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Structural controls on anomalous transport in fractured porous rock

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Abstract Anomalous transport is ubiquitous in a wide range of disordered systems, notably in fractured porous formations. We quantitatively identify the structural controls on anomalous tracer transport in a model of a real fractured geological formation that was mapped in an outcrop. The transport, determined by a continuum scale mathematical model, is characterized by breakthrough curves (BTCs) that document anomalous (or “non-Fickian”) transport, which is accounted for by a power law distribution of local transition times \( \psi(t) \) within the framework of a continuous time random walk (CTRW). We show that the determination of \( \psi(t) \) is related to fractures aligned approximately with the macroscopic direction of flow. We establish the dominant role of fracture alignment and assess the statistics of these fractures by determining a concentration-visitation weighted residence time histogram. We then convert the histogram to a probability density function (pdf) that coincides with the CTRW \( \psi(t) \) and hence anomalous transport. We show that the permeability of the geological formation hosting the fracture network has a limited effect on the anomalous nature of the transport; rather, it is the fractures transverse to the flow direction that play the major role in forming the long BTC tail associated with anomalous transport. This is a remarkable result, given the complexity of the flow field statistics as captured by concentration transitions.

1. Introduction

Modeling transport of dissolved chemicals (“tracers,” “contaminants,” and “particles”) in water-saturated fractured geological formations has been a central focus of study over several decades. Field experiments [e.g., Haggerty et al., 2001; McKenna et al., 2001; Kosakowski, 2004; Dorn et al., 2012] as well as theoretical studies [e.g., Berkowitz and Scher, 1997; Frampton and Cvetkovic, 2007; Cortis and Birkholzer, 2008; Reeves et al., 2008; Geiger et al., 2010; Kang et al., 2011; Roubinet et al., 2013] have demonstrated that anomalous (“non-Fickian”) transport is intrinsic to mass and heat transfer in fractured geological formations.

Anomalous transport most commonly manifests itself in the appearance of long tails in the spatial and/or temporal distributions of tracer concentration at given locations; in the latter case, long tails appear in tracer breakthrough curves (BTCs) integrated along a crossing surface in the domain. Such behavior cannot be captured by using continuum-level, Fickian-based advection-dispersion models, but it has been effectively quantified using continuous time random walk (CTRW) models [Berkowitz et al., 2006] and, where appropriate, associated multirate mass transfer models (MRMT) [Haggerty and Gorelick, 1995] and (limit-case) fractional advection-dispersion equations [Metzler et al., 1998; Benson et al., 2000].

Less well defined, however, are the specific structural features and/or mechanisms that control anomalous transport in fractured geological formations. For example, Cortis and Birkholzer [2008] and Geiger et al. [2010] showed that the interplay of transport in the fractures, diffusion in the matrix, and diffusion between fractures and matrix causes long tailing. Other studies indicated that diffusion between the mobile phase in the fractures and the immobile (or essentially immobile) phase in the rock matrix is a controlling factor for long tailing [e.g., Haggerty et al., 2001] or that transport in the fractures alone can be sufficient to generate tailing [e.g., Berkowitz and Scher, 1997]. Indeed, as summarized also by Kang et al. [2015b], anomalous transport in fractured domains may arise from variabilities in fluid velocities within fractures (related to fracture permeability), fracture connectivity and occurrence of preferential flow paths, and/or interactions between fractures and the host rock matrix. Geological formations containing approximately orthogonal fracture
families are well documented and studied for single phase [e.g., Belayneh et al., 2006b] and two-phase flow [e.g., Matthai et al., 2007]; see further discussion below.

This study further explores the origin of long-tailed BTCs in a fractured geological formation consisting of two relatively orthogonal families of fractures that were carefully mapped in an outcrop. The analysis leads to a robust quantification that links anomalous transport to structural and hydraulic features of the fracture network and host rock matrix. We stress that our aim in this study is to understand structural controls that lead to anomalous transport. In this context, BTCs and tracer “visitation” patterns are examined in a fractured porous medium, comparing the two measures to isolate the source of anomalous transport. Extensive application analyses of CTRW to match and predict tracer transport has been presented elsewhere [e.g., Levy and Berkowitz, 2003; Edery et al., 2014].

