Photocell Optimization Using Dark State Protection
Fruchtman, Amir; Gómez-Bombarelli, Rafael; Lovett, Brendon W.; Gauger, Erik

Published in:
Physical Review Letters

DOI:
10.1103/PhysRevLett.117.203603

Publication date:
2016

Document Version
Peer reviewed version

Link to publication in Heriot-Watt University Research Portal

Citation for published version (APA):
Photocell Optimisation Using Dark State Protection

Amir Fruchman,1 Rafael Gómez-Bombarelli,2 Brendon W. Lovett,3 and Erik M. Gauger4,5

1Department of Materials, University of Oxford, Oxford OX1 3PH, United Kingdom
2Department of Chemistry and Chemical Biology, Harvard University, Cambridge, USA
3SUPA, School of Physics and Astronomy, University of St Andrews, KY16 9SS, United Kingdom
4SUPA, Institute of Photonics and Quantum Sciences, Heriot-Watt University, EH14 4AS, United Kingdom

(Dated: October 9, 2016)

Conventional photocells suffer a fundamental efficiency threshold imposed by the principle of detailed balance, reflecting the fact that good absorbers must necessarily also be fast emitters. This limitation can be overcome by ‘parking’ the energy of an absorbed photon in a dark state which neither absorbs nor emits light. Here we argue that suitable dark states occur naturally as a consequence of the dipole-dipole interaction between two proximal optical dipoles for a wide range of realistic molecular dimers. We develop an intuitive model of a photocell comprising two light-absorbing molecules coupled to an idealised reaction centre, showing asymmetric dimers are capable of providing a significant enhancement of light-to-current conversion under ambient conditions. We conclude by describing a roadmap for identifying suitable molecular dimers for demonstrating this effect by screening a very large set of possible candidate molecules.

The operation of a solar energy harvesting device can be enhanced by clever design of a nanoscopic, quantum mechanical system [1]. Though thermodynamical considerations lead to the famous Shockley-Queisser efficiency limit for classical photocell devices [2], the ‘detailed balance’ underlying this limit can be broken by careful use of quantum interference. In particular, by carefully tailoring the interactions between two [3, 4] or more [5, 6] idealised and identical two-level energy absorbers, it is possible to prevent the re-emission of absorbed light by arranging that excitations end up in ‘dark’ – i.e. optically inaccessible – states. This allows the energy to be dissipated across a target load, rather than dissipated via spontaneous emission.

It is conjectured that nature already exploits quantum-mechanical properties in order to increase the light-harvesting efficiency of photosynthesis [7]. The most well studied system in this context is the FMO complex [3], which connects the antenna to the reaction centre in the light harvesting apparatus of green sulfur bacteria. It consists of seven (or eight [9]) bacteriochlorophyll (BChl a) molecules that are held in place by a messy protein scaffold, and surrounded by water at room temperature, resulting in non-identical BChl excitation energies. True quantum effects may seem unlikely in the ‘hot and wet’ conditions of such systems. However, the observation of quantum coherent beats in experimentally measured two-dimensional electronic spectroscopy suggests otherwise [10–16].

A good definition of the term efficiency is key to quantifying a quantum advantage. One such measure is the energy transfer efficiency, i.e. the probability of an excitation reaching the target electron acceptor after starting from a spatially localised state [17–19], but this does not capture all aspects of the process. An alternative approach is placing a system between two electrodes, and measuring the current through them [20, 23]. Dorfman et al. proposed a different canonical measure [11]: They consider the entire cycle, from absorbing a photon to extracting work, as a quantum heat engine (QHE). Procedurally, they abstract the electron acceptor to become a two-level ‘trap’, in which transferred electrons ‘fill’ the excited state before the action of driving a load resistor is mimicked by decay to the lower level of the trap. This gives a straightforward way of defining the power and the efficiency of the heat engine. In this picture, Fano interference may boost the photocurrent by 27% over that of a classical cell. Subsequently, Creatore et al. [3] reported an efficiency gain of 35% by introducing the different effect of dark-state protection using two identical dipole-coupled emitters. Further gains become possible for more than two chromophores [5, 6].

In this Letter, we use the QHE framework to determine whether a quantum advantage is achievable in non-idealized situations typical of real devices and conditions more closely resembling the photosynthetic apparatus. In particular, we consider a light harvesting device where the two constituent chromophores are not identical. Surprisingly, we will find that under realistic constraints an ‘asymmetric’ dimer may even significantly outperform previously studied systems. Presenting several example molecules that would be highly efficient...
light harvesters according to our model, we argue that the number of conceivable molecular dimers with a quantum advantage is vast.

