Temperature Transient Analysis Models and Workflows for Vertical Dry Gas Wells

Akindolu Dada*, Khafiz Muradov, David Davies

ABSTRACT

High resolution temperature sensors in downhole completions in the last decade has made high quality, transient temperature data available suitable for both qualitative and quantitative analysis. This data availability has stimulated the development of accurate models for the quantitative analysis of temperature transients. There are only a limited number of publications in the area of temperature transient analysis (TTA), the majority of which are limited to liquid production at the wellbore. One reason is that the compressible nature of gas results in a more complex mathematical problem when compared to that for incompressible liquids. The second reason is that more data is available from high precision, downhole temperature sensors installed in oil wells than from gas wells.

This work is the sequel to previous work that derived an analytical solution for the transient sandface temperature of a vertical dry gas producing well (Dada et al. 2017). We discuss the derivation of interpretation models and workflow for estimating the flow characteristics of a dry, gas producing well from transient temperature data. The developed workflow linearizes the analytical equation describing flow into the well from a dry, gas reservoir. It has been successfully applied to both a synthetic and a real well production data set.

The application area of the developed analytical solution is discussed. The two most important of the simplifying assumptions that affect the results concern (1) the impact of a gradual change in the flow rate and (2) non-Darcy inertial flow. Guidelines are developed to determine when the impact of a gradual flow rate change has died away. It was also concluded that the non-Darcy effect had little impact on the transient temperature log-time derivative, the key plot in TTA.

The developed TTA workflow has therefore been validated for many practical TTA applications, as shown by its successful application and validation against conventional pressure transient analysis (PTA) for both synthetic and real-well data sets. TTA’s unique ability to estimate the radius and permeability of a low permeability (formation damage) zone around the wellbore was also validated. This important parameter is not available from PTA. This work represents a further important step towards the development of a comprehensive PTA/TTA data analysis framework for multi-phase production wells.

1 INTRODUCTION

The regular determination of a well’s inflow performance is one of the key well surveillance tasks in production engineering. The well inflow performance depends on the permeability-thickness (kh) product contacted by the completion and the condition of the near-wellbore zone. Estimation of the

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permeability-thickness \((kh)\) and the skin values is one of the most important results from well test analysis.

Another important task of the production engineer involves monitoring the produced fluid phases and their flow rates from the combined well-reservoir system (well production allocation) as required for production optimisation, reservoir management and reporting of well reserves. Flow rate estimation involves quantifying the total volume and phase fraction of produced fluid, while production allocation determine the fraction of the total production contributed by each reservoir zone (or layer). Several methods have been developed for flow estimation, including production logging, permanent downhole flow meters, pressure drop measurement across flow constrictions, multi-rate tests, virtual flow-metering and thermal modelling (both steady state and TTA) (Konopczynski et al. 2003). Thermal modelling has great potential as a low-cost, low-risk method of obtaining this information. Further, the temperature signal propagates at a much slower rate than the traditional pressure signal. This gives TTA the unique advantage of being able to accurately probe the near-wellbore zone or profile the reservoir/inflow properties along the production interval.

High resolution downhole temperature sensors that can resolve small temperature changes for well surveillance purposes were developed in the 1970s (Completions 2008), with fibre optic technology extending the range of possible completion designs. The measurement required for TTA are now available at a reasonable cost.

However, accurate thermal models are essential when predicting the transient temperature change in the reservoir and at the sandface, during TTA. This thermal model is the basis for all analytical or numerical solutions for TTA. It is usually a partial differential equation (PDE) which describes the relationship between the fluid and rock properties and the pressure and temperature changes in the porous media. Derivation of the thermal model used to estimate the transient sandface temperature can be found in e.g. (Weibo Sui et al. 2008), which is itself based on the model by (Bird et al. 2007). This thermal model shows that the measured transient temperature change in porous media is a function of the fluid expansion, Joule-Thomson effect, heat convection and conduction. This, or similar models, were used by (Muradov & Davies 2011), (Duru & Horne 2010) and (Ramazanov et al. 2010) to estimate the transient sandface temperature analytically or numerically. The authors obtained realistic estimates of sandface temperature. (Muradov & Davies 2013) and (Duru & Horne 2010) compared their results obtained from analytical and/or numerical solutions (based on this model) with real well data.

Numerical solutions for TTA are normally used directly in inversion workflows for characterizing a formation, allocating flow rate or carrying out a near wellbore analysis. They are usually case specific, and do not produce a general solution while the analytical TTA solutions are faster and more general, as well as providing valuable insights into the problem. Table 1 lists some of the TTA publications along with their area of application.

<table>
<thead>
<tr>
<th>Author, year</th>
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Table 1: Major findings in TTA research

<table>
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<tr>
<th>Reference</th>
<th>Description</th>
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<tr>
<td>(Onur &amp; Çinar 2016)</td>
<td>Drawdown and buildup in vertical liquid producing well</td>
<td>Analytical / Numerical</td>
<td>Developed analytical transient temperature solutions for liquid drawdown and buildup</td>
</tr>
<tr>
<td>(Chevarunotai et al. 2015)</td>
<td>Vertical oil producing wells</td>
<td>Analytical</td>
<td>Transient temperature solutions for oil producing wells with large drawdowns</td>
</tr>
<tr>
<td>(Muradov &amp; Davies 2013)</td>
<td>Horizontal liquid (oil and water) producing wells</td>
<td>Analytical</td>
<td>Case studies for pressure and temperature transient analysis for liquid producing wells</td>
</tr>
<tr>
<td>(Muradov &amp; Davies 2012)</td>
<td>Horizontal liquid producing wells</td>
<td>Analytical</td>
<td>Workflows for TTA for liquids in horizontal wells</td>
</tr>
<tr>
<td>(Muradov &amp; Davies 2011)</td>
<td>Horizontal liquid producing well</td>
<td>Analytical</td>
<td>Developed analytical solutions for transient sandface temperature in liquid producing horizontal wells</td>
</tr>
<tr>
<td>(App &amp; Yoshioka 2011)</td>
<td>Oil and gas flow in vertical wells</td>
<td>Analytical / Numerical</td>
<td>Effect of Peclet number on flowing temperature change, and relationship to layer properties</td>
</tr>
<tr>
<td>(Duru &amp; Horne 2010)</td>
<td>Single and multiphase (oil and gas) flow, 1D radial flow</td>
<td>Semianalytical (operator splitting)</td>
<td>Semianalytical transient temperature solution, estimation of formation and fluid properties using real and synthetic data</td>
</tr>
<tr>
<td>(Sui et al. 2010)</td>
<td>Multilayer system with gas production</td>
<td>Numerical</td>
<td>Estimated skin, and layer properties in a multi-layered well-reservoir system, by using numerical inversion techniques</td>
</tr>
<tr>
<td>(Ramazanov et al. 2010)</td>
<td>Vertical oil producing wells with thermal wellbore storage</td>
<td>Analytical</td>
<td>Developed analytical solutions for vertical liquid producing wells</td>
</tr>
<tr>
<td>(W. Sui et al. 2008)</td>
<td>Multilayer system with liquid production</td>
<td>Numerical</td>
<td>Testing workflow for pressure and temperature in multi-layered system. Estimated skin and layer properties</td>
</tr>
</tbody>
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The estimation of zonal production rates, formation properties and the identification of the produced phases requires inversion of the forward TTA solution. This inversion is easy and fast for analytical solutions; but is slower for numerical solutions as it requires some form of optimization to minimize an objective function. (Sui et al. 2010), for example, performed the inversion by nonlinear regression using the Levenberg-Marquardt algorithm. Another advantage of an analytical solution for a system is that it explicitly describes the nature of the system’s behaviour and how its parameters relate and affect the system’s response. However, most of the analytical solutions and inversion methods developed to-date refer to liquid producing wells. This paper reports the work carried out to characterize a dry gas producing reservoir by development of inversion workflows for TTA data.

We previously developed analytical solutions for predicting the transient sandface temperature in a vertical dry gas producing well (Dada et al. 2017). We now use this solution for characterizing a reservoir; i.e. by determining the permeability-thickness product and skin. This method is used in conjunction with the well-developed PTA workflow. The combination of PTA and RTA (Rate Transient Analysis) is further used to validate the results obtained from TTA of a real-well data.
The following sections show how we linearize the analytical solution (in log-time scale) for vertical dry gas producing wells (Dada et al. 2017). This linear form of the equation is then used for characterizing the reservoir and analysing the near wellbore reservoir properties. The limits of our method stemming from our assumption of laminar flow are discussed. Our method was further successfully applied to both synthetic and real case studies, showing that it can be applied to many field situations.

