Catenary Finite Element Model Initialization using Optimization
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Abstract

The contact quality between pantograph and catenary plays a critical role on providing the required energy to power the trains traction systems. The need to study and analyze the dynamic behavior of these coupled systems as led to the development of pantograph/catenary interaction simulation software. Despite the increasing interest on the dynamical analysis of the catenary and its interaction with the pantograph, the accurate analysis of its configuration at static equilibrium becomes of most interest where its correct initial undeformed shape and correspondent undeformed mesh must be found. The initialization of the catenary, that is, the setting of the initial positions of the catenary finite element nodes have different requirements due to the different methods used in the solution of their dynamics. Furthermore, as the static configuration of the catenary provides its initial conditions for dynamic analysis it is possible that these have a significant influence on the simulation results. This is even more critical when considering the contact wire sag correct deformed shape where contact with the pantograph occurs. The work here presented proposes a catenary initialization procedure based on the definition of a fitness function to be minimized using classical gradient based optimization. The proposed methodology also opens the possibility to model catenary systems that have defects such as irregularities on its sag caused by damaged, poorly maintained or ill mounted overhead lines. Here, this irregularity can be imposed on the static equilibrium configuration of the model and the same minimization problem is set to find its corresponded undeformed shape.

Keywords: Railway dynamics; Finite elements, Pantograph/Catenary, Optimization, Structure Initialization.
1 Introduction

As railway vehicles with electrical traction are, today, the most economical, ecological and safe means of transportation. Its energy collection system is the crucial element for their reliable running. This system is generally composed by a pantograph attached to the roof of the train vehicles and an overhead electrical structure laid along the track, as represented in Figure 1. As this structural system, most commonly denominated by catenary, is in contact with the pantograph the electrical current that carries is drawn into the electrical traction system of the train. Undoubtedly this system plays a critical role concerning the ability to supply the proper amount of energy required to run the engines and maintain the trains operational speed, through the catenary-pantograph interface [1]. In fact, on present modern high-speed trains, as more electrical current is required, this issue remains one of the major limiting factors on their top operational velocity.

Figure 1: Representation of railway energy collecting system composed of a catenary and a pantograph.

Catenary systems are subjected to tight functional requirements to deliver electrical energy to trains engines, in order to ensure their reliability and to control their maintenance periods. The quality of the current collection is of fundamental importance as the loss of contact between the collector bow of the pantograph and the contact wire of the catenary with consequent arching not only limit the top velocity of high-speed trains but also imply the deterioration of the functional conditions of these mechanical equipments. A typical construction, such as the one presented in Figure 2, includes the masts (support, stay and console), serving as support for the registration arms and messenger wire, the steady arms, which ensure the correct stagger, the droppers, the contact wire and eventually, the stitch wire. The droppers here play a fundamental role in supporting the contact wire in order that its sag has the appropriate smoothness for contact with the pantograph.
The stagger, represented on Figure 3, is a requirement for almost all catenaries so that the pantographs registration strip, where the contact occurs, as an even wear across all its length. Furthermore the functionality of the catenaries impose that spans have limited length, to allow for curve insertion and that the contact and messenger wires are not longer than 1.5 Km, depending on each particular network. Therefore the catenary geometry requires overlapping between the starting and ending spans of different sections to ensure a correct transition and ensure good contact quality on pantograph entrance and exit. To ensure the smoothness of the contact wire sag, besides the droppers, the contact wire and messenger wire are mechanically tensioned at the extremities of the catenary section, usually this is done by suspended masses.
Depending on the catenary system installed in a particular high-speed railway all the elements or only some of them may be implemented. However, in all cases both messenger and contact wires are tensioned with high axial forces not only to ensure the correct geometry but also to limit the contact wire sag, presented in Figure 4. Limiting the sag not only guarantees the appropriate smoothness of the pantograph contact and ensures the stagger of the contact and messenger wires but also allows for the correct wave travelling speed to develop when contact with pantograph occurs. Of course this travelling speed must by higher than the pantograph and corresponding train speed so that the system is stable enough to guarantee good contact quality with less contact loss as possible [2,3].