We now introduce a general framework that will allow us to define three specific models shortly. We consider two, generally different, light-absorbing molecules with dipolar coupling. There is a further coupling to an abstracted reaction center ‘trap’, modelled as a two level system $|\alpha\rangle$, $|\beta\rangle$ with corresponding energies $\epsilon_\alpha$, $\epsilon_\beta$, following Ref. [1]. An excitation absorbed by the molecules can be incoherently transferred into the reaction centre via a phonon-assisted process. In what follows we restrict the dynamics to the subspace with one or zero excitations across the entire system: this approximation is valid since in realistic configurations inspired by natural photosynthentic systems, the average excitation number is very small ($N \approx 0.02$ for an energy gap of 2 eV and sunlight temperature of 6000 K). Since photoexcitation is very rare, once captured it is paramount to prevent spontaneous emission back into the environment. Hence, access to a dark state, i.e. a state that is decoupled from the photon field and thus not susceptible to spontaneous emission decay, can enhance the engine’s efficiency. By Kasha’s rule re-emission of absorbed photons will be dominated by the lowest excited state, motivating our approach of only considering a single excited level per chromophore.

We denote the excited states in the molecules as $|1\rangle$, $|2\rangle$ with energies $\epsilon_1$, $\epsilon_2$, respectively, and the ground state as $|g\rangle$ with energy $\epsilon_g$, and $J_{12}$ is the dipolar coupling between the molecules. The Hamiltonian of the system is thus of dimension five and given by (see Fig. [3]):

$$H_s = \epsilon_1 |1\rangle \langle 1| + \epsilon_2 |2\rangle \langle 2| + \epsilon_g |g\rangle \langle g| + J_{12} \left( |1\rangle \langle 2| + |2\rangle \langle 1| \right) + \epsilon_\alpha |\alpha\rangle \langle \alpha| + \epsilon_\beta |\beta\rangle \langle \beta|$$

$$= \epsilon_+ |+\rangle \langle +| + \epsilon_- |−\rangle \langle −| + \epsilon_g |g\rangle \langle g| + \epsilon_\alpha |\alpha\rangle \langle \alpha| + \epsilon_\beta |\beta\rangle \langle \beta|.$$  

(2)

In the second equation, $|±\rangle$ are the usual eigenstates diagonalising the subspace spanned by $|1\rangle$, $|2\rangle$.

In addition to the bare system we also have the solar photonic bath at $T_h = 6000$ K, which can induce spontaneous and stimulated transitions $|1\rangle \leftrightarrow |g\rangle$ and $|2\rangle \leftrightarrow |g\rangle$. Further, each molecule is embedded in its own local environment of vibrational modes, treated as infinite phonon baths at room temperature $T_c = 300$ K. A generic spin boson type interaction between excitonic states and phonon modes yields transitions between the energy eigenstates $|+\rangle \leftrightarrow |−\rangle$, $|−\rangle \leftrightarrow |+\rangle \leftrightarrow |\alpha\rangle$, $|−\rangle \leftrightarrow |\alpha\rangle$, $|\beta\rangle \leftrightarrow |\gamma\rangle$. Further, we include the reaction center decay with rate $\gamma_{\alpha\beta}$, and some leakage between $|\alpha\rangle$ and $|g\rangle$ with rate $\chi_{\alpha\beta}$. The interaction Hamiltonian is thus:

$$H_I = \hat{I}_{1g} + \hat{I}_{2g} + \hat{I}_{11} + \hat{I}_{22} + \hat{I}_{1\alpha} + \hat{I}_{2\alpha} + \hat{I}_{\beta g}$$

(3)

Here $\hat{I}_{ab} = \frac{1}{2}(|a\rangle \langle b| + |b\rangle \langle a|)\hat{\mu}_{ab}$, and $\hat{\mu}$ are operators of the coupling to the different environments: $\mu_{1g}$, $\mu_{2g}$ are dipole operators, and the rest are phonon operators [24].