2 ANALYSIS OF THE PROBLEM

Eqn. 1 is the analytical solution for the sandface temperature of a vertical dry gas well producing at a constant rate after a period of shut-in (or a step change in the flow rate). Eqn. (1) was derived for a dry gas well producing at a constant, non-zero rate and with an infinite acting boundary condition. Appendix C provides a brief description of how the solution was derived and the assumptions used in its derivation.

\[ T_{wb}(t) - T_i(t) = \varepsilon \left[ P_{(r=r_f)} - P_{wf}(t) \right] + \eta^* e^{(-2aU_o)} \left[ P_{wf}(t) - P_i \right] \]  

(1)

(The terms are listed in the Nomenclature at the end of the paper).

Note: Eqn. (1) is normally used to describe a “drawdown” test where the production rate is instantaneously increased from one constant value to a second, higher value. The derivation often assumes a zero initial rate; describing the case when well production starts after a shut-in. However, the solution is also applicable to any rate change as long as the initial temperature term in Eqn. (1) is accurate and the final flow rate is non-zero. This covers a well being placed on production and a positive or negative flow rate change, as long as the well is not shut-in. The full analytical solution for a well shut-in, or “build-up” test, is not currently available.

Gas properties are strongly temperature and pressure dependent. However, their combinations that appear in Eqn. (1) may be assumed to be constant (Dada et al. 2017) for thermal analysis. They are estimated at the initial temperature and the average pressure “P_avg” (midway between the initial wellbore pressure and the final, stabilized wellbore pressure). Note this assumption is not valid for the equivalent pressure solution for a gas well since gas properties used in the pressure model are very sensitive to the changes in pressure observed in field application. That is why an accurate, classical gas well pressure solution is used in this work [pressure is part of thermal Eqn. (1)] as far as the pressure model is concerned.

Eqn. (1) shows that the temperature change is a combination of the Joule-Thomson effect (term 1 on RHS) and the transient fluid expansion (term 2 on RHS) plus the heat convection term due to the resulting temperature gradient. Also, \(2aU_o \ll 1\) for most practical purposes, therefore \(e^{(-2aU_o)} \approx 1\).

The above allows Eqn. (1) to be reformulated as:

\[ T_{wb}(t) - T_i(t) = \varepsilon \left[ P_{(r=r_f)} - P_{wf}(t) \right] + \eta^* \left[ P_{wf}(t) - P_i \right] \]  

(2)

The pseudo-pressure method developed for gas wells by (Al-Hussainy et al. 1966) was used to obtain the pressure solution. A linear pressure - pseudo-pressure relationship [Eqn. (3)] is sufficiently accurate for our well production conditions. It can be combined with the line source solution and the
logarithmic approximation for an infinite acting reservoir producing at a constant rate (Eqn. (4))(Al-Hussainy et al. 1966).

\[ P = A + B\psi \]

\[ \psi = \psi_0 - \frac{psi_0}{2} [\gamma + \ln \left( \frac{\mu c r^2}{4\lambda kt} \right)] \]

Where “\( \gamma \)” is the Euler-Mascheroni constant.

The constants “A” and “B” in Eqn. (3) are case-specific and should be found by matching \( P \) and \( \psi \) for a given fluid composition. They can be obtained from PVT measurements of the produced fluids or calculated from its Equation-of-State as explained and illustrated in (Dada et al. 2017).

\[ \therefore T_{wb}(t) - T_i(t) = -eB \frac{psi_0 Qd}{2} \left[ \ln \left( \frac{r_w^2 + 2U_0t}{r_w} \right) \right] - \eta^* B \frac{psi_0 Qd}{2} \left[ \gamma + \ln \left( \frac{\mu c r^2}{4\lambda kt} \right) \right] \]

Eqn. (5) can be expressed explicitly as a function of time, as shown in Eqn. (6)

\[ T_{wb}(t) = T_i - \frac{BITQ_{sc}}{2kh} \left( 1 - \beta_T \frac{T}{C_p} \left[ \ln \left( \frac{r_w^2 + 2U_0t}{r_w} \right) \right] \right) + \left( \frac{\beta_T}{C_p} e^{\left( \frac{2nC_p B^2 T Q_{sc}}{r_w^2} \right) t} \right) \left[ \gamma + \ln \left( \frac{\mu c r^2}{4\lambda kt} \right) \right] \]

A plot of the transient temperature calculated from Eqn. (5) w.r.t log time is initially a curve followed by a linear portion (see Fig. 1(a), taken from the case study described in Appendix B). An equation that accurately represents the linear portion of Eqn. (5) (Fig. 1(a)) is obtained by plotting the two terms in Eqn. (5) – i.e. fluid compression and Joule-Thomson effects - on a logarithmic scale (Fig. 1(b)) to determine when each term is dominant.

Mathematical speaking, the initial nonlinear behaviour is due to the term \( \ln \left( \frac{r_w^2 + 2U_0t}{r_w} \right) \) at early times. However, the derivative of \( \ln \left( \frac{r_w^2 + 2U_0t}{r_w} \right) \) approaches that of \( \ln(t) \) as time increases. Its value can be approximated by Eqn. (8) (see Appendix D for details).

For a time \( t \geq t_j \)
∴ \( t_j = \frac{r_w^2(100+\delta)}{2U_0\delta} \) \hspace{1cm} (7)

\[
\ln\left(\frac{r_w^2+2U_0t}{r_w^2}\right) \cong \ln(t) + \ln\left(\frac{200U_0\delta}{r_w^2(100+\delta)}\right) \hspace{1cm} (8)
\]

A suitable accuracy of the \( \delta \) term above is taken as 5%.

Eqn. (9), obtained by substituting Eqn. (8) into Eqn. (5), describes the linear portion of Eqn. (5).

\[
T_{wb}(t) = T_i(t) - \varepsilon B \frac{\psi_i Q_d}{2} \left[ ln(t) + \ln\left(\frac{200U_0\delta}{r_w^2(100+\delta)}\right) \right] - \eta^* B \frac{\psi_i Q_d}{2} \left[ \gamma + \ln\left(\frac{\phi \mu c \tau_w^2}{4\lambda k t}\right) \right] \quad t \geq t_j
\hspace{1cm} (9)
\]

where \( Q_d = \frac{GTQ_{sc}}{kh \psi_i} \)

Eqn. (10) will now be used to develop workflows for characterizing transient temperature data.

\[
T_{wb}(t) = T_i + \frac{B T i Q_{sc}}{2kh} \left[ \eta^* - \varepsilon \right] ln(t) - \frac{B T i Q_{sc}}{2kh} \left[ \eta^* ln\left(\frac{\phi \mu c \tau_w^2}{4\lambda k}\right) + \varepsilon ln\left(\frac{200U_0\delta}{r_w^2(100+\delta)}\right) + \eta^* \gamma \right]
\hspace{1cm} (10)
\]

Where:

\[
\tau_T = \sqrt{r_w^2 + 2U_0 t} ; \quad \alpha = \frac{\phi \mu c}{4\lambda k} ; \quad U_0 = C v(r, t) r ; \quad u(r, t) = \frac{k d p}{\mu d r} ;
\]

\[
C = \frac{C_p \rho}{C_t} ; \quad C_t = \rho C_P = \phi C_p \rho + (1 - \phi) C_p r \rho ; \quad \varepsilon = \frac{1 - \beta T_i}{C_p \rho} ;
\]

\[
\eta^* = \eta e^{-2\alpha U_0} ; \quad \eta^* = \phi C \eta ; \quad \eta = \frac{\beta T_i}{C_p \rho}
\]

3 RESERVOIR CHARACTERIZATION AND NEAR-WELLBORE ANALYSIS

Eqn. (11) is the gradient of the linearized form of the transient temperature solution on a semi-log scale, Eqn. (10) and Eqn. (12) is the intercept. The semilog slope and intercept of the TTA signal can be estimated by fitting a straight line (of the form “\( T = a \ln(t) + b \)” to the TTA signal (Fig. 2)).

![Plot of Numerical Solution](image1)

**Figure. 2 (a) Plot of numerical transient wellbore temperature**

- **slope** \( a = \frac{B T i Q_{sc}}{2kh} \left[ \eta^* - \varepsilon \right] \)

- **intercept** \( b = T_i(t) - \frac{B T i Q_{sc}}{2kh} \left[ \eta^* \ln\left(\frac{\phi \mu c \tau_w^2}{4\lambda k}\right) + \varepsilon \ln\left(\frac{200U_0\delta}{r_w^2(100+\delta)}\right) + \eta^* \gamma \right] \) \hspace{1cm} (12)
Eqn. (11), the slope of Eqn. (10), determines the permeability-thickness “kh” and the rate “Q”, while the intercept (Eqn. (12)) evaluates the permeability “k”. Unfortunately, determining the permeability from the intercept is susceptible to the same large errors as observed in PTA (see Chapter 4).