![Contact wire sag](image)

**Figure 4:** Representation of the catenary sag, as detail A from Figure 3.

The development of computer resources led simulations to be an essential part of the design process of railway systems. The concurrent use of different computational tools allows carrying out several simulations, under various scenarios, in order to reach an optimized design. In this way, studies to evaluate the impact of design changes or failure modes risks can be performed in a much faster and less costly way than the physical implementation and test of those changes in real prototypes. Due to their multidisciplinary, all the issues involving railway systems are complex. Therefore, the use of computational tools that represent the state of the art and that are able to characterize the modern designs and predict their dynamic behaviour by using validated mathematical models is essential. Moreover, the increasing demands for network capacity, namely the increase of traffic speed, put pressure on the existing infrastructures and the effects of these changes have to be carefully considered. The European Strategic Rail Research Agenda [4] and the European Commission White Paper for Transports [5] have identified key scientific and technological priorities for rail transport over the next 20 years. One of the points emphasized is the need to reduce the cost of approval for new vehicles and infrastructure products with the introduction of virtual certification. Furthermore, the quest for interoperability of different pantographs, in existing and projected catenary systems, puts an extra demand on the ability to control their dynamic behavior [6].

This conjuncture has risen to the development of reliable, efficient and accurate computational procedures for the design and analysis of the pantograph-catenary system allowing to capture all the relevant features of their dynamic behavior [7]. Most of these software tools, until now developed, used for the simulation of the pantograph-catenary interaction are based in the finite element method (FEM) and on multibody dynamic procedures [8,9].
When modelling the catenary system with finite elements (FE), one issue that arises is the catenary initialization. This corresponds to building an undeformed catenary mesh that upon being statically loaded by gravitational and axial tension loads exhibits a correct static deformed shape, with special attention to the catenary sag. This issue of fundamental importance, as it conditions the pantograph-catenary contact quality evaluation, is a current topic of active research. Most of the methodologies used today either require complex techniques or are based on manual iteration. The work here presented proposes the use of an optimization procedure in order to achieve a correct initialization of the statically loaded catenary system. Here a fitness function is formalized where the design variables retrieved from its minimization solution are able to be used as parameters to build an undeformed finite element mesh. Upon the static analysis of this mesh its deformed shape is to follow the objective sag shape of the contact wire set on the fitness function. Hence the catenary initialization is solved. Catenary structures might appear simple at a first glance. However since the contact and messenger wires are elements subjected to very high tension loads in order to form the catenary stagger and have a reduced sag this type of systems exhibit a higher degree of complexity and have a nonlinear behavior. Also, the dropper elements act as cables which have a traction state and a null compression state in order to support the contact wire height at a static state and react as smooth as possible to a pantograph passage. As the evaluation of the fitness function is based on the static analysis of a given catenary mesh, where all the above stated effects must be addressed, this procedure constitutes to use of an optimization technique to solve a nonlinear problem.

In this work, the proposed catenary initialization is to be applied for a determined catenary model in order to form a FEM catenary mesh that in its static deformed shape will have a sag as close as the one for which the model is projected after. Furthermore, on the intend to later on evaluate pantograph–catenary contact quality degradation on damaged or ill mounted catenary lines, the same methodology is applied in order to form a catenary deformed mesh with a imposed local sag defect.

2 Catenary initialization procedure

The methodology here proposed to initialize a finite element catenary model is based on the construction stages of a catenary system where a resemblance of the procedure can be found. When mounting a catenary system, after the posts and consoles are placed, the wires need to be laid in such a way that in the end, according to the project specification, they are fully tensioned and their geometric position, namely the contact wire sag, is correct. In general this is achieved by first laying the wires in partial or full tension. As the wires are laid on the structure provisional droppers and fixes are used to hold the contact wire and fix it on the steady arm extremities. Henceforth the system is left at full tension for a period of time where later on a geometry inspection vehicle passes through the catenary. As this vehicle goes ahead, sensors on the vehicle register the sag of the catenary and the dropper lengths at the contact-wire/dropper junction are adjusted in order to meet the contact wire sag project specifications. This
last procedure may require more than one iteration until the contact wire achieves the proper deflection.

