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Rogue-soliton generation via Anderson localisation

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Abstract: We offer a new explanation of how rogue-solitons generated during the modulation instability process in optical fibres are formed. Our novel point of view is based on Anderson localisation effect assisted by an optical-event horizon.

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Modulation instability is one of the most basic and important nonlinear processes, by which an optical pulse with very narrowband ends with a massive broad supercontinuum spectrum after propagating few nonlinear-lengths inside long optical fibres \cite{1}. A huge interest has been devoted to this particular process, when Solli \textit{at al.} have shown that the output spectra contain a statistically-rare rogue solitons with large intensities and enhanced frequency redshift \cite{2}, resembling the rare rogue-waves that can appear from nowhere in oceans and lead to massive destruction. In this work, we originate the emission of rogue-solitons to the combined effects of optical-event horizon and Anderson localisation.

Using the slowly-varying envelope (SVE) approximation and neglecting higher-order nonlinearities and dispersion coefficients, pulse propagation equation in optical fibres can be written as

\begin{equation}
 i\frac{\partial A}{\partial z} - \frac{\beta_2}{2} \frac{\partial^2 A}{\partial t^2} + \frac{\omega_0}{c} \Delta n(z,t) A = 0,
\end{equation}

where $A$ is the pulse complex envelope, $z$ is the longitudinal coordinate along the fibre, $t$ is the time in a reference frame moving with the pulse group velocity, $\beta_2$ is the second-order dispersion coefficient, $n_2$ is the nonlinear refractive in units $m^2/W$, $\omega_0$ is the pulse central frequency, $c$ is the speed of light in vacuum, $\Delta n(z,t) = n_2 |A|^2/A_{\text{eff}}$ is the refractive-index modulation, and $A_{\text{eff}}$ is the effective mode area. The SVE approximation implies that $\Delta n(z,t) \approx \Delta n(t)$ over a short propagation distance. Hence, Eqs. (1) can be regarded as the temporal analogue of Anderson localisation in optical schemes that are transversely-disordered \cite{3}. Optical fibres are usually meters long, where the complex envelope can vary slowly over cm range. Therefore, light pulses could be temporally localised in different locations during propagation along the fibre via Anderson effects if the refractive index is randomly modulated.

The modulation of the refractive index induced by Kerr nonlinearity results in a temporal waveguide that travels with the optical pulse. The linear associated modes of this waveguide can show over short propagation intervals Anderson localisation, featured by the localisation time $T_{\text{loc}}$ \cite{4},

\begin{equation}
 T_{\text{loc}}(z) = \left( \int |f(z,t)|^2 \, dt \right)^2 / \int |f(z,t)|^4 \, dt,
\end{equation}

where $f$ is the eigenfunction of one of these modes that has eigenvalue $\lambda$.

Consider the propagation of a long superGaussian pulse in a solid silica-core photonic crystal fibre with zero-dispersion wavelength at 1055 nm \cite{5} in the absence of higher-order dispersion, self-steepening and Raman nonlinearities. Panels (a,b) in Fig. 1 show the temporal evolution of the nonlinear pulse and its accompanied fundamental mode. The first few modes are plotted on the top of each other in panel (c). The spatial dependency of the eigenvalue and the localisation time of the fundamental mode are shown in panel (d). When the pulse is launched inside the fibre, the background noise starts to build up and affect the pulse amplitude around $z = 6$ m due to modulation instability, resulting in a random temporal modulation of the medium refractive index. Discretising the rest of the fibre into small segments, one can observe clearly different positions of localisation of the fundamental mode demonstrated by a strong suppression of its eigenvalue and localisation time along the fibre. The track of the fundamental mode is disrupted multiple times, since each segment of the fibre is considered as a different random system. The strongest localisation occurs
Fig. 1. (a-d) In the absence of higher-order dispersion and nonlinear coefficients, (a) Temporal evolution of a superGaussian pulse with a profile \( \exp \left[ -1/2 \left( t/T_0 \right)^{10} \right] \), a central wavelength 1060 nm, \( T_0 = 3.63 \) ps (\( \equiv \) FWHM = 7 ps) and input power 100 W inside the solid silica-core photonic crystal fibre used in Ref. [5], (b,c) Temporal evolution of the fundamental mode and the first five modes of the induced temporal-waveguide, respectively. (d) The spatial dependency of the eigenvalue and localisation time of the fundamental mode. (e-h) Taking into account all the linear and nonlinear effects, (e,f) Spectral and temporal evolution of the superGaussian pulse, (g) Temporal evolution of the first five modes of the induced temporal-waveguide, (h) The spatial dependency of the eigenvalue and localisation time of the fundamental mode. Temporal contour plots are normalised to their peaks. The spectral contour plot is given in a logarithmic scale and truncated at -40 dB.

at \( z = 16.7 \) m. Panels (e,f) display the spectral and temporal evolution of the superGaussian pulse taking into consideration other linear and nonlinear effects. When modulation instability takes place, solitons are emitted and seeded by the linear modes as shown in panel (g), then decelerated due to Raman nonlinearity. During propagation solitons will receive amplification over short propagation distances via Anderson localisation. Soliton-collisions occur when their trajectories are simply intersect, however, certain type of collisions can lead to generation of a rogue soliton. An optical-event horizon [6] can be established between a leading soliton and another lagging one with a slightly higher group velocity. The leading soliton will be seen as a barrier by the trailing soliton, which will reflect back after collision. Based on the initial amplitudes of the two solitons and the group-velocity difference, the collision can result in a large of transfer of energies and rogue-soliton formation. Surprisingly, the rogue soliton is emitted at the exact position predicted by strongest Anderson localisation, which is \( z = 16.7 \) m, accompanied by strong suppression of the eigenvalue and localisation time of the fundamental mode, see panel (h).

Finally, we believe that our findings are of groundbreaking importance in the explanation of rogue-solitons formation in optical fibres and their relation to rogue-waves in open oceans.

References