically distributed (IID) with mean zero and variance $\sigma_v^2$. The noise is IID with mean zero and variance $\sigma_w^2$. These two random processes are independent from each other and known to SU. The noise uncertainty and hardware imperfections will not be considered in our investigation [10], [11].

(AS2) As the ED detection is used, the amplitude-centric FSMC model is suitable, without considering the effects from channel phase to the observations, as shown in (6).

III. Traditional Sensing-throughput Tradeoff

In this section, the regulatory constraints for sensing-throughput tradeoff under statical channel are presented, where the sampling size is constant, i.e., $M(n) = M$. The sensing performance is characterized with the probability of false alarm $p_f \doteq p(y > \xi \mid H_0)$ and the detection probability $p_d \doteq p(y > \xi \mid H_1)$. $\xi$ is the detection threshold and is chosen to obtain a certain detection probability $p_d$ owing to the regulatory constraints from some telecommunication standards. For instance, $p_d$ is chosen above 0.9 in IEEE 802.22 WRAN [12].

Based on [3], the fundamental tradeoff under statical channel can be stated as an optimization problem with an overall consideration of sensing capability and achievable throughput,

$$
\begin{align*}
\max_M U(M, \gamma), \\
s.t. \quad p_\xi(\xi, M, \gamma) &\geq p_d.
\end{align*}
$$

where $\gamma = a^2/\sigma_v^2$ is the value of $SNR_{av}$, the achievable throughput $U(M, \gamma) = \frac{T_c - Ms}{T_f} C_0 \left[1 - p_f(M, \gamma)\right] p(H_0)$ [3]. $C_0 =$
The value of $\gamma$ is commonly unknown and dynamic (e.g. due to TVF channel), and therefore, existing methods can only achieve an expected fair performance but is no longer optimal. To combat these challenges, we further design an adaptive sensing schedule which can adjust the optimal sensing duration for every frame based on the accurate real-time estimation of $\gamma$.

**IV. Adaptive Sensing Schedule**

In this section, we first formulate the sensing-throughput tradeoff under TVF channel mathematically, then analyze the relation between the optimal sampling size and $\text{SNR}_{ps}$. Finally, the adaptive sensing schedule is illustrated.

**A. Problem Formulation**

Considering the dynamic $\gamma(n)$, the achievable throughput can be written as:

$$ U(M, \gamma) = C_0 p(H_0) \sum_{n=0}^{N-1} u(M(n), \gamma(n)), \tag{9} $$

where $u(M(n), \gamma(n))$ denotes the throughput in one frame and,

$$ u(M(n), \gamma(n)) = \left[ 1 - \frac{M(n)\tau}{T_f} \right] \left[ 1 - p_f(M(n), \gamma(n)) \right]. \tag{10} $$

The optimization problem in (7) (8) can be restated as:

$$ \max_{M(n)} u(M(n), \gamma(n)), \quad n = 0, 1, \cdots, N-1, \tag{11} $$

subject to $p_d(M(n), \gamma(n)) \geq \bar{p}_d. \tag{12}$

Different from the conclusive results in [3], we can see from (10) that the achievable throughput is a joint function of sampling size as well as $\text{SNR}_{ps}$.

**Theorem 1:** The maximum point of $u(M(n), \gamma(n))$ for $M(n)$ is uniquely existing on one certain $\gamma(n)$ when $p_f(M(n), \gamma(n)) \leq 0.5$ but **sensitive to different** $\gamma(n)$.

This theorem will hold if the following two propositions can be proven.

**Proposition 1:** There is a unique maximum point of the $u(M(n), \gamma(n))$ within the interval $M\tau \in (0, T_f)$ with respect to one certain $\gamma(n)$.

**Proposition 2:** The optimal sampling size yields the highest throughput varies with respect to different $\gamma(n)$.

**Proof for Proposition 1:** For a target $\bar{p}_d$, the partial derivative of $u(M(n), \gamma(n))$ with respect to $M(n)$ can be derived as (13), where, $\beta(n) = \sqrt{2\gamma(n) + 1}Q^{-1}(\bar{p}_d) + \sqrt{\text{SNR}_r}\gamma(n)$ and $p_f(M(n), \gamma(n)) = Q(\beta(n))$.

$$ D(M(n), \gamma(n)) = \frac{\partial u(M(n), \gamma(n))}{\partial M(n)} = -\frac{\tau}{T_f} \left[ 1 - Q(\beta(n)) \right] + \frac{\gamma(n)T_f - M(n)\tau}{2T_f \sqrt{2\text{SNR}_r}} \exp\left[ \frac{-\beta(n)^2}{2} \right]. \tag{13} $$

As $Q(\cdot)$ is monotonic decreasing and $Q(0) = 0.5$, we have:

$$ \lim_{M(n)\tau \to 0} D(M(n), \gamma(n)) = +\infty, \tag{14} $$

$$ \lim_{M(n)\tau \to T_f} D(M(n), \gamma(n)) < -\frac{\tau}{T_f} \left[ 1 - Q(\beta(n) - \sqrt{\text{SNR}_r}\gamma(n)) \right] < 0. \tag{15} $$

We conclude from (14) and (15) that $D(M(n), \gamma(n))$ increases when $M(n)$ is small but decreases when $M(n)$ approaches $T_f/\tau$. Thus, there exists a maximum point of $u(M(n), \gamma(n))$ within definitional domain $M(n) \in (0, T_f/\tau)$.

