Efficient range estimation and material quantification from multispectral Lidar waveforms
Altmann, Yoann; Maccarone, Aurora; Halimi, Abderrahim; McCarthy, Aongus; Buller, Gerald Stuart; McLaughlin, Stephen

Publication date:
2016

Document Version
Peer reviewed version

Link to publication in Heriot-Watt University Research Portal

Citation for published version (APA):
EFFICIENT RANGE ESTIMATION AND MATERIAL QUANTIFICATION FROM MULTISPECTRAL LIDAR WAVEFORMS

Y. Altmann, A. Maccarone, A. Halimi, A. McCarthy, G. S. Buller∗, S. McLaughlin†

School of Engineering, Heriot-Watt University, Edinburgh, U.K.
Email: {Y.Altmann, am827, A.Halimi,A.McCarthy, G.S.Buller, S.McLaughlin}@hw.ac.uk

ABSTRACT
This paper describes a new Bayesian range estimation and spectral unmixing algorithm to analyse remote scenes sensed via multispectral Lidar measurements. To a first approximation, each Lidar waveform consists of the temporal signature of the observed target, which depends on the wavelength of the laser source considered and which is corrupted by Poisson noise. When the number of spectral bands considered is large enough, it becomes possible to identify and quantify the main materials in the scene, in addition to estimating classical Lidar-based range profiles. In this work, we adopt a Bayesian approach and the unknown model parameters are assigned prior distributions translating prior knowledge available (e.g., positivity, sparsity and/or smoothness). This prior model is then combined with the observation model (likelihood) to derive the joint posterior distribution of the unknown parameters which are inferred via a maximum a posteriori estimation. Under mild assumptions often true in practice, we show that it is possible to find a global optimizer of the problem by splitting the problem into two sequential steps estimating the unknown spectral quantities and the target ranges, respectively. The proposed methodology is illustrated via experiments conducted with real multispectral Lidar data acquired under controlled observation conditions.

Index Terms— Multispectral Lidar, Depth imaging, Spectral unmixing, Poisson noise.

1. INTRODUCTION
Light detection and ranging (Lidar) systems are particularly useful to extract spatial features from three-dimensional (3D) scenes. Using single-photon techniques, it is possible to recover structural parameters of forest canopies such as vegetation height, leaf area indices and ground slopes. Spectral information about the scene is usually extracted using passive multispectral (MSI) and hyperspectral images (HSI). Such images are particularly useful to detect spectral variations caused for instance by changes in the canopy composition or by the presence of hidden vehicles or targets for defense applications.

Combining spatial and spectral information can be achieved by coupling Lidar data and multi/hyperspectral images [1], [2]. However, as in many multimodal data fusion problems, data synchronization issues in space (alignment, resolution) and time (dynamic scene, change of observation conditions, etc.) are still open issues. To tackle these problems, multispectral Lidar (MSL), which has recently received attention from the remote sensing community [3]–[5], presents as a promising alternative. Indeed, MSL systems have the ability to fully exploit the 3D distribution of objects, in particular for scenes including semi-transparent objects (e.g., vegetation or fences). In contrast to passive hyperspectral imaging systems which integrate the spectral response along the path of each optical ray, MSL systems measure the spectral response as a function of distance, e.g. depth into a forest canopy and can be used to detect and identify objects (e.g., buildings, vehicles, human activity) hidden for instance in vegetated areas.

In [5], [6], spectral unmixing techniques were developed to analyze 3D scenes composed of multi-layered objects, assuming that the spectral signatures of the materials composing the scenes were known and assuming linear mixing processes. In this paper we extend the method proposed in [5] to account for and identify possibly deviations from the classical linear mixing model (LMM) used to estimate the abundance values of each endmember (assumed known) present in the scene. We assume that for each pixel, the photons emitted by the pulsed laser sources at different wavelength are reflected onto a single surface. This is typically the case for short to mid-range (up to dozens of metres) depth imaging where the divergence of the laser source(s) can be neglected.