Based on the described construction procedure a catenary initialization methodology can be set by formalizing a minimization problem where the objective is to find the correct lengths of each dropper in order that, when statically deformed, the corresponding FEM mesh has the intended sag at each contact-wire/dropper node.

2.1 Minimization problem formulation

Based on the catenary dropper tuning procedure, described above, a minimization problem is formulated where an objective function is set to find the correct lengths of each dropper in order that the deviation between the statically deformed shape of the finite element mesh and a prescribed objective catenary sag at each contact-wire/dropper node is minimized. The formalization of the minimization problem can be expressed as [10,11]:

\[
\min(f_i(x), g(x)) \quad i = 1..n
\]

subject to: \(1 \leq x \leq u\)

where:

\[
f_i(x) = |s_i^p - s_i(x)|
\]

\[
g(x) = |h_i^p - h_i(x)|
\]

\[
x = [d_1, d_2, ..., d_n, h]^T
\]

\[
l = [l_1, l_2, ..., l_n]^T
\]

\[
u = [u_1, u_2, ..., u_n]^T
\]

As represented on Figure 5, the function \(f_i(x)\), corresponds to the deviation of the sag, \(s_i(x)\), from the prescribed sag, \(s_i^p\), for each dropper junction \(i\), in a catenary span composed of \(n\) droppers. An identical procedure is applied on function \(g(x)\) to the catenary height, \(h_i(x)\), registered at the contact wire/steady arm junction. The sag, \(s_i(x)\), and catenary height of the deformed mesh, \(h_i(x)\), are dependent of the design variables contained in \(x\) vector. The design variables here considered are the dropper lengths, \(d_i\), accounted from the fixed messenger-wire/dropper node to the each contact-wire/dropper, plus the initial catenary height at the steady-arm/contact-wire node, \(h\). Note that the evaluation of the fitness functions \(f(x), g(x)\) implies the construction of a catenary finite element mesh according to design variables and a respective static analysis. Furthermore, \(l\) and \(u\) correspond to the lower and upper bounds of \(x\) which relate to the maximum and minimum possible lengths of the droppers and steady arm height.
Until here the minimization problem set in (1) consists of a multi-objective optimization problem. However, since from a mechanical perspective all deviations considered are of equal importance the minimization problem can be reformulated as:

\[
\min \left( \sum_{i=1}^{n} f_i(x) + g(x) \right) \\
\text{subject to: } 1 \leq x \leq u
\]  

(3)

where by reducing the problem to a single minimization its degree of complexity is decreased as also the numerical computation effort to find a solution.

Furthermore, on a particular problem where the spans of a catenary system are all geometrically identical and present vertical axisymmetric along the middle of the span, another simple reformulation of the problem is possible. This time the design variables can be reduced to the height of the catenary at the steady arm, \(h\), plus only the dropper lengths of one side of the span, as expressed by:

\[
\min \left( \sum_{i=1}^{m} f_i(x) + g(x) \right) \\
\text{subject to: } 1 \leq x \leq u \\
\text{where: } x = [d_1, d_2, ..., d_{n-1}, d_n, h]
\]  

(4)

where: \(m = \text{ceil}(n/2)\)
where when meshing the catenary model the design variables are mirrored to the other side of the span. In this case the number of fitness functions, \( f_i(x) \), is also reduced to almost half and the computational efficiency is improved.

### 3 Finite element static analysis of a catenary model

The deformed shape of the catenary is characterized by small rotations and small deformations in which, so that the optimisation procedure here presented is properly used, the slacking of the droppers is the only nonlinear effect that must be considered. The axial tension on the contact, stitch and messenger wires is constant and cannot be neglected in the analysis. Therefore, the catenary system is modelled with linear finite elements in which the dropper nonlinear slacking is modelled with compensating forces while the stress stiffening of the wires, due to their tensioning, is accounted by considering an added stiffness as function of the applied tensioning forces [12].

Using the finite element method, the static equilibrium equations for the catenary structural system are assembled as [13,14],

\[
Kd = f \tag{5}
\]

where \( K \) is the finite element global stiffness matrix of the catenary while \( d \) and \( f \) are respectively the displacements vector and the force vector containing the sum of all external applied.