2. Methods

2.1. Continuum-Scale Numerical Simulations

We performed all flow simulations in a fracture pattern that was mapped in a limestone outcrop in the Bristol Channel, UK [Belayneh, 2004]. This geometry has been used previously in simulation studies that quantified single-phase and multiphase flow in fractured geological formations [Matthai and Belayneh, 2004; Belayneh et al., 2006a; Geiger and Emmanuel, 2010; Geiger et al., 2010, 2013]. Here it allowed us to investigate, on a real domain, the relation between the BTC fit to the domain tracer flow. This was done by mapping the fracture network and representing it in a detailed simulation grid, varying the permeability in the rock matrix while keeping the fracture permeability constant.

We note that an extensive literature on transport behavior in fractured formations is based on consideration of stochastically generated discrete fracture networks (DFNs), beginning with initial studies of Smith and Schwartz [1984] and Long and Billaux [1987]. DFNs are convenient to generate and study fracture networks; however, it is difficult to generate fracture patterns with DFNs that capture geometries of natural fracture networks, because DFNs do not consider how fractures grow and interact mechanically when they propagate. Mass transfer between mobile solute in the fractures and less mobile solute in the rock matrix can be one of the factors leading to anomalous transport [e.g., Haggerty et al., 2001]. Simulating this mass transfer by DFNs, however, is often difficult because nonlocal methods are required to properly capture mass transfer between mobile and immobile regions [e.g., Carrera et al., 1998]. Moreover, natural fractures are often layer bound; roughly orthogonal fracture domains have been observed in several fractured formations [e.g., Belayneh et al., 2006b; Strijker et al., 2012; Boro et al., 2014; Afsar et al., 2014; Hardebol et al., 2015; Bisdom et al., 2016] and have been the subject of several theoretical studies [e.g., Reeves et al., 2008; Cortis and Birkholzer, 2008; Sævik et al., 2013; Kang et al., 2015a]. Hence, a mean hydraulic gradient that is aligned approximately with one set of fractures is not uncommon.

The model geometry, shown in Figure 1, was 18 × 8 m² and represents a horizontal, two-dimensional map view of the fractures in this outcrop. As the fractures are bed-bound, the thickness of individual limestone beds does not exceed 30 cm, and the limestone beds are separated by shale layers, we represent the fractures and rock matrix as a horizontal 2-D model with unit thickness. We used the discrete fracture and matrix (DFM) technique [e.g., Geiger et al., 2009; Schmid et al., 2013] and an unstructured grid comprising approximately 351,000 mixed-dimensional finite elements to resolve the complex geometrical nature of the fracture network in the simulation model and minimize numerical dispersion. The area of the finite elements in the vicinity of a fracture was below 1 mm². Permeability and porosity differed in finite elements belonging to the fractures and matrix. This high-resolution simulation grid enabled us to resolve the velocity field in great detail and capture the steep concentration gradients between fracture and matrix. Hence, we did not have to make assumptions about the rates at which fractures and matrix exchange fluids or about flow partitioning at fracture intersections; instead, the distribution of transition times that give rise to anomalous transport emerges naturally from the simulations. The velocity field \( \mathbf{v}(x) \) was obtained by solving the standard Laplace equation for incompressible, steady state fluid flow, \( \nabla \cdot \left( \frac{k(x)}{\mu} \nabla p \right) = 0 \), where \( k \) is the permeability of the fracture or matrix, \( \mu \) the fluid viscosity, and \( p \) the fluid pressure. The velocity in each finite element can be obtained readily by solving for Darcy’s law in each finite element (valid also in the fractures because the associated Reynolds numbers are relatively low even for the high velocities), using the local pressure gradient obtained from the Laplace equation.
We focused specifically on an outcrop where fractures are open and hence act as flow conduits, having permeabilities that are (possibly significantly) higher than the rock matrix. We assumed a uniform fracture aperture of 1 mm, which yielded a fracture permeability of $8 \times 10^{-12}$ m$^2$, and tested matrix permeabilities of $10^{-12}$, $10^{-13}$ and $10^{-15}$ m$^2$. The permeability was assumed to be isotropic; the porosity was assumed to be 0.25 and 1 in the matrix and fractures, respectively.