Single-photon Lidar and thus MSL systems usually record, for each pixel/region of the scene, a histogram of time delays between emitted laser pulses and the detected photon arrivals. Due to the discrete nature of detected photons, Poisson noise models are more appropriate for single-photon MSL data than Gaussian noise models classically used when analysing HSIs. Due to the design of the proposed experiments (performed indoor here) and to simplify the estimation problem, we further assume that the ambient light and dark counts can be neglected. This assumption often holds for measurements by night or for mid-range imaging applications for which the laser power can be adjusted to reduce the acquisition time and thus the background counts. In this paper, we demonstrate the possibility of efficient 3D scene analysis by exploiting geometric and spectral information contained in MSL data (33 discrete wavelengths ranging from 500nm to 820nm), under favourable observation conditions. However, the proposed method can be extended to more difficult observation conditions, as discussed in the conclusions of the paper.

Adopting a classical Bayesian approach, appropriate prior distributions are chosen for the unknown parameters of the model and the joint posterior distribution of these parameters is then derived. Here we propose to estimate the unknown model parameters via maximum a posteriori estimation. Unfortunately, the corresponding cost function to be optimized is often multimodal, which increases the risk of reaching a local as opposed to a global optimum. In this work, we show that under weak assumptions (often met in practice), it is possible to reach a global optimum using a two-step optimization scheme. More precisely, one can first estimate the mixing coefficients (abundances) involved in the spectral unmixing problem and then estimate the object ranges.

The remainder of the paper is organized as follows. Section II introduces the observation model associated with MSL returns for a single-layered object to be analyzed. Section III presents the Bayesian model associated with the spectral unmixing problem considered and the associated posterior distribution. Section IV describes the estimation strategy adopted to maximize the posterior of interest. Results of experiments conducted on real MSL data are shown and discussed in Section V and conclusions are reported in Section VI.

∗Part of this work was supported by the EPSRC via grants EP/N003446/1, EP/K015338/1, EP/M01326X/1
†Part of this work was supported by the EPSRC via grant EP/J015180/1.
II. PROBLEM FORMULATION

This section introduces the statistical observation model associated with MSL returns for a single-layered object which will be used in Section III for spectral unmixing of MSL data. We consider a 4-D array $Y$ of Lidar waveforms and of dimension $N_{row} \times N_{col} \times L \times T$, where $N_{row}$ and $N_{col}$ stand for the number of rows and columns of the regular spatial sampling grid (in the transverse plane), $L$ is the number of spectral bands or wavelengths used to reconstruct the scene and $T$ is the number of temporal (corresponding to range) bins. Let $y_{i,j,\ell,t} = [y_{i,j,\ell,1}, \ldots, y_{i,j,\ell,T}]^T$ be the Lidar waveform obtained in the pixel $(i,j)$ (i.e., $i$th row and $j$th column) using the $\ell$th wavelength. The element $y_{i,j,\ell,t}$ is the photon count within the $t$th bin of the $\ell$th spectral band considered. Let $d_{i,j}$ be the position of an object surface at a given range from the sensor, whose spectral signature (observed at $L$ wavelengths) is denoted as $\lambda_{i,j} = [\lambda_{i,j,1}, \ldots, \lambda_{i,j,L}]^T$. According to [7], [8] and assuming that the ambient illumination and dark photon counts can be neglected, each photon count $y_{i,j,\ell,t}$ is assumed to be drawn from the following Poisson distribution

$$y_{i,j,\ell,t} \mid \lambda_{i,j}, t_{i,j} \sim \mathcal{P}(\lambda_{i,j}, t_{i,j})$$