Applying a standard Born-Markov procedure [26] we arrive at a set of Pauli master equations [24]:

$$\frac{\partial}{\partial t} \vec{P} = Q \vec{P}.$$  

(4)

Here $\vec{P} = \{P_+, P_-, P_\alpha, P_\beta, P_g\}$ is a vector of the populations in the diagonal basis of the system, and $Q$ is a matrix of the different rates, respecting detailed balance for photon and phonon baths independently. Resulting transition rates are depicted in Fig. 2 and conservation of total population imposes the additional constraint $\sum \vec{P} = 1$. We give the explicit entries of $Q$ in the Supplementary Information (SI) [24], and also show that this rate equation approach is valid by direct comparison with the full Bloch-Redfield equations.

Using the concept of a photochemical voltage [27], we attribute an effective current and voltage to the reaction center [11]:

$$I = e\gamma_{\alpha\beta} P_\alpha, V = \epsilon_\alpha - \epsilon_\beta - k_BT_c \ln(P_\alpha/P_\beta).$$  

(5)

Following Refs. [1] [3–5] and [28] we now consider quantity $P = IV$ as a measure of the power generated by the system. Here $T_c$ is the (cold) phonon temperature, $k_B$ the Boltzmann constant, and $e$ the electron charge. In the absence of sunlight, the system thermalizes to the phonon bath temperature, and the voltage vanishes. Thus $V$ is a measure of the deviation from the thermal state with temperature $T_c$. We are interested in the steady-state power output, which is found by setting the LHS of Eq. 4 to zero and solving the resulting simultaneous algebraic equations.

Our Hamiltonian is static, and so there is no cycle as there would be for conventional quantum heat engines [29,31]; our device rather relies on heat flowing through the reaction centre to produce work [32,34]. Henceforth, we adopt the maximal achievable steady state power as our measure of efficiency: we treat the reaction centre as a black box optimizing its $\gamma_{\alpha\beta}$ to generate maximal power (keeping all other parameters fixed). For a representative example of the behaviour of $P$ and $I$ as a function of $V$ see Fig. S5 [24].

We now define the three specific models which we shall compare. These have contrasting molecular geometries and are depicted in Fig. 1. First, the independent model has two
identical light harvesting molecules that are not directly coupled to one another whilst each is independently coupled to the reaction centre. Specifically $\hat{\mu}_{1g} = \hat{\mu}_{2g}$; $\hat{\mu}_{1a}, \hat{\mu}_{2a}$ represent the coupling to different phonon baths, and $J_{12} = \gamma_{-+} = 0$. Further, we let $\gamma_{+g} = \gamma_{-g}, \gamma_{+a} = \gamma_{-a}, \epsilon_{+} = \epsilon_{-}$. This model does not exhibit dark state protection and serves as a benchmark for the other models.

Second, the symmetric model mirrors that described in Ref. [3] and consists of two identical, directly coupled molecules: $\epsilon_{1} = \epsilon_{2}, \hat{\mu}_{1g} = \hat{\mu}_{2g}$, and $J_{12} > 0$. This arrangement leads to a dark and bright state, $|+\rangle$ and $|--\rangle$ respectively, with $\gamma_{g} = 0$ and $\gamma_{-+} = \frac{1}{2} (\gamma_{11} + \gamma_{22})$. The molecules couple to the reaction center in anti-phase $\hat{\mu}_{1a} = -\hat{\mu}_{2a}$, rendering $\gamma_{+a} = 0$. We discuss deviations from this idealized scenario in the SI [24].

Finally, the asymmetric model is the main focus of our Letter. It comprises two non-identical molecules $\epsilon_{1} < \epsilon_{2}$ with different dipole moments $\hat{\mu}_{1g} = z \hat{\mu}_{2g}$ where $z < 1$ represents the asymmetry. The dark(er) $|--\rangle$ state has a larger overlap with molecule 1, which we imagine closer to the reaction center, and we assume molecule 2 only has negligibly small reaction center coupling ($\hat{\mu}_{2a} = 0$). We believe this that configuration should be easier to realise than the symmetric model, while allowing engineering of the energy gap $\epsilon_{+} - \epsilon_{-}$. For flat spectral densities of the environments around the transition frequencies, we find this asymmetric model exhibits a fully dark state, provided that $J_{12}, z$, and $\epsilon_{2} - \epsilon_{1}$ satisfy the relation:

$$J_{12} = \frac{2z}{1 - z^{2}} (\epsilon_{2} - \epsilon_{1}).$$  \hspace{1cm} (6)

Explicit rates for this system are given in the SI [24]. Whether or not $|--\rangle$ is indeed fully dark, we can express the resulting total excitation rate through an angle $\Phi$:

$$\gamma_{+g} = (\gamma_{1g} + \gamma_{2g}) \cos^{2} \Phi,$$ \hspace{1cm} (7)

$$\gamma_{-g} = (\gamma_{1g} + \gamma_{2g}) \sin^{2} \Phi.$$ \hspace{1cm} (8)

Thus $\tan^{2} \Phi = \gamma_{-g}/\gamma_{+g}$, and $\tan^{2} \Phi = 0$ in the presence of a completely dark state.