Boundary conditions were chosen such that the main flow direction was from left to right with a pressure gradient of approximately $5 \times 10^{-4}$ bar/m. Here we focus on how fracture network structure and matrix permeability affect tracer exchange between the mobile fracture flow and immobile matrix flow, and the subsequent impact on the anomalous nature of the BTCs. Changing the matrix permeability, rather than the fracture aperture, was examined as a means to modify the interplay of fluid flow and tracer exchange between the fractures and rock matrix. At a matrix permeability of $10^{-11}$ m$^2$, the fracture transmits approximately 30 times more fluid than the rock matrix, which is still mobile. This value increases to over 30,000 at a matrix permeability of $10^{-15}$ m$^2$ when the rock matrix can be considered stagnant. We note that the same ratios were observed even if the fracture permeability is assumed to be heterogeneous and varies locally between $10^{-9}$ and $10^{-7}$ m$^2$ [Geiger et al., 2010].

Because the fractures were well connected and permeable, changes in the overall hydraulic gradient were small. A pulse of a conservative tracer with a nominal concentration of 1 kg/m$^3$ was injected for 1, 10, and 100 days. A flux-weighted breakthrough curve was recorded at the 4.5, 9, 13, and 18 m from the inlet. All other simulation parameters are listed Table 1.

We numerically generated flux and concentration distributions as well as BTCs for the model geometry by solving the classical advection-dispersion equation

$$\phi(x) \frac{\partial c(x, t)}{\partial t} + \nabla \cdot [\mathbf{v}(x)c(x, t) - \mathbf{D}(x) \nabla c(x, t)] = 0,$$

(1)

using the DFM approach. In the above, $\phi$ is the porosity, $\mathbf{v}$ the Darcy velocity, $\mathbf{D}$ the dispersion tensor, and $c$ the concentration of a conservative tracer. We emphasize that classical dispersion was considered only at the scale below the size of a single finite element, while anomalous transport at the model scale emerged from the finely resolved velocity field [Geiger et al., 2010]. Implicit time stepping was used to integrate the solution in time. The resulting system of linear equations was solved using an algebraic multigrid method as implemented in SAMG [Stüben, 2001]. Our implementation of the DFM technique with mixed-dimensional and unstructured finite elements has been validated extensively for a range of applications, including multicomponent solute transport during single-phase and multiphase flow, by comparison to numerical solutions with analytical and reference solutions, and by detailed grid convergence analysis [Geiger et al., 2009; Schmid et al., 2011, 2012, 2013; Weis et al., 2014].
2.2. CTRW Quantification of Transport

The basic CTRW formulation considered here is described in detail, with a full exposition of the background and development, in Berkowitz et al. [2006]. The discrete (in space), temporal semi-Markov CTRW transport equations leads to the working continuum transport equation for the normalized concentration \( c(s,t) \) (in Laplace space) for an ensemble-averaged system:

\[
uc(s,u) - c_0(s) = -\hat{M}(u)[\nabla \hat{c}(s,u) - D\frac{\partial}{\partial s} \nabla \hat{c}(s,u)]
\]  

where \( \hat{M}(u) \equiv \overline{tu\psi(u)}/[1-\overline{\psi(u)}] \) is a memory function, with the Laplace transform of a function \( f(t) \) denoted by \( \overline{f(u)} \), \( t \) is a characteristic time, \( \psi(t) \) is the probability rate for a transition time \( t \) between sites, and \( \nabla \hat{c}, D\frac{\partial}{\partial s} \) are, respectively, the first and second moments of \( p(s) \), the probability distribution of the length of the transitions. The “transport velocity,” \( v_D \), is distinct from the Darcy, or “average,” fluid velocity \( v \) due to the domain heterogeneity which can lead to different average fluid and tracer velocities.