where $g_{0}(\cdot)$ is the photon impulse response whose shape can differ between wavelength channels and $t_{i,j}$ is the characteristic time-of-flight of photons emitted by a pulsed laser source and reaching the detector after being reflected onto a target at range $d_{i,j}$ and $t_{i,j}$ are linearly related in free-space propagation). Moreover, the impulse responses $[g_{0}(\cdot)]$ are assumed to be known and can usually be estimated during the imaging system calibration. We further assume that the spectral signatures of the scene surfaces can be decomposed as linear mixtures of $R$ known spectral signatures $m_r$ (also referred to as endmembers and gathered in the matrix $M = [m_1, \ldots, m_R]$)

$$\lambda_{i,j} = M a_{i,j}, \quad \forall i, j,$$

where $a_{i,j} = [a_{i,j,1}, \ldots, a_{i,j,R}]^T$ contains the abundances of the $R$ endmembers in the pixel $(i,j)$. Note that due to physical considerations the unknown abundance vectors $a_{i,j}$ are assumed to have positive entries. It is important to recall that in this work, we consider applications where the observed objects consist of a single visible surface per pixel. We do not consider cases where the photons can penetrate through objects (e.g., semi-transparent materials for which we would like to infer the internal composition) or be reflected onto multiple surfaces. This assumption allows the spectral unmixing problem to be reduced to a two spatial dimensions problem, which could be extended for distributed targets in future work. The problem addressed in this paper consists of jointly estimating the range of the targets (for all the image pixels) and solving the spectral unmixing problem (e.g., estimating the abundance vectors). The next section studies the proposed Bayesian model developed to solve the problem considered.

III. BAYESIAN MODEL

III-A. Likelihood

Assuming that the MSL waveforms $y_{i,j} = \{y_{i,j,\ell,t}\}_{t\in T}$ of a given pixel $(i,j)$ result from the photons reflected onto a surface associated with the spectrum $\lambda_{i,j}$, the likelihood associated with the pixel $(i,j)$ can be expressed as $f(y_{i,j} \mid \lambda_{i,j}, t_{i,j}) = \prod_{\ell,t} f(y_{i,j,\ell,t} \mid \lambda_{i,j}, t_{i,j})$, where assuming that the detected photon counts, conditioned on their means in all channels/spectral bands are independent. Considering that the noise realizations in the different pixels are independent, the joint likelihood can be expressed as

$$f(Y \mid A, T) = \prod_{i,j} f(Y_{i,j} \mid \lambda_{i,j}, t_{i,j}),$$

where $A = \{\lambda_{i,j}\}_{i,j}$ and $T$ is a matrix gathering the target ranges.

III-B. Prior distributions

In this work, we do not account for the potential spatial correlations between the target distances in neighbouring pixels of the scene. Thus, each target position is considered as a discrete variable defined on $T' = [t_{min}, \ldots, t_{max}]$, such that $1 \leq t_{min} < t_{max} \leq T$ (in this paper we set $(t_{min}, t_{max}) = (501, T - 500)$ for $T = 4500$, see discussion in Section IV) and assign the target ranges independent priors $p(t_{i,j}) = 1/T', \forall t \in T'$ where $T' = card(T)$. Note that more informative priors could be used, e.g., to capture potential spatial correlations affecting the range profiles, as in [8]. However, when the number of spectral bands $L$ considered and the number of detected photon are significant, the depth estimation does not require informative regularization (as the $L$ bands are used to estimate $t_{i,j}$). For this reason and for paper length constraints, we simply consider uniform priors here. Moreover, assuming prior independence between the ranges parameters $\{t_{i,j}\}$, yields

$$f(T) = \prod_{i,j} p(t_{i,j}).$$

As often assumed when addressing spectral unmixing problem, we consider applications where the number of spectral components involved in the mixture of a given pixel is likely to be smaller than the number of endmembers $R$ in the known matrix $M$. This typically occurs when the surface hit by the laser source and visible by the detector is relatively small compared to the size of the scene objects. In such cases, it makes sense to consider prior models which promote sparse estimated abundances. Similarly, when the transverse spatial sampling of the scene (using either a raster scan on a regular grid or a detector array) is dense enough, the abundance maps often present spatial structures (e.g., smoothness) that can be incorporated within a prior model. In contrast to the model proposed in [5] and which did not consider spatially correlated abundances, in this work, we consider the following priors promoting sparse and piece-wise smooth abundance maps while ensuring the abundance positivity