3.1 Workflow for Estimating Permeability-thickness or Rate

The value of the permeability-thickness or the production rate is estimated as follows:

1. Calculate the values of the fitting coefficient “B” from the PVT data or an appropriate Equation-of-State.
2. Determine “ψ_i”, the pseudo-pressure at initial reservoir conditions.
3. Determine “η” & “ε” from Eqn. (10) at \( T_i \) and \( P_{avg} \).
4. Identify the linear portion of the transient temperature profile.
5. Optional: Estimate the value of the prediction uncertainty “\( \Delta \)” (Eqn. 7) using the value of “\( t_j \)” (step 4) and “\( U_0 \)” (Eqn. (10)).
6. Determine the slope of the transient temperature data from the semi-log plot.
7. Calculate \( Q_{sc} \) or \( kh \) from Eqn. (11). \( Q_{sc} \) is calculated if an estimated value of \( kh \) is available OR \( kh \) may be calculated if \( Q_{sc} \) is measured.
8. Optional: An approximate value of \( k \) may also be estimated by substituting either \( Q_{sc} \) or \( kh \) in Eqn. (12) after determining the intercept of the straight line fitted to the linear portion of the transient temperature signal (following steps 1 to 8 above).

3.2 Workflow for Near-Wellbore Analysis

The thermal radius of investigation, \( r_T \) (Eqn. (13)), is the distance the temperature signal has travelled in the reservoir at the velocity of convective heat transfer (Ramazanov et al. 2010).

\[
r_T = \sqrt{(r_w^2 + 2U_0t)}
\]

(13)

The permeability and the radius of the damage zone can be determined from TTA. The damage zone permeability \( k_{skin} \) is solely responsible for the temperature response at early times when the thermal radius of investigation \( r_T \) is confined to the damage zone. The damage radius, \( r_d \), is determined from Eqn. (13) once “\( t_d \)”, the transition time at which the transient temperature signal slope changes due to a change in permeability, is identified (see Fig. 3(a)).
Figure 3 (a) Plot of transient temperature showing transition time. (b) Plot of curvature of transient temperature showing local maximum at transition time.

The procedure for carrying out near-wellbore analysis is shown below. This is similar to the workflow for estimating $kh$ for the virgin formation, but it also estimates the damage radius.

1. Calculate the values of the fitting coefficient “B” from the PVT data or from appropriate Equation-of-State.
2. Determine “$\psi_1$”, the pseudo-pressure at initial reservoir conditions.
3. Determine “$\eta^*$” & “$\varepsilon$” from Eqn. (10) at $T_i$ and $P_{\text{avg}}$.
4. Identify the linear portions of the transient temperature profile and estimate the transition time “$t_d$” corresponding to their intercept.
5. Optional: Estimate the value of “$\delta$” {Eqn. (7)} with the value of “$t_j$” from step 4 and “$U_0$” {Eqn. (10)}.
6. Determine the slope of the transient temperature data between “$t_j$” and “$t_d$”. This slope relates to the damage zone.
7. Evaluate the rate $Q_{sc}$ if the value of $k_{\text{skin}}h$ is known from Eqn. (11) and the slope calculated in step 6. Alternatively, estimate $k_{\text{skin}}h$ if the value of $Q_{sc}$ is known.
8. Calculate the damage radius $r_d$ from the value of $t_d$ calculated in step 4 and Eqn. (13).
9. Optional: Estimate the damage skin from the Hawkins formula if the “$kh$” of the damaged and virgin formation’s and the damage radius have been calculated.

4 SENSITIVITY TO ERRORS

4.1 Errors in Measured Temperature Data

The temperature measurement used to determine the rate, permeability-thickness ($kh$), damage permeability and damage radius will be subject to some degree of error. We have therefore carried out a sensitivity study to determine how an error of up to ±10% in the measurement translates into an error in the value of the slope and hence the values of the derived parameters.

1. The error of estimating $kh$ and/or measuring $Q_{sc}$ are almost linearly related to error in the slope (for small errors in the slope) e.g. between 0 and ± 10% {Fig. (4a) and Eqn. (14)}. The sensitivity of the temperature sensor, rather than its absolute accuracy, is thus the parameter that determines the accuracy of the $kh$ value.
2. The error of estimating $k$ alone using secondary logarithms in Eqn. (10) increases exponentially with the error in the intercept {Fig. (4b) and Eqn. (16)}. Errors of −0.001% or +0.001% in the intercept result in errors of -14 and +16 % in the estimated permeability. Determination of the intercept is also subjected to increased errors because it involves extrapolation from the measured results. Systematic sensor errors (e.g. drift) further increase the error, so the direct estimation of $k$ from Eqn. (10) is not recommended. This is normally not a problem, because $k$ is (traditionally) best determined from the $kh$ value if the formation thickness is known from well logs.
3. The error in damage permeability (or permeability-thickness) is similar to that described in “1” and “2” above. This is because the estimation of permeability-thickness $kh$ is similar to that of the virgin formation, albeit, using a different portion of the transient temperature signal. The distinct source of error in near-wellbore analysis is from the estimation of the
transition time “\( t_d \)”, and this depends on the sensitivity of the gauge and the temporal resolution used in the measurement. The higher the temporal resolution the more accurate the value of “\( t_d \)”. 

\[
\Delta k = \frac{BTTQ_{sc}}{2\Delta a} (\eta^* - \epsilon)
\]  

(14)

\[
\Delta Q_{sc} = \frac{2kh\Delta a}{BTT(\eta^* - \epsilon)}
\]

(15)

\[
\Delta k = \frac{\phi\mu c_T T_i}{4} \exp - \left[ \left( \frac{(T_i - \Delta b)2kh}{\eta^*TTQ_{sc}} \right) - \frac{\epsilon}{\eta^*} \ln \left( \frac{200U_o \delta}{T_i^2(100+\delta)} \right) - \gamma \right]
\]

(16)

Where “\( a \)” is the slope and “\( b \)” is the intercept.

![Figure. 4 (a) Plot showing sensitivity of permeability-thickness and rate estimation to errors in the slope. (b) Plot showing sensitivity of permeability alone estimation to errors in the intercept.](image)

4.2 Errors in Other Input Parameters

The analysis carried out using the workflows described above depends on inputs from several sources, e.g. sensor measurements for transient temperature, pressure and rate data, PVT lab reports or correlations for fluid properties, well logs or well test data for formation properties. All such measurements have some degree of uncertainty associated with them. It is important to quantify how the uncertainties in the inputs translate into the results of our TTA.

The effect of the uncertainty in the thermal properties, the Joule-Thomson coefficient, “\( \epsilon \)”, and the adiabatic expansion coefficient, “\( \eta^* \)”, on the estimation of \( kh \) and flow rate is described by Eqn. (17) and Eqn. (18) respectively. These thermal properties are themselves a function of other fluid and formation properties. The Fig. (5) spider plot illustrates the sensitivity of the various input parameters on the estimated values of \( kh \) and \( Q_{sc} \). The initial temperature, \( T_i \), and the thermal expansion coefficient, \( \beta_T \), have the greatest impact on the \( kh \) and \( Q_{sc} \) estimates. The gas density, \( \rho \), gas specific heat capacity, \( C_p \), pressure-pseudo pressure slope, \( B \), and TTA slope, \( a \), have a lower, linear effect on estimated values of \( kh \) and \( Q_{sc} \). The rock density, \( \rho_r \), rock specific heat capacity, \( C_{pr} \), and porosity, \( \phi \), have a negligible impact.
Figure. 5 (a) Plot showing sensitivity of permeability-thickness to different input parameters. (b) Plot showing sensitivity of rate to different input parameters.

\[
slope (a) = \frac{BT_iQ_{sc}}{2kh} [\eta^* - \epsilon]
\]

\[
k h = \frac{BT_iQ_{sc}}{2a} [\eta^* - \epsilon]
\]

\[
Q_{sc} = \frac{2akh}{BT_i[\eta^* - \epsilon]}
\]

The Monte-Carlo method and the linearized form of the analytical equation (the semi-log slope, "\(a\)", from (Eqn.11)) could also be used to prepare an uncertainty analysis. A random sample is chosen from a normal distribution of \(\epsilon\) and \(\eta^*\) and the distribution of the estimated \(kh\) values calculated, as described in chapter 5.3.2.

4.3 The Effect of a Gradual Change in the Flow Rate

The change in the sandface production rate will not always be instantaneous, despite our TTA solutions assuming a step-like change in the flow rate. This assumption is regularly violated by:

1. Gradual opening or closing a valve or choke.
2. Wellbore storage effects when the well is controlled by a surface choke.

These effects last between a few minutes and hours. Example 1 can be minimized by operating the valve at the highest allowed rate while example 2 cannot be changed and is case specific.