Unique to this study is the linking of \( \psi(t) \) (and specifically, the parameters \( \beta \) and \( t_1 \), see below) directly to characteristics of the fractured porous domain and to the effect of fracture alignment relative to the overall flow direction (as embodied in the slope and peak of the weighted residence time distribution of tracer). The key features of the CTRW distribution \( \psi(t) \) that we use, which have been successful in analyzing a number of laboratory and field observations [e.g., Berkowitz et al., 2006], are a truncated power law (TPL) distribution of the site-to-site transition times with an evolution to Fickian behavior:

\[
\psi(t) = \frac{n}{t_1} \exp\left(-t/t_2\right)/(1+t/t_1)^{1+\beta},
\]  

where \( n \equiv (t_1/t_2)^{-\beta}\exp(-t_1/t_2)/\Gamma(-\beta,t_1/t_2) \) is a normalization factor, \( \beta \) is a measure of the transition time spectrum, \( t_1 \) (= \( t \) in (2)) is a characteristic time, e.g., for median transitions between sites, \( t_2 \) is a “cutoff” time, and \( \Gamma(a,x) \) is the incomplete Gamma function [Abramowitz and Stegun, 1970]. For transition times \( t_1 \),

**Table 1. Model Parameters, Run Time and Time Step Size, and Initial and Boundary Conditions Used in the Simulations**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matrix Permeability</td>
<td>( k_m )</td>
<td>( 10^{-11}, 10^{-13}, ) or ( 10^{-15} )</td>
<td>m(^2)</td>
</tr>
<tr>
<td>Porosity</td>
<td>( \phi_m )</td>
<td>0.25</td>
<td></td>
</tr>
<tr>
<td>Longitudinal dispersion</td>
<td>( z_L )</td>
<td>0.01</td>
<td>m</td>
</tr>
<tr>
<td>Transverse dispersion</td>
<td>( z_T )</td>
<td>0.001</td>
<td>m</td>
</tr>
<tr>
<td>Diffusivity</td>
<td>( D )</td>
<td>( 10^{-10} )</td>
<td>m(^2)/s</td>
</tr>
<tr>
<td>Fractures</td>
<td>Aperture</td>
<td>( a )</td>
<td>0.001</td>
</tr>
<tr>
<td>Permeability</td>
<td>( k_f )</td>
<td>( 8.33 \times 10^{-8} )</td>
<td>M</td>
</tr>
<tr>
<td>Porosity</td>
<td>( \phi_f )</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>Longitudinal dispersion</td>
<td>( z_L )</td>
<td>0.01</td>
<td>m</td>
</tr>
<tr>
<td>Transversal dispersion</td>
<td>( z_T )</td>
<td>0.001</td>
<td>m</td>
</tr>
<tr>
<td>Diffusivity</td>
<td>( D )</td>
<td>( 10^{-10} )</td>
<td>m(^2)/s</td>
</tr>
<tr>
<td>Initial Conditions</td>
<td>Viscosity</td>
<td>( \mu )</td>
<td>0.001</td>
</tr>
<tr>
<td>Concentration</td>
<td>( c )</td>
<td>0.0</td>
<td>kg/m(^3)</td>
</tr>
<tr>
<td>Pressure gradient</td>
<td>( \Lambda p/\Lambda x )</td>
<td>( 5-7.5 \times 10^{-4} )</td>
<td>bar/m</td>
</tr>
<tr>
<td>Boundary Conditions</td>
<td>Inflow concentration</td>
<td>( c_{i\rightarrow o} )</td>
<td>1.0</td>
</tr>
<tr>
<td>Outflow concentration</td>
<td>( c_{o\rightarrow i} )</td>
<td>( \partial c/\partial x = 0 )</td>
<td></td>
</tr>
<tr>
<td>Inflow rate</td>
<td>( q_{i\rightarrow o} )</td>
<td>( 2 \times 10^{-5} )</td>
<td>m/s</td>
</tr>
<tr>
<td>Outflow pressure</td>
<td>( p_{o\rightarrow i} )</td>
<td>10</td>
<td>bar</td>
</tr>
<tr>
<td>Simulation Settings</td>
<td>Simulation time</td>
<td>( T )</td>
<td>600</td>
</tr>
<tr>
<td>Tracer pulse</td>
<td>( t_{tracer} )</td>
<td>1</td>
<td>d</td>
</tr>
<tr>
<td>Time step</td>
<td>( \Delta t )</td>
<td>0.005</td>
<td>d</td>
</tr>
</tbody>
</table>