$$f(A \mid \lambda_{TV}) \propto \left\{ \begin{array}{ll} \exp[-\lambda_{r} ||A_{r}||_{1,1} - \lambda_{TV} \sum_{r} TV(A_{r})] & \text{if } a_{i,j,r} \geq 0, \forall (i,j) \\ 0 & \text{else} \end{array} \right.,$$

where $A_r$ is an $N_{row} \times N_{col}$ matrix such that $[A_r]_{i,j} = a_{i,j,r}$. $||A_{r}||_{1,1} = \sum_{r} ||a_{r,i,j}||$, and $TV(A_{r})$ denotes the total-variation (TV) regularization [9], [10]. The positive parameters $\lambda_{r}$ (resp. $\lambda_{TV}$) control the prior sparsity (resp. smoothness) of each abundance map $A_{r}$. In this work, we set $\lambda_{r} = \lambda_{TV}$ and these parameters are assumed to be fixed $(\lambda_{r}, \lambda_{TV}) = (10, 100)$ in the results of Section VI. Although the estimation of these hyperparameters is out of scope of this paper, note that they can be tuned via cross validation or adjusted via sequential optimization (see e.g., [11]).

Finally, assuming prior independence between the $R$ abundance maps yields

$$f(A \mid \lambda_{1}, \lambda_{TV}) = \prod_{r=1}^{R} f(A_{r} \mid \lambda_{1}, \lambda_{TV}).$$

III-C. Joint Posterior distribution

From the joint likelihood and prior model specified in Sections III-A and III-B, we can now derive the joint posterior distribution for $T$ and $A = \{A_{r}\}_{r=1}^{R}$, given the observed waveforms $Y$ and the value of the fixed hyperparameters $\Phi = (\lambda_{1}, \lambda_{TV})$. Using Bayes’ theorem, and assuming prior independence between $T$ and $\{A_{r}\}$, the joint posterior distribution associated with the proposed Bayesian model is given by

$$f(T, A \mid Y, \Phi) \propto f(Y \mid T, A) f(A \mid \Phi) f(T).$$

This model can be used to recover the abundance maps $A_{r}$ and the target ranges $T$ from the observed waveforms $Y$.
IV. ESTIMATION STRATEGY

The posterior distribution (5) models our complete knowledge about the unknowns given the observed data and the prior information available. In a similar manner to [5], we exploit this posterior to perform joint depth estimation and spectral unmixing of the MSL data. However, while a minimum mean squared error (MMSE) estimator was used in [5], here we consider the following joint maximum a posteriori (MAP) estimator

\[ \hat{\Theta}, \hat{\Lambda} = \arg \max_{\Theta, \Lambda} \log f(M, \Theta, \Lambda | M, \Theta, \Lambda), \]

which can also be obtained by minimizing the negative log-posterior \(-\log f(M, \Theta, \Lambda | M, \Theta, \Lambda)\). Although is can be shown that \(f(M, \Theta, \Lambda | M, \Theta, \Lambda)\) is log-concave with respect to (w.r.t.) \(\Lambda\), computing the estimator in (6) is generally challenging, mainly because \(f(Y | M, \Theta, \Lambda)\) can be multimodal w.r.t. \(\Theta\) and \(\Lambda\). In [5] a simulation method was used to handle the possibly multimodal likelihood (3) based on the full 4D data cube. As will be shown below, the joint depth estimation and spectral unmixing problem is solved with a reduced computational cost using efficient optimization methods (under mild conditions discussed below). Indeed, we show that the estimator in (6) can be obtained by first computing \(\hat{\Lambda}\) and then \(\hat{\Theta}\).