The comprehensive numerical model of mass and heat transfer around a wellbore [described in(Dada et al. 2017)] was used to generate the data required for studying the variable rate effect. The variable rate effect (Fig. (6)) produces a transient temperature signal which has a similar appearance to the skin effect. This presents difficulties when carrying out a near-wellbore analysis, making it necessary to differentiate whether a particular feature is caused by a rate variation, the skin effect or a combination of both. Fig. (6) shows that the transient temperature signal returns to the base (step rate change) value after a time period that is proportional to the duration of the rate change. The time required for measuring data suitable for carrying out a meaningful TTA depends on the rapidity with which the change to the sandface flowrate is achieved.
Fig. 7(a) shows the relative change in transient temperature derivative (Eqn. 19) for different ramp-up times (the time required for the change in sandface rate to occur). The relative differences between the derivatives compared to that for a step rate change reduces with time. Fig 7(b), a plot of the settling time (the time it takes for the relative error in the slope to reduce to 5%) , shows that it is ~ 12 times greater than the ramp-up time. An allowable relative error of 5% was chosen since the TTA is more sensitive to errors in other parameters, such as the PVT properties and surface rate measurement.

\[
\text{rel. error} = \frac{\frac{dT}{d\ln(t)}_{\text{ramp}} - \frac{dT}{d\ln(t)}_{\text{step}}}{\frac{dT}{d\ln(t)}_{\text{step}}}\tag{19}
\]
It was observed that the slope of the plots in Fig. 8(a) showed a downward trend with increasing $\Gamma_{TQc, TWkhSG}$. The settling time (for the ramp-up effect to stop masking the step-like rate change solution) can be estimated by using this trend (Fig. 8(b)) if an estimate of $\Gamma_{TQc, TWkhSG}$ is also available. Note that Fig. 8(b) refers to the specific gas properties chosen, but a similar trend is observed for other gas properties (Fig. 8(c)).

The settling time in a particular case can be quantified from the product of $\frac{dt_{settling}}{dt_{ramp}}$ (read off the y-axis of Fig. 8(b)) and $t_{ramp}$. More qualitatively, Fig. 8(b) indicates that the settling time of the TTA slope is less than 15 times the ramp-up time for the example chosen. Therefore, TTA requires that:

1. The duration of data measurement must be sufficiently long that the transient temperature signal has unambiguously (within a relative error < 5%) returned to the base value.
2. Investigation of the near-wellbore skin effect requires that the ramp-up time must be short enough for the transient signal to return to the base value within the time it takes for the transient temperature disturbance to travel out of the near-wellbore region. This time can be estimated with Eqn. (13).
Note that the “Ramp-Up Time” as used here refers to the time for the sand face flow rate to stabilise. This may be very different from the actual stroke time required to move a surface choke.

4.4 Limitations Due to Non-Darcy Effects

Our analytical solution assumes the gas flow obeys Darcy’s law. However, inertial effects which lead to the gas flow deviating from Darcy’s law at the higher velocities are often observed in the field. (Sui et al. 2010) included this non-Darcy effect in their numerical models while the analytical Forchheimer’s equation [Eqn. (20)] adds an additional pressure drop term \( \beta \rho |v|v \). The dimensionless number \( r_{ND} \) represents the relative importance of the non-Darcy effect (the ratio of the pressure gradients due to the non-Darcy and the Darcy flow). Note that \( r_{ND} \) is defined here in terms of the gas flow rate measured at standard conditions.

\[
\begin{align*}
    r_{ND} &= \frac{\beta \rho |v|k}{\mu} \\
    r_{ND} &= \frac{Q_{sc} \beta}{2 \pi r_w k_w} \frac{\beta k}{\mu}
\end{align*}
\]

\( r_{ND(crit)} \) is the critical non-Darcy ratio at which the resulting errors are still acceptable compared to the errors in other input parameters. An acceptable error of 5% in the TTA slope is suggested. The corresponding critical Darcy velocity \( v_{(crit)} \) [Eqn. (23)] and critical Darcy surface rate \( Q_{sc(crit)} \) [Eqn. (24)] below which the non-Darcy effects can be neglected are:

\[
\begin{align*}
    |v_{(crit)}| &= \frac{\mu r_{ND(crit)}}{\beta pk} \\
    Q_{sc(crit)} &= \frac{\mu r_{ND(crit)} 2 \pi r_w k_w}{\beta \rho_s k}
\end{align*}
\]

Our analysis methods may thus be confidently applied to velocities or surface flow rates smaller than \( v_{(crit)} \) and \( Q_{sc(crit)} \) respectively. The valid application area of our TTA methodology is well specific. It depends on the completion geometry and the reservoir properties. An accurate estimation of \( Q_{sc(crit)} \) also depends on a good knowledge of the value of \( \beta \). (Wang & Economides 2009) have published a compilation of non-Darcy coefficient correlations.

An approach that will be useful to well surveillance engineers is to calculate the specific critical surface flowrate by dividing \( Q_{sc(crit)} \) by the length of the well’s completion or the thicknesses of the reservoir sand. This term can then be applied to different wells within the same field/reservoir.

\[
Q_{scn(crit)} = \frac{Q_{sc(crit)}}{2 \pi r_w k_w} = \frac{\mu r_{ND(crit)}}{\beta \rho_s k}
\]

Alternatively, the effect of non-Darcy flow on transient temperature can be investigated by considering the relationship between \( r_{ND} \) and the additional transient temperature drawdown due to non-Darcy flow. The dimensionless number \( T_{ND} \) is the ratio of the additional temperature drawdown due to the non-Darcy flow effect to the temperature drawdown due to Darcy flow.
\[ T_{nD} = \frac{T_w(Darcy) - T_w(non-Darcy)}{T_i - T_w(Darcy)} \]  
\[ P_{nD} = \frac{P_w(Darcy) - P_w(non-Darcy)}{P_i - P_w(Darcy)} \]  

(26a)  
(26b)

Fig. 9 illustrates the effect of non-Darcy flow on the transient well temperature and pressure for the case study described in Appendix B. It clearly shows that the non-Darcy effect cannot always be neglected during TTA as it is responsible for between 10% - 30% of the measured temperature change. Application of our analytical solutions with a reasonable accuracy therefore requires verification that the non-Darcy effect is either negligible or can be corrected for.

It is important to note that in all cases the effect on the slope of the transient temperature signal due to non-Darcy flow is relatively small. Hence our TTA methodology (of estimating \(kh\) from the slope) is valid in many practical applications, even if the critical rate is by far exceeded. For example, Fig. 10(b) shows that the error in the slope of the transient temperature is about one order of magnitude less than the value of \(r_{nD}\).
We also studied the non-Darcy effect on the transition time between the damaged and virgin formation. The Appendix B case study with the addition of a damaged zone of reduced permeability was used. Fig. 11 shows that the transition time is independent of the non-Darcy effect despite there being a significantly greater change in temperature in the non-Darcy case (Fig. 11(b)).

![Figure 11: (a) Plot of transient temperature showing the transition time for pure Darcy flow](image)

![Figure 11: (b) Plot of transient temperature showing the transition time for non-Darcy flow](image)

This means we can determine the damage radius using the Darcy’s law TTA solution without loss of accuracy due to the non-Darcy effect. The transition time was also shown to be independent of the value of $\beta$.

## 5 CASE STUDIES

### 5.1 A Synthetic Well Model

This synthetic case history concerns the application of TTA to constant rate production from a vertical dry gas production well. The TTA data [Fig. 12(a)] was generated by a numerical simulation model based on the Appendix B data (full details can be found in Appendix E). Two of these case studies are discussed in this section, refer to Appendix E for details of the other case studies.

![Figure 12 (a) Transient temperature prediction with the linear interval and slope highlighted. (b) Transient temperature with slope of damage region and clean formation indicated](image)
5.1.1 Estimating the Rate and Permeability Thickness

Fig. 12(a) is a semi-log plot of the transient, downhole temperature versus time (Case study 1, full details in Table E1 and E2 of Appendix E1). The first and last data points (blue stars) of the well-fitted ($R^2 = 0.9998$) straight line are $t_j = 1.5\ hr$ and $t_s = 120\ hr$ respectively. The analysis can thus be applied a few hours after the rate change. $t_j$ can be estimated using Eqn. (7) for different test conditions, but a short duration well test not only requires a low value of $t_j$, but also requires good quality data (lack of noise etc.). However, a longer test period will normally ensure a more accurate value for the slope.

The estimated values of either $kh = 380\ mD.\ ft$ or Flow Rate $= 7.37\ MMScf/day$ are within 5% of the Appendix B input values of $kh = 390\ mD.\ ft$ for and Flow Rate $= 7.02\ MMScf/day$.

5.1.2 Estimating the Magnitude of a Near-Wellbore Damaged Zone

A near-wellbore formation damage zone with a 50% reduction in permeability for a distance of 2.89 ft from the wellbore wall was added to the Appendix B numerical model (Case study 1, full details given in Table E3 and E4 of Appendix E2) i.e. $r_d = 3.28\ ft$ and $k_{skin}h = 195\ mD.\ ft$. TTA using the skin estimation workflow (Chapter 3.2) utilising the temperature derivatives and the thermal investigation radius, gave estimates within 6% of the above input values {estimated $k_{skin}h = 202.4\ mD.\ ft$ and estimated $r_d = 3.08\ ft$}.