*Note that matrix permeabilities below \( 10^{-15} \) m\(^2\) lead to identical breakthrough curves for this particular model geometry [Geser et al., 2010] and hence are not investigated.*

*Fracture permeability is computed from the parallel plate law with \( k_f = a^2/12 \). Fracture and matrix permeability are assumed to be isotropic.*

*Small variations in pressure gradient are due to the different matrix permeabilities.*

*A small time step \( \Delta t \) was chosen to minimize numerical dispersion. Test runs were performed to identify the appropriate size of \( \Delta t \).*
<t < t₂, ψ(t) behaves as a power law ∝ (t/t₁)⁻¹⁻β while for t > t₂, ψ(t) decreases exponentially; thus a finite t₂ enables evolution from non-Fickian to Fickian transport.

Solutions of (2), to analyze tracer BTCs, are readily available in Cortis and Berkowitz [2005], for both step and pulse inputs. The TPL has been used to determine β and t₂, in conjunction with determination of the distribution of times for, e.g., the North Sea field site [Di Donato and Blunt, 2004]. More recently, Edery et al. [2014] provided insights on the relationship between main features of tracer transport and structural properties of a heterogeneous porous medium, using tracer particles to interrogate particle “visitation” in different regions of the medium. This approach enabled identification of a direct link between the statistics of the hydraulic conductivity field and β. The analysis also demonstrated how consideration of a TPL, in the context of a single domain realization, can approximate the average residence time pdf associated with many realizations of the domain.

A similar approach is employed here to link anomalous transport behavior to structural properties of the fractured porous domain under consideration. As we will show, the effect of fracture orientation is key, and the relationship to β is investigated via consideration of the residence time distribution.

3. Results and Discussion

The BTCs for matrix permeabilities of 10⁻¹¹, 10⁻¹³, and 10⁻¹⁵ m² with TPL fits can be seen in Figures 2a, 2b, 2c, respectively. In this study, we focused on matrix permeability of 10⁻¹³ m² (Figure 2b). The 1 day (86,400 s) pulse is apparent in the peak concentration. The most obvious feature is the drop in concentration after the pulse ended, and the long tailing that followed. Such tailing is fundamentally characteristic of anomalous transport. In this context, the CTRW formulation, with the TPL (3), matches the BTC. For all rock permeabilities tested here, we obtained β ≈ 1.2 (indicating highly anomalous transport), while t₁, the median time, was of the order of 10 s and t₂, the cutoff time where transport evolves to Fickian, is 10⁷.³ s, nearly where the tailing stops. These parameters are key to understanding the nature of the transport in terms of local temporal statistics. To further test the fitting procedure, the same parameter estimates were also used in BTC fits with (a) varying distances of 4.5, 9, and 13 m from the inlet face (Figure 3a), in addition to the fit here at 18 m; (b) different pulse values of 1, 10, and 100 days (Figure 3b) with the same parameters. The fact that for all permeabilities, locations, and pulse durations, BTCs are fitted with the same parameters suggests that the anomalous tailing is insensitive to the rock formation under pulse conditions; otherwise, tracer interchange with the rock matrix would adjust the tail accordingly.

In the context of our analysis, we recognize that BTCs provide essentially no information on spatial evolution of a tracer plume. Limitations in experimental laboratory and field studies of tracer transport in porous media lead to a general focus on BTCs as a characteristic measure. Because a BTC quantifies the local tracer concentration at a single control plane location, over time, it represents an average over the spatial distribution of tracers.