In order to accurately account for the stress stiffening of the catenary structure due to the tension stress state caused by the line tensioning with high axial tension forces the beam finite element used for the messenger, stitch and contact wire, designated as element \( i \), is written as,

\[
K^e_i = K^e_L + T K^e_G \tag{6}
\]

in which \( K^e_L \) is the linear Euler-Bernoulli beam element stiffness matrix, \( T \) is the axial tension and \( K^e_G \) is the element geometric stiffness matrix [15]. The droppers and the registration and steady arms are also modelled with the same beam element but disregarding the geometric stiffening. In order to ensure the correct representation of the deformed shape, 4 to 6 elements are used in between droppers to appropriately model the contact and messenger wires [8]. There is no special requirement on the number of elements required to model each dropper, registration or steady-arm. The global stiffness matrix, \( K \), is then built by assemblage of the matrices of the elements according to the catenary model mesh. The force vector, \( f \), containing the sum of all external applied loads is evaluated as:

\[
f = f_{(g)} + f_{(t)} + f_{(d)} \tag{7}
\]

which contains the gravity forces, \( f_{(g)} \), the line tensioning forces, \( f_{(t)} \), and the dropper slacking compensating terms, \( f_{(d)} \). The gravity forces in vector \( f_{(g)} \), are accounted with
not only all elements masses but also with the mass of the gramps that attach the droppers to the wires being modelled here as lumped masses.

Although the droppers perform as a bar during extension their stiffness during compression is either null or about 1/100th of the extension stiffness, to represent a residual resistance to buckling. As the droppers stiffness is included in the stiffness matrix, $K$, as a bar element, anytime one of them is compressed such contribution for the catenary stiffness has to be removed, or modified. Much close to how a dynamic analysis would be performed [16], in order to keep the analysis linear the strategy pursued here is to compensate the contribution to the stiffness matrix by adding a force to vector $f(d)$ equal to the bar compression force

$$f_{d(t)} = K_{\text{dropper}} B d$$

(8)

where the Boolean matrix $B$ simply maps the global nodal coordinates into the coordinates of the dropper element. Note that, for the static analysis of a determined catenary mesh model, if the dropper compensation forces are not null, its calculation will require a corrective iteration procedure. In this case, considering a null compression stress state of the droppers at the first iteration, the static equilibrium equation is to be solved iteratively until a target convergence is reached. Such that for the $i^{th}$ iteration $|d_i - d_{i-1}| < \varepsilon_d$, being $\varepsilon_d$ a user defined tolerance. Note that the criteria of convergence of the nodal displacements must imply convergence of the force vector also, this is, the balance of the equilibrium equation right-hand side contribution of the dropper slacking compensation force with the left-hand-side product of the dropper stiffness by the nodal displacements in equation (5).

4 Application of the catenary initialization procedure

In order to demonstrate the catenary initialization procedure here proposed two cases are presented. Both relate to the same chosen catenary model where in one a regular catenary initialization is evaluated for a prescribed objective sag and other for a prescribed sag with local defect on one of the spans.

4.1 Catenary model

The model here proposed for the initialization procedure is based on a realistic model built to serve as a reference model for pantograph-catenary simulation software, for validation and comparison purposes [7]. The software used to process the static analysis and initialization procedure is PantoCat [16] from IST, Portugal.

The proposed catenary model is similar to the French LN2 and the Italian C270 systems but with modified span length and contact wire tension parameters, its main geometry and material properties of its elements are presented in Table 1.
Benchmark catenary model

| Catenary height [m] | 6.28 | Contact wire height [m] | 5.08 |
| Number of spans | 28 | Number of droppers/span | 9 |
| Nº spans at contact wire height | 20 | Inter-dropper distance [m] | 6.75 |
| Span length [m] | 55 | Stagger [m] | ±0.2 |