Several mechanisms may cause deviation from a fully dark state in both coupled models: First, different local environments would generally entail differing reorganization energy shifts and thus excitation energies. For example, the FMO complex consists of seven identical BChl units, embedded in a protein scaffolding, resulting in on-site energies spanning a range of 25 meV [10]. Second, the two dipoles may be at an angle $\varphi$ instead of parallel [3], breaking the interference needed for a completely dark state. Third, the coherent coupling $J_{12}$ depends on both the distance between the two molecules, and the angle $\varphi$: $J_{12} = J_{12}^{0} \cos \varphi$, where $J_{12}^{0}$ is the coupling with parallel dipoles. Taking all this into account we get in the general case, i.e. for all models,

$$\tan^{2} \Phi = \frac{\Omega_{R}(1 + z^{2}) - (\epsilon_{2} - \epsilon_{1})(1 - z^{2}) - 2zJ_{12} \cos \varphi}{\Omega_{R}(1 + z^{2}) + (\epsilon_{2} - \epsilon_{1})(1 - z^{2}) + 2zJ_{12} \cos \varphi},$$ \hspace{1cm} (9)

with $\Omega_{R} = \sqrt{(\epsilon_{2} - \epsilon_{1})^{2} + J_{12}^{2}}$ being the Rabi frequency of the bare system between sites 1 and 2. Further discussion about deviation from the fully dark state, and details on the coupling to the reaction centre are given in the SI [24].

The performance of the (a)symmetric relative to the independent models will be assessed by using our simulations to determine the ratio of the respective maximum powers, found by varying $\gamma_{a/\beta}$ in each case [see Eq. (5)]. For a fair comparison, we keep $\gamma_{+g} + \gamma_{-g}, \epsilon_{-}$, and $\gamma_{1a}$ equal across all three models.

Fig. 3 presents the enhancement achievable by dark-state protection. Here we have optimized $J_{12}$ as well as $\gamma_{a/\beta}$ with all other parameters fixed. We also constrain $J_{12}$ to below 30 meV as an upper limit of realistic coupling strength. For the asymmetric model the dark state criterion, Eq. 6 informs an appropriate dipole asymmetry $z$ for a given value of $J_{12}$. Quantum enhancement is only possible when the transfer into the reaction center is relatively slow and constitutes a bottleneck in the cycle. In the limit of $\gamma_{1a} \to 0$ an upper bound to the enhancement emerges. Within a reasonable parameter range this limit grows with increasing $J_{12}$ for the symmetric and with $\epsilon_{2} - \epsilon_{1}$ for the asymmetric case. As strong coupling is harder to realise than site energy mismatch, asymmetric dimers might more easily achieve high performance. We note that deeper into the slow transfer limit the potential enhancement factors can significantly exceed the values of up to 50% reported by Refs. 1, 3 and 5. By contrast, for fast transfer rates dark-state protection offers no advantage: absorbing photons at an energy higher than the extraction energy (i.e. at reduced thermal photon occupancy), combined with $\gamma_{2a} = 0$, is now detrimental.

Fig. 4 shows the relative power enhancements of the asymmetric model as a function of the energy difference $\epsilon_{1} - \epsilon_{2}$ and of the coupling $J_{12}$. We also plot the equivalent enhancement given by the symmetric model, and show there is a parameter regime, with boundaries marked by a black line, for
two reasons: First, in the regime at finite coupling and energy difference. This happens for \( \epsilon_2 = \epsilon_1 \) equal. Parameters are as in Fig. 3 and with the symmetric equivalent (solid blue). The black thick line is the contour where the symmetric and asymmetric power ratios are equal. Parameters are as in Fig. 5 and with \( \gamma_1 = 6 \times 10^{-7} \text{eV} \).