Figure 2. BTC (dots) from simulations in the Bristol formation, with rock permeability of (a) 10⁻¹¹ m², (b) 10⁻¹³ m², and (c) 10⁻¹⁵ m², with corresponding CTRW fit (solid curve) with parameter values νᵢ = 3.1×10⁻⁷ m/s, Dᵢ = 1.58×10⁻⁴ m²/s, β = 1.2, t₁ = 12 s, t₂ = 10⁷ s for all figures.
To gain further insight, we present the total concentration of tracer visitation per site (where a "site" is equivalent to a finite volume where the tracer concentration is preserved) throughout the simulation (Figure 4). This is presented with a cutoff for normalized concentrations lower than $10^{-2}$ (the minimum concentration recorded is $10^{-4}$). Concentrations below $10^{-1}$ are related solely to the rock matrix, emphasizing the fact that the host rock is generally avoided by the tracer during the overall fluid flow and transport process, and, somewhat surprisingly, that the rock matrix does not contribute to the anomalous nature of the transport, neither through mobile-immobile mass transfer nor when there is flow in the rock matrix at high matrix permeabilities. The anomalous BTC tail in Figure 2b breaks at $c_o \approx 10^{-4}$ and rapidly approaches zero. This part of the curve represents low tracer concentrations leaving the rock; the rock tracer concentration is smaller than $10^{-1}$, and $c_{\text{rock}}/c_o = 15.925 < 10^{-5}$, where $c_o$ refers to the cumulative concentration.

Hence, the question arises: what is the source of the tailing? Careful examination of Figure 4 shows that the fractures approximately horizontal to the overall flow direction accommodate cumulative concentrations higher than $10^{-3}$ kg/m$^3$; thus, most of the tracer migrates through the horizontal fractures with almost no interaction with the host rock. This explains the sharp drop in concentration after the pulse ends but not the tailing, seen in Figure 2b. The tail can be explained by the tracer concentrations in the fractures transverse to the overall flow direction, which range from 5 to $10^3$ kg/m$^3$; it is these concentrations that can be related to the concentrations of the tail. [We note that in the context of fracture networks (impermeable rock), the key role of low velocity, transverse fractures in generating long tailing was noted by, e.g., Berkowitz and Scher [1997]. Clearly, results similar to those presented here would not arise if there are few fractures transverse to the overall head gradient, and/or the matrix permeability is large relative to that of the fractures.] This empirical analysis is a useful means to intuitively understand the relationship between the tail and the transverse fractures, but it lacks quantification that can relate the residence time distribution embodied in the TPL 3 to the tracer transport and distribution in the simulation.

**Figure 3.** BTCs from simulations in the Bristol formation, with rock permeability $10^{-13}$ m$^2$, at (a) distances of 4.5, 9, and 13 m from the inlet boundary, corresponding to blue circles, red triangles, and green plus sign, respectively. Corresponding CTRW fits (solid, dashed, and dashed-dotted line, respectively) with parameter values $v_w = 3.1 \times 10^{-3}$ m/s, $\beta = 1.2$, and $t_1 = 10^{-3}$ s for all fits while $D_w = 1.58 \times 10^{-3}$ m$^2$/s, $t_1 = 1$ s, for the solid line, $D_w = 3.1 \times 10^{-4}$ m$^2$/s, $t_1 = 1$ s, for the dashed line, and $D_w = 1.58 \times 10^{-4}$ m$^2$/s, $t_1 = 12$ s, for the dotted line, and (b) 1, 10, and 100 day pulses (blue circle, red plus sign, and green cross, respectively); and corresponding CTRW fits (solid, dotted, and dashed line, respectively) with parameter values $v_w = 3.1 \times 10^{-3}$ m/s, $D_w = 1.58 \times 10^{-3}$ m$^2$/s, $\beta = 1.2$, $t_1 = 12$ s, $t_2 = 10^{-2}$ s.