From (1) it can be seen that

\[ \tilde{y}_{i,j,t} \propto \mathcal{P}(\lambda_{i,j,t}) \tilde{g}_{i,j,t} \]

where \(\tilde{g}_{i,j,t} \equiv \sum_{m} g_{i,j,m} \tilde{a}_{m,i,j} \) and \(\tilde{a}_{m,i,j} = \sum_{t} a_{m,i,j} \). That is, the integrated waveform (summed over the time bins) for each pixel and wavelength follows a Poisson distribution whose mean only depends on spectral parameters \(\lambda_{i,j,t} = m_c a_{i,j} \) scaled by \(\tilde{g}_{i,j,t}\) which contains only information about the range of the target. The additional assumption considered in this work concerns the values of \(\tilde{g}_{i,j,t}\). More precisely, we assume that \(\tilde{g}_{i,j,t}\) is constant for all possible values of \(t_{i,j} \in \mathcal{T}\). This occurs in practice when \(\mathcal{T}\) is far from the boundaries of \((1,T)\) compared to the spread of the impulse responses \(g_{i,j,m}\). In our experiments, the supports of the instrumental responses are shorter than 500 time bins and we ensured the histograms of time-of-flights were long enough not to clip any target peak. This motivates our choice of \((t_{min}, t_{max})\) in Section III-B. Under this additional assumption, we have

\[ \log \left( \prod_{i,j} f(y_{i,j}, \lambda_{i,j,t}, t_{i,j}) \right) = \log \left( \prod_{i,j} f(y_{i,j,t} | \lambda_{i,j,t}, t_{i,j}) \right) + c_{i,j,t} \]

where \(c_{i,j,t}\) is a constant which does not depend on \(\lambda_{i,j,t}\). Note that \(c_{i,j,t}\) depends on \(t_{i,j}\) though and should be noted \(c_{i,j,t}(t_{i,j})\). However, we do not explicit this dependency and use \(c_{i,j,t}\) to lighten the notations. Combining (5), (6) and (8) yields

\[ (\hat{\Theta}, \hat{\Lambda}) = \arg \max_{\Theta, \Lambda} \log f(\Lambda) - \log f(M) - \sum_{i,j} \log f(y_{i,j,t} | \lambda_{i,j,t}) + c_{i,j,t} \]

where we notice that the optimization w.r.t. \(\Lambda\) does not depend on the value of \(\Theta\). Thus, we can first estimate \(\Lambda\) (it reduces to estimating the marginal MAP estimator of \(f(\Lambda | Y, \Phi)\) and then compute \(\Theta\) which also maximizes the conditional \(f(M | Y, \Theta, \Lambda)\). In addition to splitting to estimation of \(\Lambda\) and \(\Theta\) into two simple sequential steps, one of the main advantages of the proposed approach (over the method considered in [5]) is its low computational complexity while providing a global optimum of (5). Indeed, the estimation of \(\Lambda\) is achieved using only a reduced number of summarizing features (i.e., \(\{\tilde{g}_{i,j,t}\}\)) from the original data. The next paragraph details how to sequentially compute \(\hat{\Lambda}\) and \(\hat{\Theta}\).

IV-A. Estimating \(\hat{\Lambda}\)

Eq. (7) can be rewritten in matrix form as

\[ \hat{Y} \sim \mathcal{P}(\hat{\Lambda}) \]

where \(\hat{Y}\) is an \(L \times N_{row} N_{col}\) matrix gathering the elements \(\tilde{y}_{i,j,t}\). \(\hat{\Lambda}\) corresponds to the endmember matrix whose columns have been scaled by \(\tilde{g}_{i,j,t}\), \(\Lambda\) and is the \(R \times N_{row} N_{col}\) reshaped abundance matrix. Consequently, computing \(\hat{\Lambda}\) reduces to unmixing the integrated waveforms \(Y\) under Poisson noise assumption, using the prior model/regularization described in Section III-B and estimating \(\hat{\Theta}\) via MAP estimation. The corresponding minimization problem

