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The Lifetime Performance Prediction of Fractured Horizontal Wells in Tight Reservoirs

Mojtaba MoradiDowlatabad, Mahmoud Jamioiahmady, Heriot-Watt University

Abstract

Multiple fractured horizontal wells (MFHWs) are recognised as the most effective stimulation technique to improve recovery from tight and shale gas assets. The performance of MFHWs depends on series of flow regimes developed during production. Understanding of the complex flow behaviour and the proper interpretation of these flow regimes are necessary to obtain information about the reservoir and to predict the performance of these wells.

There are several models available for simulating the early transient linear flow behaviour in unconventional reservoirs. However, they involve over-simplifying assumptions about the hydraulic fracture geometry and the reservoir. Furthermore, these models fail to appropriately consider the compound linear flow regime, the interference effect and/or the transitional periods in between different flow regimes that are expected to be important in such low permeability formations.

In this paper, the aim is to investigate the key characteristics of the transient (unsteady-state) flow periods and accordingly propose a practically attractive tool for performance prediction of the MFHWs in tight reservoirs. To achieve this objective, first, series of simulated well test data are discussed to identify the key practical flow regimes that can be expected for various practical MFHWs designs with different fracture spacing, length of fractures etc. Following this investigation, new analytical models are proposed to predict the performance of MFHWs under different dominant transient flow regimes.

Moreover, integrating the new models with the pseudo steady state (PSS) productivity index formulation proposed previously by the authors, a new approach is presented that covers the identified flow regimes, i.e. the formation linear, the compound linear and PSS flow regimes and the transition periods in between. The outcome of this study can be used for tasks such as well testing, production performance analysis, forecasting and optimisation of MFHWs design.

1. Introduction

Conventionally the formations with permeability varying between 1µD and 0.1 mD are classified as tight reservoirs. In these reservoirs, enlarged drainage area by the horizontal well
with multiple transvers fractures increases the well productivity significantly. This is why multiple fractured horizontal wells are considered as the most effective stimulation technique to improve recovery from such low permeability reservoirs. Understanding of the complex flow behaviour and predicting the performance of these wells are vital. The available production analysis methods that have, so far, been used for unconventional reservoirs are: 1) Straight-line (or flow-regime) analysis, 2) Type-curve, 3) Empirical, 4) Semi-analytical and numerical simulation and 5) Hybrid (e.g. analytical and empirical) methods.


Nobakht et al. (2010) presented a hybrid approach to forecast production from shale reservoirs by applying analytical equations for the transient period and using hyperbolic decline curves for the boundary dominated flow period.

Analytical modelling has been typically used to a) generate type-curves; b) characterise the system; and c) to generate production forecasts. Meyer et al. (2010) presented an analytical methodology to predict performance of MFHWs based on trilinear (Lee and Brockenbrough 1986) and pseudo steady-state resistivity models (Meyer and Jacot 2005) of fluid flow towards vertical fractures in shale reservoirs. Azari et al. (1990) presented constant-pressure, semi-analytical solutions using the Duhamel’s theorem and Laplace transformations for the constant-rate solution, when considering only bilinear, linear and pseudo-radial flow regimes. Furthermore, in many of these models are based on over-simplifying assumptions about the hydraulic fracture geometry and the reservoir. For example, some assume that all fractures must extend to reservoir boundaries. Such assumptions cause, for example, the linear flow to be immediately followed by a boundary-dominated flow, which means they lack to appropriately consider the compound linear flow regime, the interference effect and/or the transitional periods in between different flow regimes that are expected to be dominant in such low permeability formations. In the following, first, analysis of simulated well test data is presented to identify the key practical flow regimes that can be expected for various practical MFHWs designs. Following this investigation, new analytical models are proposed to predict the performance of MFHWs under various dominant transient flow regimes. Finally, integrating the new models with the pseudo steady state (PSS) productivity index formulation proposed previously, a new approach is presented that covers the identified flow
regimes, i.e. the formation linear, the compound linear and PSS flow regimes and the transition periods in between.