**Figure 4.** Cumulative tracer concentration per location for the entire duration of the simulation, for the Bristol domain with the same parameters as in Figure 2. The concentration is presented on a logarithmic scale (color bar) and has a cutoff $10^{-1}$ kg/m$^3$ for presentation purposes.
The link between the concentration evolution and the BTC is through the residence time distribution in the domain. Tracer at each location in the domain has a different residence time, according to the local permeability and head. This can be presented as a histogram on a log-log scale (Figure 5, blue points), where a strong effect of long residence times, in terms of frequency, can be observed. The dominance of the long residence time emerges because the rock matrix with the slowest velocities holds the largest fluid volume; short residence times occur in fractures aligned (approximately horizontal) with the overall flow direction, while transverse fractures have increasingly large residence times, depending on their distance from the horizontal fractures. This system gives rise to a continuous distribution of residence times that coincides with the TPL residence time (with appropriate weighting). To convert this residence time distribution (histogram) into a temporal pdf defining \( \psi(t) \) (3), we divide each frequency in the histogram by the residence time in the associated element, based on the Darcy velocity (as done by Edery et al. [2014]); see Figure 6 (blue dots). Here we see that the probability for the short residence times in the horizontal fractures is substantially greater than the probability of short residence times in the transverse fractures and rock matrix. Even so, this pdf does not coincide with the TPL found in the BTC fit (Figure 2b). To further treat the distribution of residence times, we normalize the frequency (Figure 5, red points) by the tracer concentration passing through each element and then transform it to a pdf as before (Figure 6, red dots). It then becomes clear that the transverse fractures and rock matrix display a long tailing (power law on a log-log scale) with exponent of \( -2.2 \) (black line in Figure 6). This result fits remarkably with the TPL residence time power law, as can be seen, it does not capture the full length of the distribution.

We note also that Siirila-Woodburn et al. [2015] presented 3-D high-resolution simulations of tracer migration in synthetic domains with geostatistically based, heterogeneous hydraulic conductivities. Particle tracking simulations were used to generate BTCs; they demonstrated the appearance of multiple (local) peaks in single realizations, and showed that the number and slopes of local peaks depend on statistical anisotropy and travel distance. In this context, the local peaks seen in Figure 6, for the \( \psi(t) \), are analogous.

As shown previously (Figure 4), most of the domain is not sampled by the tracer?
tracer because it corresponds to the large matrix volume. There is a clear correspondence between the directionality of the fracture relative to the overall flow direction and the tracer concentration. A fracture aligned with the flow direction tends to experience higher volumetric flow rates, and vice versa. Particularly for inlet pulse conditions, the host rock has a limited contribution to the anomalous transport. Even so, the (mean spatial, unweighted) tracer concentration provides only the partial answer: the anomalous tailing can be attributed to the tracer residence time at each site (Figure 7). Here we see the inverse relation between the residence time and the fracture alignment relative to the overall direction of flow. Tracer in fractures more transverse to the flow displays longer residence times, which further contribute to the tailing effect, because tracer entering and exiting low residence time locations is controlled by the fractures aligned with the flow direction. This creates a cascade of tracer concentrations from low residence time area (Figure 7, blue color) to dead-end locations with high residence time (Figure 7, red color).

4. Conclusions

The main, and possibly nonintuitive, conclusion of this analysis is that the anomalous nature of transport is not controlled by the rock matrix; time scales associated with tracer diffusion within the rock matrix are significantly larger than those associated with transport in the fractures, and are not visible in the BTC plots shown here. The effect of diffusion on the anomalous tail is negligible because the matrix diffusivity is $10^{-10}$ m$^2$/s; this implies that it takes orders of magnitude longer to transport a particle by diffusion into the rock matrix, compared to transporting a particle advectively in the fractures at the average fracture velocity of 0.005 m/s.

Recalling Figure 6, a simplistic look at the residence time histogram with no weighting (as well as the permeability) can lead to the incorrect conclusion that the rock matrix with its extremely long residence times are the controlling aspects of the BTC tailing. In fact, it is the continuous transition between horizontal and transverse fractures (relative to the overall flow direction) that leads to the anomalous tailing fitted by the TPL. This latter conclusion has important implications for contamination in fractured media, because most of the domain is not sampled by tracer pulses in the permeable rock matrix where diffusion dominates, in contrast to the general case wherein tracer can visit all locations in the domain. The same applies to the tracer leaving the rock matrix: the time scales required for tracer to leave the (less permeable) rock matrix appear to increase due to the low velocities in transverse fractures.

References