<table>
<thead>
<tr>
<th>Contact Wire</th>
<th>Messenger Wire</th>
<th>Droppers</th>
<th>Steady Arms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Section [mm²]</td>
<td>150</td>
<td>120</td>
<td>12</td>
</tr>
<tr>
<td>Mass [kg/m]</td>
<td>1.35</td>
<td>1.08</td>
<td>0.117</td>
</tr>
<tr>
<td>Young modulus [GPa]</td>
<td>100</td>
<td>0.97</td>
<td>-</td>
</tr>
<tr>
<td>Axial Stiffness EA [kN]</td>
<td>-</td>
<td>-</td>
<td>200</td>
</tr>
<tr>
<td>Bending Stiffness EI [N.m²]</td>
<td>195.0</td>
<td>131.7</td>
<td>-</td>
</tr>
<tr>
<td>Tension [kN]</td>
<td>22</td>
<td>16</td>
<td>-</td>
</tr>
<tr>
<td>Claw with:</td>
<td>dropper</td>
<td>dropper</td>
<td>-</td>
</tr>
<tr>
<td>Claw mass [kg]</td>
<td>0.195</td>
<td>0.165</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 1: Geometric and element material properties of the benchmark catenary model.

Relative to each span of the catenary model here considered, the prescribed sag \( s_i \) at each dropper \( i \)/contact wire connection at the local span position \( x_i \) is presented on Table 2. On this table, the nominal length of the droppers, \( L_d \), is also presented. Note that this lengths are only indicative and given in approximation.

<table>
<thead>
<tr>
<th>Dropper ( i )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_i [m] )</td>
<td>4.5</td>
<td>10.25</td>
<td>16.0</td>
<td>21.75</td>
<td>27.5</td>
<td>33.25</td>
<td>39.0</td>
<td>44.75</td>
<td>50.5</td>
</tr>
<tr>
<td>Sag ((s_i) [mm])</td>
<td>0</td>
<td>24</td>
<td>41</td>
<td>52</td>
<td>55</td>
<td>52</td>
<td>41</td>
<td>24</td>
<td>0</td>
</tr>
<tr>
<td>( L_d [m] )</td>
<td>1.017</td>
<td>0.896</td>
<td>0.810</td>
<td>0.758</td>
<td>0.741</td>
<td>0.758</td>
<td>0.810</td>
<td>0.896</td>
<td>1.017</td>
</tr>
</tbody>
</table>

Table 2: Required sag at dropper positions on the span.

A general FE model representation of the catenary model in its static deformed state is presented in Figure 6.
4.2 Regular catenary initialization

For the regular catenary initialization of the proposed catenary model the minimization problem equated in (4) is formulated where the symmetric feature of the span is used. The design variables are set to build the finite element mesh of all the catenary spans except the spans belonging to the overlap section at the ends. Also the evaluation of the fitness functions is defined for only the central span. To solve the minimization problem `fmincon` function from Matlab [17] is used with interior point algorithm. The initial design is set to the nominal dropper lengths presented in Table 2 and the upper and lower bounds set to plus and minus 0.6 m off the initial design. For the stopping criteria both termination, constraint violation and fitness function tolerances are set do $10^{-9}$ m.

The best solution found to the minimizing problem is presented on Table 3 where it is also possible to examine, for each design variable the deviation between the initial design and the found optimal solution.
### Table 3: Initial solution and optimal solution found for the current minimizing problem.

| Design variables | Initial solution ($x_0$) | Optimal solution ($x^*$) | $|x^*-x_0|$ |
|------------------|--------------------------|--------------------------|----------------|
| $d_1$ [m]        | 1.017000                 | 1.021524                 | 0.004524       |
| $d_2$ [m]        | 0.896000                 | 0.903236                 | 0.007236       |
| $d_3$ [m]        | 0.810000                 | 0.817690                 | 0.007690       |
| $d_4$ [m]        | 0.758000                 | 0.768343                 | 0.010343       |
| $d_5$ [m]        | 0.741000                 | 0.750242                 | 0.009242       |
| $h$ [m]          | 5.080000                 | 5.080653                 | 0.000653       |

In order to analyse the solution found, the fitness function evaluations, $f_i(x)$ and $g(x)$, at the initial design and optimal solution are presented on Table 4. Note that, for a set of given design variables, $x$, this functions relate to the deviation between the objective contact wire sag and the sag evaluated on the static analysis of a mesh generated from the design variables. By evaluating the fitness functions presented, between the initial and optimal designs it is possible to evaluate the effectiveness of the applied methodology. Considering the sum of all of these deviations, also presented in Table 4, it is possible to see that its value is in the order of hundredths of millimetre which is a very low deviation for the specified problem and implies a well obtained solution. From the discriminated fitness functions it is also possible to observe that the sag on the first dropper and the catenary height are the most problematic objectives to reach.