\[ \min_{\Lambda > 0} C(\Lambda) + \lambda_1 \sum_{r} ||\Lambda_r||_1 - \lambda TV \sum_r TV(\Lambda_r) \]

is can be solved using any state-of-the-art convex optimization algorithm since \(C(\Lambda) = -\log(f(Y | \Lambda))\) is proper, lower semicontinuous and convex [12] (in practice \(M\) has positive entries and is full-rank). Here we used an instance of alternating direction method of multipliers (ADMM) similar to PIDAL-TV [12] but other alternative algorithms could have been used (e.g., SPIRAL [13]). Comparison of algorithms for solving (11) is out of scope of this paper and the interested reader is invited to consult [10], [12] for details about the ADMM implementation using TV and \(t_1\) regularization. It is interesting to recall here that although solving (9) requires the whole observation matrix \(Y\), estimating \(\hat{\Lambda}\) only requires the integrated waveforms, which drastically reduces the computational complexity of the problem.

IV-B. Estimating \(\hat{\Theta}\)

As discussed above, \(\hat{\Theta}\) can be obtained by maximizing \(f(\Theta | Y, \hat{\Lambda}, \Phi)\). Moreover, it can be seen for (5) that

\[ f(\Theta | Y, \hat{\Lambda}, \Phi) = \prod_{i,j} f(t_{i,j} | y_{i,j}, \hat{M}_{i,j}) \]

where \(f(t_{i,j} | y_{i,j}, \hat{M}_{i,j}) \propto f(y_{i,j,t} | \hat{M}_{i,j}) \). Consequently, the elements of \(\hat{\Theta}\) can be estimated independently (and in a parallel manner). Since \(t_{i,j}\) is assumed to be discrete and can only take value in finite set \(\mathcal{T}\), the estimation of \(\hat{\Theta}\) is straightforward. Note that since we compute the values of \(f(t_{i,j} | y_{i,j}, \hat{M}_{i,j})\) for all possible values of \(t_{i,j} \in \mathcal{T}\), we can derive measures of uncertainty about the ranges (see Section V).

V. SIMULATION RESULTS

Fig. 1. Examples of instrumental impulse responses measured with an acquisition of 100s at different wavelengths (500, 550, 600, 650, 700, 750 and 800nm)
V-A. Experiment description

We assess the performance of the proposed method to analyse the depth and spectral profiles of a 5 × 5 cm scene (see Fig. 2 (a)) composed of 8 objects made of polymer clay and mounted on real tree leaves and fixed onto a dark-grey backboard at a distance of 1.8m from a time-of-flight scanning sensor, based on time-correlated single photon counting (TCSPC). The transceiver system and data acquisition hardware used for this work is broadly similar to that described in [14]–[18], which was previously developed at Heriot-Watt University. The measurements have been performed indoor, in the dark to limit the influence of ambient illumination. The scene has been scanned using a regular spatial grid of 190 × 190 pixels and \( L = 33 \) regularly spaced wavelengths ranging from 500nm to 820nm. The histograms consist of \( T = 3000 \) bins of 2ps, which represents a depth resolution of 300µm per bin. The power of the supercontinuum laser source has been adjusted from preliminary runs and the per-pixel acquisition time is 10ms for each wavelength.

The instrumental impulse responses \( g_{0,1}(t) \) (partly depicted in Fig. 1) were estimated from preliminary experiments by analysing the distribution of photons reflected onto a Lambertian scatterer placed at a known distance over a long period of time (100s here). Fig. 1 illustrates the fact that the response of imaging system can change in amplitude and shape, depending on the wavelength considered due to the wavelength-dependent characteristics of its different elements (e.g., supercontinuum laser source, detector, lenses). Notice also the delays between the different peaks mainly due to the different (and wavelength-dependent) path lengths of the light in the imaging system. These delays can be compensated for as part of the calibration and do not have a significant influence on the imaging performance.