Fig. 13(a) illustrates the sensitivity of the transient temperature signal for formation damage radii of 1.5 ft and 3.0 ft. The radius of the formation damage, the time at which the slope of the transient temperature changes, is even more conspicuous in the derivative plot (Fig. 13(b)).

The TTA signal increases as the level of formation damage increases. This case history illustrates how the TTA signal travels through the formation damage zone at a rate that is several orders of magnitude smaller (4.2 hr) than the corresponding pressure signal (10 s). This slow transmission of the transient temperature signal gives TTA its unique ability to recognise permeability changes in the near wellbore formation; information which cannot be obtained from PTA.
### 5.2 Analysis of Real-Field Data

We will now apply the workflows developed in this paper to transient temperature downhole data recorded in a real, vertical, gas producing well (Fig. 14 and Table 2) producing ~100 MMscf/day. A permanent downhole gauge installed some distance above the producing layer measures pressure, temperature and surface production rate every 30 minutes. There are 3 well start-ups during the 3½ month data acquisition period (Fig 14). Two usable drawdown periods (highlighted in red) were identified from the raw, production measurements.

![Rate against time](image1.png)

**Figure. 14** Surface production rate and downhole temperature

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Viscosity $\mu$ at $P_{avg}$ &amp; $T_i$ [cP]</td>
<td>0.01373</td>
</tr>
<tr>
<td>Specific gravity $s. G$</td>
<td>0.605</td>
</tr>
<tr>
<td>Thermal expansion coefficient of gas $\beta_T$ at $P_{avg}$ &amp; $T_i$ [$^{o}F^{-1}$]</td>
<td>0.0044</td>
</tr>
<tr>
<td>Specific heat capacity of gas $\beta_T$ at $P_{avg}$ &amp; $T_i$ [Btu/(lbf °F)]</td>
<td>682.14</td>
</tr>
<tr>
<td>Density of gas $\rho$ at $P_{avg}$ &amp; $T_i$ [lbm/ft$^3$]</td>
<td>3.995</td>
</tr>
<tr>
<td>Porosity of formation $\phi$</td>
<td>0.15</td>
</tr>
<tr>
<td>Specific heat capacity of the formation rock $\rho_r$ at $P_{avg}$ &amp; $T_i$ [Btu/(lbf °F)]</td>
<td>219.74</td>
</tr>
<tr>
<td>Density of formation rock at $P_{avg}$ &amp; $T_i$ [lbm/ft$^3$]</td>
<td>156.07</td>
</tr>
</tbody>
</table>

**Table 2:** Gas and formation properties (estimated at 1370 psi and 141.3°F)

TTA assumes a constant flowrate before and after the step rate change in question. The useable drawdown periods were selected based on their having a preceding period of constant flowrate (a well shut-in) and a sufficiently long drawdown flow period, so the effects of wellbore warm-up die-out while still providing sufficient data for TTA.

The transient temperature signal (Fig. 15(a)) shows an initial warmup period during which the sensor temperature increases as hotter produced fluid arrives at its location above the producing zone. This warm-up effect dies-out after 6 hrs, after which the signal shows the same behaviour as observed in the ideal model case studies, a linear slope on a semilog plot.
We showed earlier that, the effect on the slope of the transient temperature signal due to non-Darcy flow is relatively small even when Darcy’s law is not valid. \( r_{nD} \), the ratio of the additional pressure drop due to the non-Darcy effect to the Darcy pressure drop, describes the importance of this effect. (Geertsma 1974) illustrated how to determine \( r_{nD} \) from Eqn. (22).

The non-Darcy effect results in underestimating \( kh \) because it increases the slope of the TTA signal. Numerical simulation indicated that the non-Darcy effect on the value of the TTA slope is approximately one tenth of the value of \( r_{nD} \). The value of \( r_{nD} \) was found to be about 0.5 for this case, implying an error of 5% in the slope of the transient temperature signal. The relative increase in slope due to the non-Darcy effect is thus relatively small in this case. Hence, we can apply a “Darcy-based” workflow with sufficient accuracy for this illustration of TTA.

The accuracy of the \( kh \) value estimated from TTA data also depends on both the methodology used to choose the data points and the resulting number of data points available for analysis. We therefore examined the impact (Fig. 15(b)) on the results from this real field case of two procedures for selecting the data points based on the following criteria:

- Have the transient effects from the initial wellbore warmup and varying rates died-out before the first data point is selected?
- Is the flow still in the infinite acting regime?
- Is the transient temperature in the linear region and Eqn. 9 is applicable?

**Method 1 (a fixed number of fitting points and a variable end offset):** The number of selected data points were kept constant and the data points are offset from the last data point. The slope is calculated for each offset.

**Method 2 (zero end offset and a variable number of fitting points):** the selected data points are not offset, but the number of selected data points is increased and the slope is calculated in each case.

Fig. (16) and Table 3 summarise the results from the TTA of the mean permeability-thickness product and its variance using a different number of data points. We also repeated the analysis with a fixed number of data points while offsetting the data points (i.e. changing the data points used for the analysis). The results of this study are shown in Fig. (17) and Table 4.

The results obtained from the two methods are similar with a standard deviation of less than 10% from the mean value of the estimated permeability-thickness. However, the results from drawdown
period 1 have a lower standard deviation, probably because the linear portion of the transient temperature signal is longer (~ 10 hr versus ~ 8.5 hr). This demonstrates the potential of using TTA for gas well and reservoir characterization. Alternatively, if kh is known, this method can be used to estimate the production rate from one or more layers by using the transient sandface temperature measurement.

![Graphs](image)

(a) drawdown period 1  
(b) drawdown period 2  

Figure. 16: Plots of surface production rate, downhole transient temperature and fitted lines for different offsets and 12 data fitting points

<table>
<thead>
<tr>
<th>Production rate (MM scf/day)</th>
<th>Drawdown 1</th>
<th>Drawdown 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimated mean kh (mD ft) with 12 fitting points</td>
<td>34,100</td>
<td>35,300</td>
</tr>
<tr>
<td>(Incremental offset from 1 to 5 data points)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standard deviation of estimated kh (mD ft) using 12 data points</td>
<td>± 1,690</td>
<td>± 3,430</td>
</tr>
</tbody>
</table>

Table 3. TTA estimated kh for drawdowns 1 and 2

<table>
<thead>
<tr>
<th>Production rate (MM Scf/day)</th>
<th>Drawdown 1</th>
<th>Drawdown 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimated mean kh (mD ft) with zero offset</td>
<td>32,000</td>
<td>36,500</td>
</tr>
<tr>
<td>(using 12 to 17 data points)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standard deviation of estimated kh (mD ft) using 12 data points</td>
<td>± 795</td>
<td>± 2210</td>
</tr>
</tbody>
</table>

Table 4. TTA estimated kh for drawdowns 1 and 2

(a) drawdown period 1  
(b) drawdown period 2  

Figure. 17: Surface production rate and downhole transient temperature for different numbers of data points, zero offset
5.3 Validation of TTA Results

The above results from the TTA workflow have been compared with those from both pressure and rate transient analysis (RTA) using the Fig. 18 data.

5.3.1 Case Study No. 2: Pressure Transient Analysis

There are no distinct pressure build-up periods in the measured downhole pressure data (presumably because of non-ideal well shut-in); hence it is only possible to use the drawdown periods, the same ones as used for TTA. Our PTA in a gas well uses the pressure drawdown solution for infinite acting radial inflow (Eqn. 4 or equivalent pressure drawdown solution). Eqn. 27 describes the semilog slope for the pressure. The PTA results for this particular case are quite inaccurate.

\[
\frac{dP_{wf}(t)}{d[\ln(t)]} = m = \frac{BITTQ_{sc}}{2kh}
\]  

(27)
The value of $kh$ was also estimated with commercial software using the same input data.

The results obtained from TTA, PTA (Fig. 19 and Table. (5)) and a more suitable RTA (rate-transient analysis) {Table. (6)} are close and are summarised in Table. (7), though the estimate from drawdown 2 using PTA deviates from the other results. Also note that the TTA estimates are consistently lower than those from PTA and RTA, possibly due to the previously explained increasing (ramp-up) rate effect leading to an underestimate of the $kh$ and an overestimation of the flow rate. The $kh$ estimates from the two drawdown periods are more consistent with TTA, while PTA shows the largest discrepancy. The discrepancy between the two drawdowns in the PTA is due to the fact that the second drawdown is not starting from stabilized pressure. This can be verified from Fig. (18); which shows a gradual reduction in pressure (despite the surface choke being shut) before the start of the second drawdown event. This decreasing pressure trend during the shut-in periods is
probably due to the gas trapped in the wellbore loosing heat into the cooler surroundings and thus contracting. This results in the pressure change that can have two possible effects; on one hand it masks the ‘ideal’ buildup pressure data. On the other hand, cooling of the gas column in the wellbore leads to contraction which can result in some afterflow, which in turn affects the drawdown analysis after the shut-in period. A similar decreasing temperature trend has been observed by (Izgec et al. 2007), they recommended placing the gauge close to the sandface to ensure sufficient data quality.