### Table 4: Evaluation of the fitness functions at the initial design and optimal solution.

<table>
<thead>
<tr>
<th></th>
<th>Initial solution ($x_0$)</th>
<th>Optimal solution ($x^*$)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_1(x)$ [m]</td>
<td>4.1543E-03</td>
<td>1.2925E-05</td>
<td></td>
</tr>
<tr>
<td>$f_2(x)$ [m]</td>
<td>5.0272E-03</td>
<td>1.3307E-07</td>
<td></td>
</tr>
<tr>
<td>$f_3(x)$ [m]</td>
<td>5.0643E-03</td>
<td>9.0724E-07</td>
<td></td>
</tr>
<tr>
<td>$f_4(x)$ [m]</td>
<td>6.0558E-03</td>
<td>1.8839E-07</td>
<td></td>
</tr>
<tr>
<td>$f_5(x)$ [m]</td>
<td>5.5779E-03</td>
<td>9.5609E-07</td>
<td></td>
</tr>
<tr>
<td>$g(x)$ [m]</td>
<td>3.3994E-03</td>
<td>5.2702E-05</td>
<td></td>
</tr>
<tr>
<td>sum [m]</td>
<td>2.9279E-02</td>
<td>6.7812E-05</td>
<td></td>
</tr>
<tr>
<td>cpu time [s]</td>
<td>876 (for Intel i7 2600K)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

With the optimal design variables it is then possible to generate a catenary finite element mesh where its static deformation is very close to the objective sag. On Figure 7 is possible to observe the sag on a catenary span that resulted from the use of the optimal solution.
4.3 Catenary initialization with local sag defect

In order to proceed with the initialization of a finite element catenary model with a localized sag defect on one of its spans a new objective sag is set as presented in Table 5. Note that, compared with Table 2, the new objective sag has a local defect of 9 mm introduced on the third dropper. Contrary to the regular section initialization, here the axisymmetry of the span cannot be used where the minimization problem is set as formalized in equation (3). Also the design variables to build the mesh are only used on the span which contains the sag defect. As for the other spans the design variables found on the optimal solution for a regular catenary initialization are used. To solve the minimization problem the same function, \texttt{fmincon} from Matlab is used within the same conditions except for the initial design where the optimal solution for the regular catenary initialization is chosen instead.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|}
\hline
Dropper $i$ & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
\hline
$x_i$ [m] & 4.5 & 10.2 & 16.0 & 21.7 & 27.5 & 33.2 & 39.0 & 44.7 & 50.5 \\
\hline
Sag ($s_i$) [mm] & 0 & 24 & 50 & 52 & 55 & 52 & 41 & 24 & 0 \\
\hline
\end{tabular}
\caption{Required sag at dropper positions, with a local defect included.}
\end{table}

The best solution found to the minimizing problem is presented on Table 6. It is possible to examine how much deviation is associated from the initial solution to the found optimal for each design variable. As expect the higher deviation observed relates to the design variable that is closely related to the local defect inserted on the sag.
**Design variables**

| Design variable | Initial Solution ($x_0$) | Optimal Solution ($x^*$) | $|x^*-x_0|$ |
|-----------------|--------------------------|--------------------------|------------|
| d1 [m]          | 1.021524                 | 1.021408                 | 0.000116   |
| d2 [m]          | 0.903236                 | 0.902764                 | 0.000473   |
| d3 [m]          | 0.817690                 | 0.839094                 | 0.021404   |
| d4 [m]          | 0.768343                 | 0.767827                 | 0.000516   |
| d5 [m]          | 0.750242                 | 0.749998                 | 0.000244   |
| d6 [m]          | 0.768343                 | 0.768096                 | 0.000247   |
| d7 [m]          | 0.817690                 | 0.817466                 | 0.000225   |
| d8 [m]          | 0.903236                 | 0.903062                 | 0.000174   |
| d9 [m]          | 1.021524                 | 1.021437                 | 0.000087   |

Table 6: Initial solution and optimal solution for the initialisation of a finite element catenary model with local sag defect.