2. Practical Flow Regimes around MFHWs in Tight Reservoirs

It is important to identify the flow regimes occurring during the MFHWs production life. The theoretical flow regimes that could be established around MFHWs were first introduced by van Kruysdijk (1989) and are shown in Figure 1. However, some of the expected flow regimes may not occur in tight reservoirs because of the practical developed development strategies. Therefore, series of simulations have been performed for various MFHW arrangements and configurations using the reservoir model, generated by a commercially available reservoir simulator, described below and shown in Figure 2. This Figure shows the considered flow system and two magnified images of the fractures that intersect the well and that of the surface areas surrounding the vicinity of the well and fractures.

2.1 Base Case Model Description

In this study, a 3D Cartesian grid model with 151*151*10 cells with dimension of 40*40*10 ft in the X, Y and Z directions, respectively, has been set-up to simulate a tight gas reservoir. The gridding was selected based on a sensitivity analysis on the global grid size to avoid numerical dispersion, while keeping run time reasonable. Due to much more complex flow behaviour around a MFHW compared to that around a conventional well, the local grid refinement (LGR), which explicitly defines hydraulic fractures in the simulation, is required to properly capture the variation of flow parameters as fluid travels from the matrix to the fractures and then to the wellbore. Another sensitivity analysis on the grid refinement was carried out to determine the optimum number of grids around each fracture. The optimum LGR around each fracture used in this study divided each parent grid into 9 sub grids in the X, 4 sub grids in the Y and 1 grid in the Z directions.

The hypothetical tight gas reservoir produces from a horizontal well, placed in the centre of the model. The dry gas flows within a reservoir with an initial reservoir pressure of 7,500 psi, porosity of 0.15 and an average effective reservoir permeability ($K_m$). Table 1 provides more information on the model’s properties. To establish the scenarios, the following additional assumptions have been made, unless otherwise stated:

1) The reservoir rock is homogeneous.
2) The fluid is single-phase and slightly compressible.
3) Darcy Law governs the flow of fluid towards fractures and within the matrix.
4) Pressure loss along the horizontal section of the wells is assumed negligible.
5) The fractures are identical in term of physical properties such as conductivity and have been positioned vertically with constant spacing along the well and penetrating the whole reservoir thickness.
6) Considering MFHWs with cased/perforated completion has been used in this study, the flow to the wellbore is only through hydraulic fractures.
7) No geomechanics model is included in this study as it is expected that the impact not to be significant for the considered range of permeability. In other words, the formation and fracture properties do not change during a simulation.

The impacts of pertinent parameters were considered in a pre-screening sensitivity exercise to identify the parameters considerably affecting the performance of MFHWs from those with minimal effects. For instance, it was observed that within the rock compressibility ($C_f$) range of $1E^{-7}$ to $3E^{-4}$ l/psi, $C_f$ did not affect the performance of MFHWs if rock permeability varies between 0.001 and 0.1 mD.

![Figure 1: Schematic diagram illustrating the theoretical flow regimes sequence for a MFHW.](image)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
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<td>Initial Reservoir Pressure</td>
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<td>psi</td>
</tr>
<tr>
<td>Reservoir Temperature</td>
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<td>Reservoir Porosity</td>
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<td>Rock Compressibility</td>
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<td>psi</td>
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</table>

Table 1: Reservoir Parameters.
The log-log derivative plot was used to identify main flow regimes in this study. Table 2 and Table 3 show the flow regimes for different ratios of the well length to the reservoir (drainage) length in the X-direction ($L_w/2X_e$) and those of the reservoir half-length in the Y-direction to fracture half-length ($Y_e/X_f$) with $N_f=7$ in the reservoirs with $K_m=0.1$ and 0.001 mD, respectively. These Tables show that some of the theoretical flow regimes, listed in Figure 1, are not present. For instance, if the reservoir half-length in the Y-direction (or spacing between wells) is equal to the half-length of the fractures (i.e., $Y_e/X_f=1$) and the well is drilled through the whole reservoir (i.e. $L_w/2X_e=1$), the early linear flow and boundary-dominated flow are the only expected flow regimes if the very short-lived fracture linear flow is ignored.