**Proof for Proposition 2:** The mixed partial derivative of $u(M(n), \gamma(n))$ is shown in (22), which is impossible to become zero. Hence, we conclude that the optimal $M(n)$ satisfying $D(M(n), \gamma(n)) = 0$ will be associated with the varying $\gamma(n)$.

**Note** that, since the channel gain $a(n)$ evolves as a FSMC process and $a(n) \in A$, we can have that $\gamma(n) = a(n)^2 \sigma_s^2/\sigma_w^2$ evolves also as an FSMC process with the same TPM $P_a$, and $\gamma(n) \in R = \{R_0, R_1, \cdots, R_{K-1}\}$ where $R_0 = \sigma_s^2/\sigma_w^2$. Continue without changing paragraph, the reconfiguration relation between the optimal sampling size $M^*_n$ and $R_k$ can be presented as $\Delta = \left[ [R_0, M^*_0], [R_1, M^*_1], \cdots, [R_{K-1}, M^*_k] \right]$ which should meet the condition that $D(M^*_n, R_k) = 0$. The key conception of our proposed adaptive sensing schedule is that the sampling size $M(n)$ of each frame will be adapted with regards to the current $\gamma(n)$, which is determined via the reconfiguration relation above, i.e. $M(n) \in M = \left[ M^*_0, M^*_1, \cdots, M^*_K \right]$.

**B. Adaptive-Joint Sensing Algorithm**

In order to accomplish the dynamic reconfiguration of the sampling size, the channel gain estimation and the PU state detection need to be jointly implemented. The proposed adaptive-joint sensing algorithm is iterative and accomplished by the following three steps.

1) Sampling size determination: Inside one frame $n$, the SU is fed with an initial number of optimal sample size $M(n)$ which is determined based on the predictive $\text{SNR}_{ps}$, i.e., $\gamma(n)(n-1)$, with the reconfiguration set $\Delta$. As mentioned before, the $\text{SNR}_{ps}$ related to channel state may transmit only
in the transition frames but stay static in the remaining frames. Therefore, \( \gamma(n | n - 1) \) is obtained by:

\[
\gamma(n | n - 1) = \begin{cases} 
\arg\max_{y(n) \in \mathcal{Y}} p(y(n) | \hat{y}(n - 1)), & \text{transition,} \\
\hat{y}(n - 1), & \text{remaining,}
\end{cases}
\]

where the prior probability \( p(y(n) | \hat{y}(n - 1)) \) is obtained based on the TPM \( \mathbf{P}_a \) and the estimated \( SNR_{ps} \) of the previous frame.

The reconfiguration is executed by establishing the optimal sampling size \( M(n) \) as \( M^*_k \) if the predictive \( SNR_{ps} \) value \( \gamma(n | n - 1) \equiv R_k \), where \( k \in \{0, 1, \cdots, K - 1\} \). After reconfiguring the sampling size, the observation \( y(n) \) can be obtained according to (6).

2) Channel gain estimation: According to our past investigation focus on the channel estimation in uncertain CR system, i.e., [5], the algorithm for channel estimation contains two main steps as coarse detection and channel estimation by different mechanisms.

The objective of coarse detection is to obtain a rough detection of PU state and facilitate different strategies to estimate channel gain. It is derived via maximum a posteriori probability (MAP) criterion with inaccurate channel state assumption \( a^i \) which is the minimum of channel gain set \( \mathcal{A} \):

\[
x(n)^i = \arg \max_{x(n) \in \mathcal{H}_0, \mathcal{H}_1} p[x(n) | y(n), a^i].
\]

For different results of coarse detection, the channel estimations will be implemented respectively according to different mechanisms. Specifically, when \( x(n)^i = \mathcal{H}_0 \), there is little observed information that can be utilized. So, we obtain the estimation based on the prior transition property,

\[
\hat{a}(n) = \begin{cases} 
\arg\max_{a(n) \in \mathcal{A}} p[a(n) | \hat{a}(n - 1)], & \text{transition,} \\
\hat{a}(n - 1), & \text{remaining.}
\end{cases}
\]