In order to analyse the solution found, the evaluation of the fitness function for the initial and optimal solution are presented on Table 7. It is also presented the sum of all its deviations. As described earlier it is possible to evaluate the effectiveness of the applied methodology by analysing the values of the fitness function. The sum of the deviations between the objective sag and the optimal found sag are in the order of hundredths of millimetres which is low deviation for problem here considered and a good solution is considered to be obtained. From the discriminated fitness functions evaluations it is also possible to observe that the sag on the first dropper is the most problematic objective to reach much as in the regular catenary initialization.

<table>
<thead>
<tr>
<th>Penalty function</th>
<th>Initial Solution ($x_0$)</th>
<th>Optimal Solution ($x^*$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>f1(x) [m]</td>
<td>1.2925E-05</td>
<td>3.9003E-05</td>
</tr>
<tr>
<td>f2(x) [m]</td>
<td>1.3307E-07</td>
<td>1.0154E-06</td>
</tr>
<tr>
<td>f3(x) [m]</td>
<td>8.9991E-03</td>
<td>1.5248E-07</td>
</tr>
<tr>
<td>f4(x) [m]</td>
<td>1.8839E-07</td>
<td>1.1503E-06</td>
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<td>f5(x) [m]</td>
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<td>f6(x) [m]</td>
<td>2.0486E-07</td>
<td>3.0443E-07</td>
</tr>
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<td>f7(x) [m]</td>
<td>9.4020E-07</td>
<td>5.7032E-07</td>
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<td>f8(x) [m]</td>
<td>1.8250E-07</td>
<td>1.0374E-06</td>
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<td>f9(x) [m]</td>
<td>1.2859E-05</td>
<td>1.9073E-06</td>
</tr>
<tr>
<td>sum (x)</td>
<td>9.0275E-03</td>
<td>4.7797E-05</td>
</tr>
<tr>
<td>cpu time [s]</td>
<td>1856 (for Intel i7 2600K)</td>
<td></td>
</tr>
</tbody>
</table>

Table 7: Evaluation of the penalty functions that compose the formulated fitness function for the initial and found optimal solutions.

With the optimal design variables it is then possible to generate a catenary finite element mesh where its static deformation is very close to the objective sag with local
defect. On Figure 8 is possible to observe the sag on the catenary span with the prescribed defect.

![Figure 8: Finite element representation of the resulting sag with, local defect, from the mesh constructed from the optimal solution.](image)

5 Conclusions

In this work, an implementation of a catenary initialization procedure is presented. This method is based on solving a minimization problem by means of a classical gradient based optimization. The results reveal that this methodology is effective and provides accurate finite element catenary models for further dynamic behaviour analysis studies. The proposed methodology also opens the possibility to model catenary systems that have defects such as irregularities on its sag caused by damaged, poorly maintained or ill mounted overhead lines. Here, this irregularity can be imposed on the static equilibrium configuration of the model and the same minimization problem is set to find its corresponded undeformed shape.

One disadvantage of this method is the time cost to execute this procedure. However, there is the possibility to reduce the size of the problem in question with further investigation and development of the procedure. Also, once obtained the optimised results for the catenary initialisation are be stored and can be used anytime with no need to solve the same minimization problem again unless some of the catenary characteristic are altered.

Although good solutions for the initial catenary positions have been obtained with the approach used here there is no guarantee that a good solution for the initial catenary configuration is always obtained for which a careful analyses of the results is critical as well as setting the bounds of the design variables. Furthermore it is of importance to further investigate about the sensitivity of other design parameters and relate the fitness functions to other aspects of the catenary geometry besides de sag.
References