Al Ahmadi et al. (2010) discovered that none of the 400 wells in unconventional reservoirs analysed exhibited fracture linear or bi-linear flows. In this study, as shown in Table 2, the latter flow regime was only observed, when very low permeability fractures (about 1 D) were induced in the formation with $K_m=0.1$ mD (the highest $K_m$ considered as a tight reservoir), but not for the other higher values of permeability. It should be noted that for the lower formation permeability value of 0.001 mD, only the early linear flow was observed, (Table 3). In addition, these Tables also show that:

- Early radial flow does not exist in any of the cases studied.
• Compound linear flow do not exist when $Y_e/X_f = 1$ or 3 and $L_w/2X_e = 0.6$ or higher but such designs are more likely to be present in shale reservoirs based on the common industry practice (either available in open literature or suggested by our industrial partners).

• Late radial flow is only observed in the case with $Y_e/X_f = 30$ and $L_w/2X_e = 0.02$, however, such a design (with small fracture half-length and spacing) may not be practical in tight reservoirs based on the common industry practice (either available in open literature or suggested by our industrial partners).

Table 2: Flow regime sequence for MFHWs when $K_m=0.1$ mD and $N_f=7$.

<table>
<thead>
<tr>
<th>$S_f$</th>
<th>$L_w/2X_e$</th>
<th>$Y_e/X_f$</th>
<th>Fracture</th>
<th>Bilinear</th>
<th>Early Linear</th>
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*: when very low permeability fractures (about 1 D) were induced.

Table 3: Flow regime sequence for MFHWs when $K_m=0.001$ mD and $N_f=7$.

<table>
<thead>
<tr>
<th>$S_f$</th>
<th>$L_w/2X_e$</th>
<th>$Y_e/X_f$</th>
<th>Fracture</th>
<th>Bilinear</th>
<th>Early Linear</th>
<th>Early Radial</th>
<th>Compound Linear</th>
<th>Late Radial</th>
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6
Furthermore, mostly a transition flow period has been observed after the early linear flow, and not the expected radial flow. During this transition period, fractures start to interfere with each other and deplete the area around the fractures. Simultaneously, the establishment of a complete compound linear flow regime from beyond fractures toward the set of fractures is also happening in most of examined cases. There is also another transitional period between the compound linear flow and boundary dominated flow. This period could only be exhibited as a late radial flow regime if the well length is too small compared to the length of the reservoir (or expected drainage area). These transitional periods, which may represent a few log-cycles in production time or in other words during a significant bulks of the total production, do not exist in the flow regime sequence presented by (Raghavan et al. 1997, van Kruysdijk 1989).

In summary, the early formation linear, the compound linear and the boundary-dominated flow regimes are exhibited in most of the cases considered practical in tight reservoirs. However, due to the slow pressure propagation in tight reservoirs, there are long transitional periods between the mentioned flow regimes. These periods contribute to a large bulk of production and hence, need to be considered in any production forecasting technique. The results of this study could be used for quickly identifying the expected flow regimes for a specific MFHW design in a tight reservoir.

3. **New Analytical Model for Predicting MFHWs Performance**

As already discussed, the main flow regimes could be recognised as the following:

a) **Transient Period:**

1. Early-time flow period (the early formation linear or bi linear flow)
2. First transitional period
3. Intermediate-time flow period (compound linear flow)
4. Second transitional period

b) Boundary Dominated Flow Period:
5. Late-time flow period (pseudo-steady state flow)

In the following sections, the single-phase flow governing equations for each of these five periods and a methodology to model the two transitional periods are discussed. It should be noted that the fracture linear flow period has been ignored in this study due to its short life span in comparison to the production time.

3.1 Transient Period: Early Time Flow Period

In general, the linear flow regime is defined as parallel flow lines that move toward a plane, orthogonally, as it shown in Equation 1. These flow regimes may be diagnosed by a half-slope in the derivative on a log-log diagnostic plot or by a straight line on a square root of time (linear flow specialized) plot. In general, Equation 1 has been commonly used for describing the linear flow regimes:

\[
P_{wf} = P_i - \left( \frac{8.128qB}{A} \sqrt{\frac{\mu t}{K_m \Phi C_t}} \right)
\]

Equation 1

where A is the exposed area to the linear flow, h is the formation thickness, t is the time, \( P_i \) is the initial pressure and q is the production flowrate.

(a) linear flow within a fracture (b) linear flow to a fracture (c) linear flow toward a HW

Figure 3: An illustration of various linear flow.