FIG. 4. (color online) Orange surface: relative power enhancement of the asymmetric model, as a function of the energy difference \( \epsilon_2 - \epsilon_1 \) and coupling \( J_{12} \). Gray surface: relative power enhancement of the symmetric model (independent of \( \epsilon_2 - \epsilon_1 \)). Black dashed line: asymmetric model enhancement for a fixed \( \epsilon_2 - \epsilon_1 = 90 \text{meV} \). The dashed line is projected onto the \( \epsilon_2 - \epsilon_1 = 0 \) plane, for comparison with the symmetric equivalent (solid blue). The black thick line is the contour where the symmetric and asymmetric power ratios are equal.

which the asymmetric model outperforms the symmetric one. The asymmetric model displays a peak power enhancement at finite coupling and energy difference. This happens for two reasons: First, in the regime \( \epsilon_+ - \epsilon_- \lesssim k_B T \), the rate \( |\rangle \rightarrow |\rangle \) is non-negligible, and so the dark state is not protected. Second, if \( \epsilon_+ - \epsilon_- \gg J_{12} \), the rate \( |\rangle \rightarrow |\rangle \) becomes negligible, and the dark state is rarely populated. Note that in the limit \( J_{12} \rightarrow 0 \), the asymmetric model gives a smaller power than the independent benchmark. This is because we set \( \gamma_1 = 0 \) for the asymmetric model, effectively making it a single antenna setup benchmarked against two antennae. We examine further realistic imperfections, including the presence of additional dephasing mechanisms, in the SI [24].

To assess the feasibility of generating suitable asymmetric molecules for power enhancement, we use a library containing quantum-chemically predicted properties of organic light-emitting diode molecules [35] to identify systems that minimize \( \tan^2 \Phi \) in Eq. (9). The SI [24] provides a full account of how quantum chemical calculations lead to promising molecular dimer candidates through a rigorous multistage process. Our donor candidates feature strongly allowed optical transitions (\( \mu \) of 3.5 atomic units) and site energies between 3.5 to 2.5 eV to optimally absorb sunlight. As required by the model, acceptable acceptor compounds must have \( \epsilon_1 < \epsilon_2 \) and possess lower transition dipole moments with \( z \approx 0.2 \) to deliver \( \tan^2 \Phi \lesssim 0.05 \). For simplicity, we assume fully aligned transition dipole moments and center-to-center distances between donor and acceptor moieties of 1 nm (approximately corresponding to the size of a small aromatic bridging group), resulting in an inter-site coupling of up to 15 meV. Importantly, we analyse the properties of our dimers for both ground and excited state equilibrium geometry to identify systems whose relevant properties are robust to vibrational relaxation effects accompanying optical absorption and emission.

The predicted properties of a selection of molecular pairs are reported in Table S1 [24] and an illustration of some molecules is shown in Fig. S1 [24]. These examples provide evidence that the chemical regime required for dark-state protection is readily available in ordinary molecular systems. A full implementation of the proposed model would also require an additional molecular system to act as a trap, as well as control over orientation and distance between donor and acceptor. Whereas the chemical synthesis of such a complex structure is challenging, our results show that matching fundamental components for such a system is entirely feasible.

In conclusion, we have presented a general model of light absorption by an asymmetric pair of coupled chromophores, finding that it can outperform both the symmetric dimer and a pair of independent molecules in realistic parameter regimes of operation for a solar cell device. Not relying on identically matched coupled chromophores, this approach is more robust to deviations from the delicate conditions required by its symmetric counterpart. Moreover, we have shown that an abundance of real pairs of molecules have the required asymmetric properties, and indeed, such asymmetry is an integral part of natural photosynthetic systems.

The reason our asymmetric model works so well is that it enables arbitrarily large energy gaps between the bright and dark states, thus preventing phonon-assisted promotion from the dark to the bright state. In the regime where excitations are rare and the transfer into the reaction center is very slow, this translates into better protection of the excitations, thus increasing the overall efficiency of the device.

The research data supporting this publication can be accessed, see Ref. [50].

We thank Alex Chin, Ahsan Nazir, Simon Benjamin, Alán Aspuru-Guzik, and Jorge Aguilera-Iparraguirre for stimulating discussions. This work was supported by the Leverhulme Trust (RPG-080). EMG is supported by the Royal Society of Edinburgh / Scottish Government. RGB thanks Samsung Advanced Institute of Technology for funding. AF thanks the Anglo-Israeli association and the Anglo-Jewish association for funding.

* Corresponding author, e.gauger@hw.ac.uk


[24] See Supplementary Information, which includes Refs. [27][22].


[36] http://dx.doi.org/10.17630/e901ecd8-e8d0-4062-a5b1-2ce2bfa5c09c.