<table>
<thead>
<tr>
<th></th>
<th>Drawdown 1</th>
<th>Drawdown 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimate of kh from TTA [mD. ft]</td>
<td>34,100</td>
<td>35,300</td>
</tr>
<tr>
<td>Estimate of kh from PTA [mD. ft]</td>
<td>48,649</td>
<td>232,564</td>
</tr>
<tr>
<td>Estimate of kh from RTA [mD. ft]</td>
<td>40,900</td>
<td>55,400</td>
</tr>
</tbody>
</table>

Table 7. Estimated kh for drawdowns 1 and 3 obtained using TTA, PTA and RTA

### 5.3.3 Uncertainty Estimation

The sensitivity of kh estimation to different input parameters has been considered in a previous section. It was observed that the thermal expansion coefficient, \( \beta_T \), has the highest impact on the estimation of \( k_h \) while gas density, \( \rho \), the gas specific heat capacity, \( C_p \), the pressure-pseudo pressure slope, \( B \), and the TTA slope, \( a \), rank after \( \beta_T \) in terms of importance. It can be observed from Eqn. (17) that the fluid properties are all lumped into 2 main terms, “\( \varepsilon \)” and “\( \eta^* \)”. The impact of the uncertainty in these two properties has been investigated using the Monte-Carlo method.

The uncertainty in the value of the formation averaged adiabatic coefficient and Joule-Thomson coefficient was investigated by creating normal distributions with a standard deviation of 5% around the initial temperature and the average pressure (see Table 8 and a plot of the probability density function, Fig. (20)).

\[
\mu_{\eta^*} = \eta^*(T_i, P_{avg}) \\
\sigma_{\eta^*} = 0.05\mu_{\eta^*} \\
\mu_\varepsilon = \varepsilon(T_i, P_{avg}) \\
\sigma_\varepsilon = 0.05\mu_\varepsilon \\
P_{avg} = \frac{P_i+P_t}{2}
\]

<table>
<thead>
<tr>
<th></th>
<th>Drawdown 1</th>
<th>Drawdown 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu_{\eta^*} )</td>
<td>1.1003 \times 10^{-7}</td>
<td>1.1003 \times 10^{-7}</td>
</tr>
<tr>
<td>( \sigma_{\eta^*} )</td>
<td>1.3754 \times 10^{-8}</td>
<td>1.3709 \times 10^{-8}</td>
</tr>
<tr>
<td>( \mu_\varepsilon )</td>
<td>-2.6376 \times 10^{-6}</td>
<td>-2.6448 \times 10^{-6}</td>
</tr>
<tr>
<td>( \sigma_\varepsilon )</td>
<td>3.2970 \times 10^{-7}</td>
<td>3.3060 \times 10^{-7}</td>
</tr>
<tr>
<td>Sample size</td>
<td>5000</td>
<td>5000</td>
</tr>
</tbody>
</table>

Table 8. Normal distribution parameters for fluid thermal properties
The distribution of \( k_h \) obtained from the sampled values of \( \eta^* \) and \( \varepsilon \) is shown in Fig. 21. A normal distribution function was also fitted to the results and the mean and standard deviation was estimated (Table 9). The standard deviation of the \( k_h \) distribution is about 12% of the mean value, as opposed to the 5% standard deviation value for the input properties. This gives an indication of how much confidence can be placed on the estimated \( k_h \) value and quantifies how less certain input data will lead to a much poorer estimation of the \( k_h \) value.

### Table 9. Distribution of \( k_h \) estimate due to normal distribution in fluid thermal properties

<table>
<thead>
<tr>
<th>Sample size</th>
<th>( k_h ) (95% confidence)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5000</td>
<td>( 0.8 \times 10^{-11} ) to ( 1.3076 \times 10^{-11} )</td>
</tr>
<tr>
<td>5000</td>
<td>( 0.8997 \times 10^{-11} ) to ( 1.4615 \times 10^{-11} )</td>
</tr>
</tbody>
</table>

6. CONCLUSION

This work has developed the mathematical background and a practical workflow for Temperature Transient Analysis in a vertical dry gas producing well. It is proven to:
• Be an effective alternative to the well-known Pressure Transient Analysis workflow in this type of well for determining the near-wellbore reservoir properties.
• Have the unique ability to identify the permeability and the depth of the near-wellbore formation damage.

This was achieved by:

1. Developing a (semi-log plot) linear form of the analytical solution transient sandface temperature in a vertical dry gas producing well. This linear analytical solution was subsequently incorporated in workflows for estimating the “permeability-thickness” of the producing layer, including determining the properties of a formation damage zone present in the near-wellbore region.

2. Verifying that a workflow based on Darcy flow can be confidently applied in many practical cases since the impact of non-Darcy effects on the slope of the transient temperature–time semi-log plot were minimal. The critical flow rate, above which the non-Darcy flow effects notably affect the temperature signal, is well and reservoir specific, but also depends on the (chosen) acceptable level of error.

3. Analysing the uncertainty in the results that is inherent in the workflow and the errors in the values of the input parameters.

4. Applying the developed workflow to both synthetically generated and field measured temperature data. The analysis of these data sets demonstrated the value of the proposed Temperature Transient Analysis workflow for estimating the properties of a gas producing layer when the flowrate is known or, alternatively, determining the flowrate when the properties of the producing reservoir are known.

ACKNOWLEDGEMENTS

We wish to thank the sponsors of the “Value from Advanced Wells” Joint Industry Project at Heriot-Watt University, Edinburgh, United Kingdom for providing financial support for one of the authors. We also wish to acknowledge the OpenFOAM community and developers for providing free access to their libraries.

NOMENCLATURE

\( \alpha \): Defined in Eqn.C15 and Eqn.D7
\( \beta \): Non-Darcy coefficient
\( \beta_T \): Thermal expansion coefficient
\( \gamma \): Euler-Mascheroni constant
\( \delta \): Deviation of analytical solution from logarithmic approximation
\( \varepsilon \): Joule-Thomson coefficient
\( \eta \): Adiabatic coefficient
\( \eta^* \): Formation averaged adiabatic coefficient
\( \lambda \): Constant term
\( \mu \): Viscosity of fluid
\( \mu_\eta^* \): Mean value of \( \eta^* \)
\( \mu_\varepsilon \): Mean value of \( \varepsilon \)
\( \mu_{kh} \): Mean value of \( kh \)
\( \rho \): Density of fluid
\( \rho_r \): Density of rock
\( \sigma_{\eta^*} \): Standard deviation of \( \eta^* \)
\( \sigma_{\varepsilon} \): Standard deviation of \( \varepsilon \)
\( \sigma_{kh} \): Standard deviation of \( kh \)
\( \phi \): Porosity
\( \psi \): Pseudo-pressure
\( \psi_i \): Pseudo-pressure at initial conditions
\( \Gamma^* \): Constant term
\( c \): Isothermal compressibility
\( d \): Molar density
\( k \): Permeability
\( r \): Radius
\( r_{nD} \): Ratio of non-Darcy pressure drop component to Darcy pressure drop component
\( r_T \): Thermal radius of investigation
\( t \): Time
\( t_j \): Time at which transient temperature becomes linear
\( v \): Velocity
\( A \): Constant term in pressure pseudo-pressure relationship
\( B \): Coefficient in pressure pseudo-pressure relationship
\( C \): Ratio of gas heat capacity to averaged formation heat capacity.
\( C_p: \) Specific heat capacity of fluid  
\( C_{pr}: \) Specific heat capacity of rock  
\( C_t: \) Total formation volumetric heat capacity  
\( L_w: \) Length of well-reservoir contact  
\( P: \) Pressure  
\( Q_d: \) Dimensionless pressure  
\( T: \) Temperature  
\( T_{nD}: \) Ratio of temperature change due to non-Darcy effect to temperature change due to Darcy effect  
\( U_o: \) Velocity of convective heat transfer

**SUBSCRIPTS**

\( crit: \) Critical condition  
\( d: \) Damage zone  
\( i: \) Initial conditions  
\( r: \) Rock  
\( t: \) Time  
\( sc: \) Surface conditions  
\( skin: \) Damage region / skin region  
\( T: \) Thermal  
\( w: \) Well  
\( wb: \) Wellbore  
\( wf: \) Well flowing

**REFERENCES**


Appendix A: Gas Properties

To determine the gas properties over the range of pressure and temperature in the reservoir, the following properties (Table A1) were used along with the Benedict-Webb-Rubin (BWR) EOS.