At the early time flow period, the early linear flow is the main flow regime in most of MFHWs in tight reservoirs. For this flow regime, as the exposed area to the flow is the cross section of a fracture \(2X_hh\), the corresponding equation is as follows:
\[ P_{wf} = P_i - \left( \frac{16.52 q^2 B^2 \mu}{h^2 \phi C_t} \sqrt{\frac{\tau}{K_m X_f^2}} + \frac{141.2qB}{K_m h} (S_D + S_c) \right) \]  
Equation 2

where \( S_D \) is the damage skin, \( S_c \) is the convergence skin in a fractured horizontal well. For a horizontal well with multiple fractures in a tight reservoir, the total production rate, \( q_t \), in the early-time flow period can be determined by the following Equation:

\[ q_t = \sum_{i=1}^{N} q_i \]  
Equation 3

Assuming the constant fracture spacing and identical properties of the fractures, the total production rate of the well can be calculated by multiplying the production rate from a single fracture by the number of fractures during the early-time flow period. In other words, \( q \) in Equation 2 is \( (q_t/N_f) \).

The early linear flow regime ends when pressure perturbations from the neighbourhood fractures reach each other (i.e. fracture interference starts). The corresponding time can be calculated by the following Equation:

\[ t_{el} = 237 \frac{\phi \mu C_t S_f^2}{K_m} \]  
Equation 4

where \( \phi \), \( \mu \), \( C_t \) and \( t_{el} \) are the porosity, viscosity, total compressibility and the time of interference respectively. It should be noted that the time of fracture interference depends on the reservoir properties and fracture spacing (\( S_f \)).

If bi-linear flow exists rather than early linear flow, Equation 2 should be replaced by:

\[ P_{wf} = P_i - \left( \frac{44.13q B \mu \sqrt{k}}{h \sqrt{K_f W_f} \phi C_t K_m} + \frac{141.2q B}{K_m h} (S_D + S_c) \right) \]  
Equation 5

where \( W_f \) and \( K_f \) are the width and permeability of the fractures, respectively.

### 3.2 Transient Period: Intermediate Time Flow Period (Compound Linear Flow)

As mentioned before, for many cases, the compound linear flow regime was observed. To further highlight the importance of this flow regime, the results of some of the simulations described above have been used to generate sets of dimensionless pressure (\( P_D \)) versus dimensionless time (\( t_{Dxf} \)) type curves, which are shown in Figures 4-6 for \( X_f=1020, 500 \) and 100 ft., respectively. These type curves were generated based on Equation 6 and Equation 7 for a range of fracture spacing of 40 to 840 ft.
\[ P_D = \frac{K_m h (P_i - P_{wf})}{141.2 \mu B} \]  

Equation 6

\[ t_{Dxf} = \frac{0.00633 K_m t}{\mu C_t X_f^2} \]  

Equation 7

where \( P_D \) is the dimensionless pressure, \( t_{Dxf} \) is the dimensionless time and \( P_{wf} \) is the flowing bottom-hole pressure.

These Figures illustrate that the type curves for different values of \( S_f \) overlap each other at the early time (linear) and late time (boundary dominated) flow conditions; highlighting that the intermediate time between these two flow periods is important for characterising the systems.

Figure 4: PD versus \( t_{Dxf} \) type curves for MFHWs with \( X_f=1020 \text{ ft} \) at different \( S_f \) values.
Figure 5: PD versus $t_{Def}$ type curves for MFHWs with $X_f=500$ ft at different $S_f$ values.

Figure 6: PD versus $t_{Def}$ type curves for MFHWs with $X_f=100$ ft at different $S_f$ values.
Thompson et al. (2012) used the definition of the compound linear flow regime by van Kruysdijk (1989) and proposed an analytical solution for it. Van Kruysdijk identified this flow regime “as the well response, as if, it drains a linear reservoir with a width equal to the distance between the outer fractures”. Thompson et al. proposed that if \( X_f \) in Equation 8, which was proposed by Wattenbarger et al. (1998) as a reduced, dimensionless form of Equation 2 is replaced by the half-length of the well \( (L_w/2) \), the solution for the compound linear flow regime, Equation 9, could be expressed by:

\[
P_D = \sqrt{\frac{\pi t_{Dxf}}{X_f}} \quad \text{Equation 8}
\]

\[
P_D = 2 \frac{X_f}{L_w} \sqrt{\frac{\pi t_{Dxf}}{X_f}} \quad \text{Equation 9}
\]

In this formulation, it is assumed that the well is drilled through the entire length of the drainage area i.e. \( L_w = 2X_e \), as shown in Figure 7. Thus, Equation 9 could not always be valid because the well may not go through the whole reservoir and/or the contribution of the reservoir beyond the stimulated area to the total production is significant.