Otherwise, when \( x(n)^i = \mathcal{H}_1 \), the joint estimation algorithm will be implemented via the MAP criterion and accumulative modification mechanism [5].

\[
\hat{a}(n)^{\text{MAP}} = \arg \max_{a(n) \in \mathcal{A}} p[a(n) | \hat{a}_{\text{pre}}, x(n)^i, Y(n), G(n)].
\]

where \( \hat{a}_{\text{pre}} \) denotes the estimated channel of the previous channel coherent time, i.e., \( \hat{a}_{\text{pre}} = \hat{a}(n | T_f | -1) \). The accumulation observation and counter \( \hat{Y}(n), G(n) \) can be updated as:

\[
\{Y(n), G(n)\} = \begin{cases} 
\{y(n), 1\}, & \text{transition,} \\
\{\hat{Y}(n - 1) + y(n), G(n - 1) + 1\}, & \text{remaining.}
\end{cases}
\]

3) PU state Detection: Once the channel gain has been updated, the estimated \( SNR_{ps} \) can be updated as \( \hat{\gamma}(n) = \hat{a}(n)^2 \sigma_n^2 / \sigma_a^2 \). Then, the real-time threshold \( \xi(n) \) is:

\[
\xi(n) = \sigma_n^2 \left[ \sqrt{M(n)(2\hat{\gamma}(n) + 1)}Q^{-1}(\hat{p}_d) + M(n)(\hat{\gamma}(n) + 1) \right].
\]

The detection result is derived via the Neyman-Pearson (N-P) rule, i.e., \( y(n) \notin \xi(n) \). It’s clear from (27) that the detection threshold is real-time and specified in every frame based on the current sampling size and estimated \( SNR_{ps} \). Therefore, the detection result is accurate enough to guarantee the target detection probability \( \hat{p}_d \).

C. Implementation

Based on the elaborations above, the schematic implementation of the new adaptive schedule is illustrated by Fig.2. The sampling size \( M(n) \) in the sensing duration of current frame is adjusted firstly according to the predictive \( SNR_{ps} \) and the proposed reconfiguration relation \( \Delta \). Then, the observation signal \( y(n) \) will be obtained and used for both channel estimation and PU state detection. The channel gain is estimated relying on different mechanisms. Then the real-time threshold can be updated for making the final decision on the PU state.

V. NUMERICAL RESULTS AND DISCUSSIONS

In this section, computer simulation results and discussions are presented to evaluate the sensing-throughput tradeoff performance of proposed schedule. We choose \( a_0 = a_1 \), and hence, \( p(\mathcal{H}_1) = 0.5. \hat{p}_d = 0.9, T_f = 0.1 \text{ms}, N = 10000, f_s T_f = 0.01, \) and \( K = 8. SNR_{ss} = 20 \text{dB} \), and hence, \( C_0 = 6.6582 \). The PU emitted signal is assumed to be QPSK modulated with bandwidth of 6MHz [3] and the additive noise is a zero-mean CSCG process.

We first show the joint impact of the sampling size \( M \) and the channel gain \( a \) on the SU’s achievable normalized throughput which is defined as \( \Upsilon^a(M, a) \). The optimal sampling size \( M^*(a) \), and channel gain \( a \) which has been proven in Section IV.A, the intersection line of its partial derivative surface with the zero-flat indicates the optimal achievable throughput \( \Upsilon^a(M, a) \). The results of Fig.3 and Fig.4 indicate that the optimal sampling size maximizing the achievable throughput is unique for one channel gain but varies with different channel gains. Therefore, the simulated results match to the theoretical results of Theorem 1 very well.

In Fig. 5, we then compared the maximum throughput of different configurations of the sensing duration in time-varying
channels. It is seen that the throughput performance of our adaptive sensing schedule, with the jointly estimated channels, may approach the ideal performance with the known channels, which also validate our designed joint channel estimation algorithm. From Fig. 5, its performance significantly outperforms the static sensing schedule schemes.

Finally, we investigate the estimation mean square error (MSE) performance of TVF channel under different $f_D = 100Hz, 200Hz$. Moreover, the effects of channel estimation error on the detection performance as well as system throughput performance are further studied. It’s clear from Fig.6 that the decrease of channel estimation error enhances the system throughput. Since accurate channel estimation can help to reduce the probability of false alarm while still guarantee the target detection probability, as shown in Fig.7.

VI. CONCLUSIONS

In this correspondence, we consider the sensing-throughput tradeoff problem in dynamic environments and design an adaptive sensing schedule scheme. Particularly, we show the optimal sensing duration is closely related with dynamic channel gains. A joint sensing algorithm with adaptive sensing duration is then proposed. Simulation results are provided to validate the designed scheme, with which the significant improvement in throughput can be attained.

REFERENCES