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pseudo critical temperature</td>
<td>$T_{pc}$</td>
<td>-116.59°F</td>
<td></td>
</tr>
<tr>
<td>Pseudo critical pressure</td>
<td>$P_{pc}$</td>
<td>676.22 Psi</td>
<td></td>
</tr>
<tr>
<td>Thermal conductivity</td>
<td>$K_T$</td>
<td>0.982</td>
<td>Btu/(hr.ft²·°F/ft)</td>
</tr>
<tr>
<td>Molal specific heat capacity of natural gas at ideal conditions</td>
<td>$C_{p,o}$</td>
<td>8.1</td>
<td>Btu/(lb.mol°F)</td>
</tr>
<tr>
<td>Universal gas constant</td>
<td>$\bar{R}$</td>
<td>1.987</td>
<td>Btu/(lb.mol°F)</td>
</tr>
<tr>
<td>Specific gas constant</td>
<td>$R$</td>
<td>124.12</td>
<td>Btu/(lbm°F)</td>
</tr>
<tr>
<td>Specific gravity of gas</td>
<td>$S.G_f$</td>
<td>0.605</td>
<td></td>
</tr>
<tr>
<td>Viscosity at initial reservoir pressure</td>
<td>$\mu_i$</td>
<td>0.0152 cP</td>
<td></td>
</tr>
<tr>
<td>Mass fraction of H₂S in natural gas</td>
<td></td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Mass fraction of CO₂ in natural gas</td>
<td></td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Mass fraction of N₂ in natural gas</td>
<td></td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

Table A1: Natural gas properties

Appendix B: Case Study Description

The case study used here describes a typical gas producing well and is taken from (ERCB 1979)

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thermal conductivity</td>
<td>$K_T$</td>
<td>0.982</td>
<td>Btu/(hr.ft²·°F/ft)</td>
</tr>
<tr>
<td>Porosity</td>
<td>$\phi$</td>
<td>0.15</td>
<td></td>
</tr>
<tr>
<td>Specific heat capacity of gas</td>
<td>$C_{p,f}$</td>
<td>723.70</td>
<td>Btu/(lbm°F)</td>
</tr>
<tr>
<td>Ratio of specific heat</td>
<td>$R$</td>
<td>124.12</td>
<td>Btu/(lbm°F)</td>
</tr>
<tr>
<td>Specific gas constant</td>
<td>$R$</td>
<td>124.12</td>
<td>Btu/(lbm°F)</td>
</tr>
<tr>
<td>Specific heat capacity of rock</td>
<td>$C_{p,r}$</td>
<td>219.74</td>
<td>Btu/(lbm°F)</td>
</tr>
<tr>
<td>Density of rock</td>
<td>$\rho_r$</td>
<td>156.07 lbm/ft³</td>
<td></td>
</tr>
<tr>
<td>Specific gravity of gas</td>
<td>$S.G_f$</td>
<td>0.605</td>
<td></td>
</tr>
<tr>
<td>Pseudo-pressure at initial reservoir pressure</td>
<td>( \psi_i )</td>
<td>3.366 \times 10^{14}</td>
<td>Psi²/cP</td>
</tr>
<tr>
<td>---------------------------------------------</td>
<td>-------------</td>
<td>----------------</td>
<td>--------</td>
</tr>
<tr>
<td>Viscosity at initial reservoir pressure</td>
<td>( \mu_i )</td>
<td>0.0152</td>
<td>Cp</td>
</tr>
<tr>
<td>Total formation compressibility at initial condition</td>
<td>( C_f i )</td>
<td>6.015 \times 10^{-4}</td>
<td>/Psi</td>
</tr>
<tr>
<td>Gas flow rate at standard conditions</td>
<td>( Q_{sc} )</td>
<td>7.0216</td>
<td>MMScf/day</td>
</tr>
<tr>
<td>Pressure at standard conditions</td>
<td>( P_{sc} )</td>
<td>14.7</td>
<td>Psi</td>
</tr>
<tr>
<td>Temperature at standard conditions</td>
<td>( T_{sc} )</td>
<td>60.53</td>
<td>°F</td>
</tr>
<tr>
<td>Initial reservoir pressure</td>
<td>( P_{i} )</td>
<td>2030.5</td>
<td>Psi</td>
</tr>
<tr>
<td>Initial reservoir temperature</td>
<td>( T_{i} )</td>
<td>119.93</td>
<td>°F</td>
</tr>
<tr>
<td>Reservoir permeability</td>
<td>( k )</td>
<td>10</td>
<td>mD</td>
</tr>
<tr>
<td>Reservoir thickness</td>
<td>( h )</td>
<td>39</td>
<td>ft</td>
</tr>
<tr>
<td>Well radius</td>
<td>( r_w )</td>
<td>0.39</td>
<td>ft</td>
</tr>
<tr>
<td>Reservoir boundary radius</td>
<td>( r_e )</td>
<td>1000</td>
<td>ft</td>
</tr>
<tr>
<td>Thermal expansion coefficient of gas</td>
<td>( \beta_T )</td>
<td>0.009396</td>
<td>/°F</td>
</tr>
<tr>
<td>Constants in pressure solution (ERCB 1979)</td>
<td>( \Gamma )</td>
<td>0.0292</td>
<td>Psi/°F</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table B1: Case study for numerical simulation and analytical solutions

<table>
<thead>
<tr>
<th>( r_{nD} = 0.1 )</th>
<th>( Q_{sc} ) [MM scf/day]</th>
<th>( k ) [mD]</th>
<th>( \beta ) [1/m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Darcy</td>
<td>1.513</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>Non-Darcy</td>
<td>1.513</td>
<td>10</td>
<td>4.21 \times 10^9</td>
</tr>
<tr>
<td>( r_{nD} = 0.2 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Darcy</td>
<td>3.026</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>Non-Darcy</td>
<td>3.026</td>
<td>10</td>
<td>4.21 \times 10^9</td>
</tr>
<tr>
<td>( r_{nD} = 0.3 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Darcy</td>
<td>4.540</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>Non-Darcy</td>
<td>4.540</td>
<td>10</td>
<td>4.21 \times 10^9</td>
</tr>
<tr>
<td>( r_{nD} = 0.4 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Darcy</td>
<td>6.053</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>Non-Darcy</td>
<td>6.053</td>
<td>10</td>
<td>4.21 \times 10^9</td>
</tr>
<tr>
<td>( r_{nD} = 0.5 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Darcy</td>
<td>7.567</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>Non-Darcy</td>
<td>7.567</td>
<td>10</td>
<td>4.21 \times 10^9</td>
</tr>
</tbody>
</table>

Table B2: Case study for non-Darcy effect

**Appendix C: Analytical Solution**

The analytical solution Eqn. 1 was derived from the thermal model by (Weibo Sui et al. 2008).

\[
\rho C_p \frac{\partial T}{\partial t} - \beta_T \frac{\partial \rho}{\partial t} - \Phi C_f \left( P + \rho_T C_{Pr} T \right) \frac{\partial P}{\partial t} = -\rho v v \cdot \nabla T + \beta_T v \cdot \nabla P - v \cdot \nabla P + K_T \nabla^2 T \tag{C1}
\]

The thermal model is Eqn.(C1) describes the transient temperature change in a porous media, and it includes the effects of transient fluid and rock expansion (second and third terms on lhs), Joule-Thomson effect (second and third terms on rhs), heat convection (first term on rhs) and heat conduction (fourth term on rhs).

Where: \( C_p \) and \( C_{Pr} \) are the specific heat capacity of the gas and formation rock respectively, \( \rho_T \) is the density of the formation rock \( C_f \) is the formation compressibility, \( v \) is velocity, \( \beta \) is the thermal expansion coefficient, \( K_T \) is the thermal conductivity, \( T \) is the temperature and \( \rho C_p \) is the mean formation heat capacity.
The analytical solution Eqn.1 was derived using the method of characteristics, with the following assumptions:

1. Conduction and heat exchange with the surround rocks effects are negligible.
2. The existing Line Source Pressure Solution (at constant temperature) for gas flow in porous media can be used to calculate pressure.
3. The relationship between pressure and pseudo-pressure can be represented by a straight line. This is normally valid within the range of pressure between the initial reservoir pressure and the bottom hole flowing pressure.
4. The term $\exp\left(-\frac{\phi \mu c r^2}{4 \lambda k t}\right)$ can be assumed to equal unity for $r < 3$ m (a typical investigation distance in TTA) if very early times ($t < 0.5$ hrs) are excluded. This is shown graphically in Figure 12.

\[ \exp\left(-\frac{\phi \mu c r^2}{4 \lambda k t}\right) = \exp\left(-\frac{ar^2}{t}\right) \approx 1 \]

5. Non-Darcy effects are neglected.
6. There is instantaneous thermal equilibrium between the rock and the flowing fluid.

Further assumptions about the gas properties are as follows:

7. The reservoir and well temperature are always higher than the critical temperature of the gas and below the Joule-Thomson inversion temperature.
8. The gas behaviour can be adequately modelled using the real gas compressibility factor (z-factor).