![Diagram of a multi-fractured horizontal well](image)

**Figure 7: A multi-fractured horizontal well drilled through the whole reservoir length.**

Here, three different cases with various well lengths are discussed to demonstrate the limitation of Equation 9 in tight reservoirs. Figure 8 compares the bottom-hole pressure of the various cases obtained by the dimensionless equations suggested by Thompson et al. (2012) with those obtained from reservoir simulations. The Figure shows that as the \( L_w/X_e \) ratio decreases from 2 to 0.6, the accuracy of the prediction from the dimensionless equations decreases drastically. Therefore, new equations are required, which have been developed as described in the next section.
3.2.1 New Definition of the Compound Linear Flow

To achieve the desired solution, a new and wider definition of the compound linear flow is introduced in this study, where the exposed area to the flow is dynamic. In other words, the area changes with time and is not limited to the cross-section of the well as assumed by previous researchers. The area starts expanding from the stimulated area as soon as the interference of fractures occurs while the flow behaviour is still linear, (half-slope in the derivative on a log-log diagnostic plot). For example, Figure 9 (a, b and c), shows the pressure profiles at different times (t=2, 7 and 10 years) of production, when a well with 7 fractures in the reservoir with $K_m=0.001 \text{ mD}$ produced under compound linear flow. The dominance of compound linear flow is confirmed using well testing analysis with corresponding data shown in Figure 10. This Figure shows the derivative of pressure of this flow regime is linear (slope=0.5) for a considerable amount of time; however, the area is not limited to the well area.

Here, Equation 10 is proposed to calculate the dynamic area by considering the pressure propagation beyond the stimulated volume over time.

Figure 8: Comparison of the simulation results with the results obtained from the available equations, $K_m=0.1 \text{ mD}$, $X_f=500 \text{ ft}$ for various $L_w/X_e$ values of 2, 1.1 and 0.6.
\[ A = A_1 + A_2 + 2h \int_{t_{scL}}^{t_{ecL}} \frac{\eta}{948} dt \] \hspace{1cm} \text{Equation 10}

where \( A_1 \) is the cross area of well (\( L_w h \)) as shown in Figure 2, \( A_2 \) is the side area of the well (\( 2X_f h \)), \( \eta \) is the formation diffusivity, \( t_{scL} \) and \( t_{ecL} \) are the start and the end times of the compound linear flow and \( t \) is the time elapsed from the end of the early linear flow (\( t_{el} \)).

In the third term of the equation, the concept of radius of investigation has been used to account for the additional dynamic area beyond the fractured area over time.

This definition of the exposed area allows using the general form of linear flow equation for calculating pressure profile of the compound linear flow regime as follows:

\[ P_{wf} = P_i - \left( \frac{8.128 q_t B}{A} \sqrt{\frac{\mu t}{K_m \phi C_t}} \right) \] \hspace{1cm} \text{Equation 11}

where, \( q_t \) is the total production rate of the well and \( A \) is calculated at any time by Equation 10. It should be noted that if the compound linear flow does not disappear after reaching the first boundary, the \( 2X_f h \) should be used for \( A \) instead of the output of the Equation 10 as the pressure has reached the reservoir boundaries.
Figure 9: A schematic of the compound linear flow propagating over time, $K_m=0.001$ mD.
3.3 Transitional Flow Periods