If a single wavelength was to be used to estimate the depth profile, the variations of the peak shape could make the choice of the most relevant wavelength difficult as the depth estimation accuracy mainly depends on the amplitude (reflectivity estimation) and width (depth estimation) of the peak. Of course, the depth estimation performance also depends on the spectral signatures of the objects of the scene (e.g., some objects can have a low reflectivity at a given wavelength and are thus hardly detectable). By considering several wavelengths to estimate the depth profile, we can expect a more robust depth estimation (each object needs to be visible at at least one wavelength) as we benefit from potential redundancy between the different spectral bands.

V-B. Unmixing results

Fig. 3 shows the spectral signatures of the \( R = 9 \) endmembers manually extracted from the the data (based on the known position of the different objects) and Fig. 4 depicts the corresponding estimated abundance maps. Although the leaves and most clay objects present close shades of green in Fig. 2 (a), Fig. 3 shows that the leaf spectra are significantly different from the green clay spectra which have similar shapes (and thus makes the unmixing problem particularly difficult). Nevertheless, the estimated abundances are generally in good agreement with the RGB image as it is possible to identify the regions where the different materials are present.

V-C. Depth estimation

Fig. 2 (b) shows the depth/range image estimated using the proposed method (i.e., after having estimated the abundance maps) which is in very good agreement with the structure of the scene in Fig. 2 (a) (the reference range being set to the range of the backboard). In particular, it is possible to detect subtle depth variations (e.g., central veins of the leaves, depth gap between the leaves and the board). To evaluate the quality of the depth estimation, we compute for each pixel the posterior probability \( f(t_{i,j}|y_{i,j}, \hat{M}_{\hat{a}_{i,j}}) \) in (12). The corresponding probability map in Fig. 2 (c) illustrates the high concentration of

\[
f(t_{i,j} | y_{i,j}, \hat{M}_{\hat{a}_{i,j}}) \approx (\hat{t}_{i,j} - t_{i,j})^{2}
\]

which translates an accurate depth estimation. Indeed, \( f(t_{i,j} = \hat{t}_{i,j} | y_{i,j}, \hat{M}_{\hat{a}_{i,j}}) \) is higher than 95\% in most pixels, the lower probabilities being associated with regions where the surface orientations yields lower photon counts and thus higher uncertainties about the object ranges. Note however that \( f(t_{i,j} \in (\hat{t}_{i,j} - 1, \hat{t}_{i,j} + 1) | y_{i,j}, \hat{M}_{\hat{a}_{i,j}}) \) is higher than 99\% for more than 99\% of the pixels, leading to confidence intervals at 99\% smaller than 0.9mm for almost all estimated ranges.
VI. CONCLUSION

We have proposed a new Bayesian model and a fast joint depth estimation and spectral unmixing algorithm for 3D scene analysis from MSL data. Assuming the ambient illumination can be neglected, the spectra of the scene surfaces visible by the imaging system were decomposed into linear mixtures of known endmembers, potentially corrupted by sparse deviations/anomalies. Adopting a Bayesian approach, prior distributions were assigned to the unknown model parameters; in particular sparsity and smoothness promoting priors were used to encode the spatial organization of the abundance maps. Including ambient illumination and dark count levels in the observation model (as in [8], [18]–[20]) is the obvious next step from a more general application of the proposed method. This can be done easily by including additional background terms in (1). In future work, especially for remote sensing applications, it will be crucial to account for the presence of distributed (multi-layered) targets and anomalies (e.g., objects present in isolated pixels and/or presenting spectral signatures which differ from the main objects of the scene), which would yield multiple returns in the MSL data. This could potentially allow the estimation of real 3D abundance profiles.

VII. REFERENCES