As already mentioned, the transitional flow periods contribute to a large bulk of MFHW production and hence, need to be considered in the production forecasting. The following Lagrange polynomial approximation is proposed to capture the MFHW performance during these periods properly:

\[
P_D = \exp \left( f(t_{Dxf}) \right) \tag{12}
\]

where:

\[
f(t_{Dxf}) = \sum_k \ln P_{Dk} \prod_{i \neq j} \frac{\ln t_d - \ln t_{Dj}}{\ln t_{Dk} - \ln t_{Dj}} \tag{13}
\]

To use Equation 12 and Equation 13, a number of data points from the solutions of the early linear, compound linear and PSS are required to be used to find the best curve that fits the data before the equation is used to interpolate the other points in the corresponding transition zones. That is, if this equation is applied for the first transitional period (between the early and the intermediate flow periods), \( t_{Dk} \) and \( t_{Dj} \) are a number of points in the early linear and compound linear flow periods, respectively, with known solutions \( P_{Dk} \).
For the second transitional period, $t_{Dk}$, $t_{Dj}$ are a number of points in the intermediate and PSS times with known solutions of $P_{Dk}$.

For a given number (k) of discrete points, there are many possible interpolating functions; however, the Lagrange polynomial, first introduced by Waring (1779), finds the one unique function with degree of (k-1). In addition, the durations of the transition periods are expected to be long, which cause a large variation in known solutions of flow regimes before and after the transition periods. In mathematics, these are called the functions with high condition number. The Lagrange polynomial are recommend to solve such problems (Gautschi 1974).

It should be noted that when constructing the polynomials, there is a trade-off between a better fit and a smooth well-behaved fitting function. The more data points that are used in the interpolation, the higher the degree of the resulting polynomial, and therefore the greater oscillation it will exhibit between the data points. Therefore, a high-degree interpolation may be a poor predictor of the function between points; however, the accuracy at the data points will be perfect.

3.4 Results and Discussion

Various MFHWs scenario, which were not used before, were simulated using the same simulator to investigate the reliability of the new model for capturing the unsteady state performance of MFHWs in the transient period. It has to be also added that the equations are valid for cases that primarily meets the underlying assumptions.

Figure 11 shows the bottom-hole pressure profiles that have been obtained from various modelling approaches for a MFHW with 7 fractures and the half-length of 500 ft at a spacing of 600 ft in a reservoir with $K_m=0.001$ mD. For this case, the well has produced under early linear flow regime for 75 days without fractures interference followed by 10 years of compound linear flow as shown in Figure 10. Compared with the numerical simulation results, the Figure shows that the commonly used formulations (i.e. Thompson et al.’s equations) significantly underestimates the bottom-hole pressures in the transient flow period. However, using the concept of dynamic area has significantly improved the prediction of the pressure profile.

Another MFHW design (with $N_f=7$, $X_f=500$ and $S_f=320$) and $K_m=0.1$ mD was considered. The bottom-hole pressure profile obtained from the new method for this case has been compared with those obtained from numerical simulation and the commonly used equations as shown in Figure 12. The Figure shows a considerable improvement in the predictions.
when using the new approach has been used compared to those predicted by the Thompson’s equation.

Figure 11: Comparison of bottom-hole pressure in transient period obtained from various approaches (\(K_m=0.001 \text{ mD, } N_f=7, S_f=600 \text{ ft and } X_f=500 \text{ ft}\)).

Figure 12: Comparison of bottom-hole pressure in transient period obtained from various approaches (\(K_m=0.1 \text{ mD, } N_f=7, S_f=320 \text{ ft and } X_f=500 \text{ ft}\)).
Figure 13: Comparison of bottom-hole pressure in compound linear flow obtained from various approaches (K_m=0.05 mD, N_f= 10, S_f= 200 ft and X_f=100 ft).

Figure 14: Comparison of bottom-hole pressure in compound linear flow obtained from various approaches (K_m=0.01 mD, N_f= 5, S_f= 400 ft and X_f=300 ft).
Figure 15: Comparison of the results from Lagrange polynomial approximation (red line) with the reservoir simulation (blue dots) for the first transition period using the known solutions (black dots).

Figure 13 and Figure 14 show the bottom-hole pressure profiles of other MFHWs examples producing under compound linear flow regimes. Figure 13 shows the performance of a MFHW design with $N_f=10$, $X_f=100$ ft, $S_f=200$ ft and $K_m=0.05$ mD while Figure 14 shows the bottom-hole pressure profiles of another case with $N_f=5$, $X_f=300$ ft, $S_f=400$ ft, $K_m=0.01$ mD. These data demonstrate that compared to results of the numerical simulation and commonly used equations, the new equation has significantly improved the prediction of the pressure profiles when using the dynamic area concept.

Figure 15 shows the application of the procedure proposed for predicting the transition periods. In this Figure, a number of known points from the early linear and compound linear flow regime solution have been used to estimate the performance of the first transition zone. The Figure also confirms the good predictive capability of Equation 10, in terms of matching the reservoir simulation outputs and when compared to the predictions of the Thompson’s equation.

It should be noted if optimising the performance of well based on a short-term objective (i.e. before the interference time) is aimed, these new equations could also be used for the targeted time.
3.5 Well Lifetime Performance Prediction

Combining the developed transient time models with a model suitable for the boundary dominated flow enable us to simulate the MFHWs lifetime performance. For this purpose,, the PSS based PI equation developed for MFHWs in tight reservoirs by the authors (MoradiDowlatabad and Jamiolahmady 2015) is used.

The time for the pressure to establish a boundary-dominated flow could be estimated by:

\[ t_{pss} = 3790 \frac{\Phi \mu C_o A}{K_m} t_{DApss} \]  
Equation 14

where \( t_{pss} \) is the PSS time and \( t_{DApss} \) is the shape factor value, which depends on the geometry and well placement.

In addition, by writing the material balance for a slightly compressible single-phase fluid, the average reservoir pressure at PSS time could be obtained by the following:

\[ P_r = P_t - \left(1 - \frac{N_P}{N} \right) \left(\frac{B_{oi}}{B_o}(1 + C_r)\right) \]  
Equation 15

where \( P_r \) is the average reservoir pressure at PSS time, \( N \) is the original fluid-in-place, \( N_P \) is the cumulative fluid production up to the PSS time and \( B_{oi} \) and \( B_o \) are the fluid formation volume factors at initial and PSS times, respectively.

After PSS time, the reservoir pressure at a specific time could be calculated by applying the material balance and using the fact that the pressure gradient over time is constant for the reservoir at pseudo steady state condition. Therefore, as:

\[ \frac{dp}{dt} = - \frac{0.23396qB}{C_t A h \Phi} \]  
Equation 16

The average reservoir pressure could be calculated using the equation below:

\[ P_{r,i+1} = P_{r,i} - \frac{0.23396qB \Delta t}{C_t A h \Phi} \]  
Equation 17

where \( P_{r,i+1} \) and \( P_{r,i} \) are the average reservoir pressures at time steps \( i \) and \( i+1 \) after PSS time and \( \Delta t \) is the time difference between the two time-steps. Using Equation 18, the well performance can be obtained:

\[ P_{wf,i+1} = P_{r,i+1} - \frac{q}{PI_t} \]  
Equation 18

where \( PI_t \) is the MFHWs productivity index. Therefore, by combining Equation 17 and Equation 18, the flowing bottom-hole pressure (\( P_{wf} \)) could be calculated by:

\[ P_{wf,i+1} = \left( P_{r,i} - \frac{0.23396qB \Delta t}{C_t A h \Phi} \right) - \frac{q}{PI_t} \]  
Equation 19
Various MFHWs scenarios were simulated to investigate the reliability of the new model for capturing the full well-life performance of MFHWs. Figure 17 shows the bottom-hole pressure profiles that have been obtained from two modelling approaches (numerical reservoir simulation and the new analytical solution) for a MFHW (with $N_f = 7$, $X_f = 500$ and $S_f = 600$ ft) in a reservoir with $K_m = 0.1$ mD that produces at a constant rate of 500 MScf/Day for 1000 days. It is noted that for this case, the well produces under unsteady state flow conditions for almost three months. The interference time for this case is about 18 hours, while the well produces under the compound linear flow for 70 days. Figure 16 shows that 8 data points from the compound linear flow and 10 points from the boundary-dominated flow have been used to create the polynomials, which model the performance of the second transitional period. Similar procedure has been followed for the first transition period with points taken from the early linear and compound linear flow regimes. PI has been calculated to be 6.92 MScf/Day.psi. Figure 17 shows a very close agreement between pressure profiles from the new developed approach and numerical simulation.

![Figure 16: Comparison of the results from Lagrange polynomial approximation (black line) with the reservoir simulation (Red dots) for the second transition period using the known solutions (Blue dots).](image-url)
The approach was applied for modelling the lifetime performance of another MFHW case (with $K_m=0.01$ mD, $N_f=5$, $S_f=400$ ft and $X_f=300$ ft) which produced 500 MScf/day for 10 years. The results showed although the early linear flow lasted only 4 days, the well production reached the pseudo steady state condition after 977 days (2.7 years) due to the long (800 days) compound linear flow (Figure 14) and the transition periods. The PI of the well was calculated as 0.40 Mscf/Day.psi at PSS conditions.

Figure 18 compares the flowing bottom-hole pressure profiles of the well obtained by the new approach and reservoir simulation for this case. It should be noted that the observed difference between the profiles at PSS conditions is recognised to be due to the small difference between the calculated PI value and the actual value (obtained by the reservoir simulation (PI=0.385 Mscf/day.psi)).

![Figure 17: Comparison of bottom-hole pressure (both transient and boundary dominated flow conditions) obtained from the new developed approach with numerical simulation, ($K_m=0.1$ mD, $N_f=7$, $S_f=600$ ft and $X_f=500$ ft).](image-url)
Figure 18: Comparison of flowing bottom-hole pressure (both transient and boundary dominated flow conditions) obtained from the new developed approach with numerical simulation, (\(K_m=0.01\) mD, \(N_f=5\), \(S_f=400\) ft and \(X_f=300\) ft).

3.6 Summary and Conclusions

In this study, the key features of transient flow around MFHWs in tight reservoirs were studied.

1. It was demonstrated that although development of flow regimes around MFHWs depend on the fracture geometry and reservoir properties, the early linear, compound linear and boundary-dominated flow regimes are expected to be observed in most of the practical cases.

2. The presented dimensionless type-curves illustrated the importance of the flow behaviour in the intermediate time, mostly dominated by the compound linear, for characterising the systems, as the early time linear flow and pseudo-steady-state parts overlapped at many different prevailing conditions.

3. The analyses showed that the common formulation of linear flow is not applicable in tight reservoirs where the well may not go through the whole reservoir and/or the contribution of the reservoir beyond the stimulated area to the total production is significant.
4. A new, general definition of the compound linear flow regime was introduced and used to describe the flow during the compound linear flow period. The proposed definition considers the exposed area to the flow to be dynamic (Equation 10) such that the area changes with time.

5. A new approach based on the Lagrange polynomials, was also proposed to model the two long transition periods between the three main flow regimes.

6. Combining these and the previously developed PI formulation, a simple, analytical and practical model for forecasting the full lifetime performance of MFHWs was proposed.

7. This analytical models can be used for tasks such as well testing, production performance analyses, forecasting and optimisation of MFHWs design.

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Nomenclature:

- \( B \): Formation volume factor
- \( h \): Formation thickness
- \( \text{HWs} \): Horizontal wells
- \( K_m \): Matrix permeability
- \( K_f \): Fracture permeability
- \( \text{LGR} \): Local grid refinement
- \( K_{mf} \): Matrix permeability
- \( \text{MFHWs} \): Multiple fractured horizontal wells
- \( N \): Original fluid-in-place
- \( P_i \): Initial Reservoir pressure
- \( \bar{P}_r \): Reservoir pressure
- \( Q_g \): Gas production rate
- \( r_e \): Drainage radius
- \( r_w \): Wellbore radius
- \( S_c \): Convergence skin
- \( S_f \): Fracture spacing
- \( W_f \): Fracture width
- \( X_e \): Drainage half-length in \( X \) direction
PI  Productivity index

$Y_e$  Drainage half-length in $Y$ direction

$P_{wf}$  Flowing Bottom-hole pressure

$\mu$  Viscosity of the fluid

References


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2. The presented dimensionless type-curves illustrate the importance of the flow behaviour in the intermediate time, mostly dominated by the compound linear, for characterising the systems, as the early time linear flow and pseudo-steady-state parts overlapped at many different prevailing conditions.

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