The following assumptions are required when using the line source, pressure solution (Ahmed 2010):

9. The reservoir is infinitely acting.
10. The well is producing at a constant flow rate.
11. The wellbore, radius $r_w$, is situated at the centre of the reservoir.

Applying the assumptions above the thermal model Eqn. C1 can be simplified to Eqn. C2. Eqn. C2 is then solved by the method of characteristics to obtain the analytical solution Eqn. C3, full details of the derivation is provided in a separate paper.

\[
K_1 \frac{\partial T}{\partial t} - K_2 \frac{\partial P}{\partial t} = K_3 \cdot \frac{\partial P}{\partial r} \cdot \frac{\partial T}{\partial r} - K_4 \left(\frac{\partial P}{\partial r}\right)^2 \quad \text{(C2)}
\]

Eqn. (C2) can be expressed as Eqn. (C3) below

\[
\frac{\partial T}{\partial \tau} - \frac{K_3}{K_1} \cdot \frac{\partial P}{\partial \tau} \cdot \frac{\partial T}{\partial r} = \frac{K_2}{K_1} \frac{\partial P}{\partial t} - \frac{K_4}{K_1} \left(\frac{\partial P}{\partial r}\right)^2 \quad \text{(C3)}
\]

We can apply the method of characteristics to Eqn. (C3), and express it as show in Eqn. (C4) to Eqn. (C6).

\[
\frac{\partial T}{\partial \tau} = -\epsilon \frac{\partial P}{\partial \tau} + \eta^* \frac{\partial P}{\partial \tau} \quad \text{(C4)}
\]

\[
\frac{\partial t}{\partial \tau} = 1 \quad \text{(C5)}
\]

\[
\frac{\partial r}{\partial \tau} = -\frac{K_3}{K_1} \frac{\partial P}{\partial r} \quad \text{(C6)}
\]
Where \( K_2 = \frac{\phi \beta T}{\rho C_p} = \eta^* \) and \( K_4 = \frac{(\beta T - 1)}{\rho C_p} = -\varepsilon \). A similar solution as that derived by (Ramazanov et al. 2010), can be applied here also.

\[
T_{wb}(t) = T_i(t) + \varepsilon \left[ P_{(r=r_T)} - P_{wf}(t) \right] + \eta^* \int_0^t \frac{dp}{d\tau} |_{(r=r_T)} \, d\tau ;
\]

(C7)

The solution of the integral term on the rhs of Eqn.(C7) gives \( e^{(-2\alpha U_o)} [P_{wf}(t) - P_i] \), therefore the complete solution to Eqn.(C7) is given below as Eqn.(C8). While the solution of Eqn.(C5) and Eqn.(C6) gives the characteristic curves (Eqn.(C9) and Eqn.(C10))

\[
T_{wb}(t) = T_i(t) + \varepsilon \left[ P_{(r=r_T)} - P_{wf}(t) \right] + \eta^* e^{(-2\alpha U_o)} [P_{wf}(t) - P_i]
\]

(C8)

\[
\tau = t
\]

(C9)

\[
s = \sqrt{r^2 + \frac{2K3B\psi Q_d t}{K1}}
\]

(C10)

Where \( U_o = \frac{K3}{K1} B\psi_i Q_d = \frac{\rho C_p k}{\rho C_p \mu} r \frac{\partial p}{\partial r} \); and \( r_T = \sqrt{r_w^2 + 2U_o t} \) and the line source pressure solution for the infinite actng radial ssystem is used to determine \( P_{wf}(t) \) and \( P_{(r=r_T)} \).

\[
P = A + B\psi
\]

(C11)

\[
\psi = \psi_i - \frac{\psi Q_d}{2} \left[ \gamma + \ln \left( \frac{\phi \mu c \nu^2}{4\lambda k t} \right) \right]
\]

(C12)

The pressure terms in Eqn. (C8) can be rewritten as Eqn. (C13), (C14) & (C15) by using Eqn. (C11) & (C12);

\[
P_{(r=r_T)} = A + B \left( \psi_i - \frac{\psi Q_d}{2} \left[ \gamma + \ln \left( \frac{\phi \mu c \nu^2}{4\lambda k t} \right) \right] \right)
\]

(C13)

\[
P_{wf} = A + B \left( \psi_i - \frac{\psi Q_d}{2} \left[ \gamma + \ln \left( \frac{\phi \mu c \nu^2}{4\lambda k t} \right) \right] \right)
\]

(C14)

\[
P_i = A + B\psi_i
\]

(C15)

Eqn. (C8) can now be written as shown below in Eqn. (C15)

\[
T_{wb}(t) - T_i(t) = \varepsilon \left( -B \frac{\psi Q_d}{2} \left[ \ln \left( \frac{r_T^2}{r_w^2} \right) \right] \right) + \eta^* \left( -B \frac{\psi Q_d}{2} \left[ \gamma + \ln \left( \frac{\phi \mu c \nu^2}{4\lambda k t} \right) \right] \right)
\]

(C15)

Where: \( r_T = \sqrt{(r_w^2 + 2U_o t)} \)

\[
Q_d = \frac{\Gamma Q_{sc}}{kh\psi_i}
\]

\[
\alpha = \frac{\phi \mu c}{4\lambda k}
\]

\[
U_o = C\nu(r, t) r
\]

\[
C = \frac{c_f \rho_f}{C_t}
\]

\[
C_t = \phi C_f \rho_f + (1 - \phi) C_T \rho_T
\]
The analytical solution assumes constant gas properties, however gas properties change with pressure and temperature, therefore we determined as suitable condition (pressure and temperature) for estimating the gas property to be used in the analytical equation. This condition was selected to minimize the error in the solution.

The volumetrically averaged properties provide the closest match to the numerical solution for the case considered. The averaged property is calculated at the initial reservoir temperature and the average pressure defined by Eqn. (C11) where $P_s$ is the stabilized pressure and $P_i$ is the initial pressure.

$$P_{avg} = \frac{P_i + P_s}{2}$$  \hspace{1cm} (C11)

The stabilized pressure is the pressure at which (i) the radius of investigation equals the external reservoir radius or (ii) when the transient pressure effect is felt at the reservoir boundary (ERCB 1979). The time required for stabilization can be determined from the equation $t_s = \frac{\phi c r^2}{4 \lambda k}$.

**Appendix D: Linearization of Analytical Solution**

$$\therefore \ T_{wb}(t) - T_i(t) = -\varepsilon B \frac{\psi_i Q_d}{2} \left[ \ln \left( \frac{r_w^2 + 2 U_o t}{r_w^2} \right) \right] - \eta^* B \frac{\psi_i Q_d}{2} \left[ \gamma + \ln \left( \frac{\phi c r^2}{4 \lambda k t} \right) \right]$$  \hspace{1cm} (D1)

In Eqn. (D1) above, we used the logarithmic approximation for the exponential integral function.

The plot of the transient temperature calculated using Eqn. (D1) w.r.t log time shows a nonlinear portion, followed by a linear portion as shown in Fig.(D1a). This figure was obtained from numerical simulation, using the data in Appendix B , it describes a vertical well with constant rate production with the well temperature measured at the mid-perforation point. We would like to obtain an equation that accurately represents the linear portion of Eqn. (D1).

The two terms in Eqn. (D1) can be plotted in the logarithmic scale as shown in Fig.(1b) to outline when each one is dominant.
The nonlinear behaviour is due to $\left( \frac{r_w^2 + 2\mu_o t}{r_w^2} \right)$, however as time increases the derivative of $\ln \left( \frac{r_w^2 + 2\mu_o t}{r_w^2} \right)$ approaches that of $\ln(t)$. We can approximate the value of $\ln \left( \frac{r_w^2 + 2\mu_o t}{r_w^2} \right)$ as shown below.

$$\lim_{t \to \infty} \left[ \ln \left( \frac{r_w^2 + 2\mu_o t}{r_w^2} \right) \right] = \ln(t) + \Delta$$

(D2)

Where $\Delta$ is a shift added to $\ln(t)$.

$$\Rightarrow \lim_{t \to \infty} \frac{d}{dt} \left( \ln \left( \frac{r_w^2 + 2\mu_o t}{r_w^2} \right) \right) = \frac{d}{dt} (\ln(t))$$

However for practical purposes, we can obtain a time $t_j \neq \infty$ at which the value of $\frac{d}{dt} \left( \ln \left( \frac{r_w^2 + 2\mu_o t}{r_w^2} \right) \right)$ is sufficiently close to that of $\frac{d}{dt} (\ln(t))$. We can determine a time $t_j$ such that the percentage difference between these two is less than $\delta$ (where $\delta \leq 1\%$)

$$\frac{d}{dt} \left( \ln \left( \frac{r_w^2 + 2\mu_o t}{r_w^2} \right) \right) = \frac{2\mu_o}{r_w^2 + 2\mu_o t}$$

$$\frac{d}{dt} (\ln(t)) = \frac{1}{t}$$

For a time $t \geq t_j$

$$t_j = \frac{r_w^2 (100 + \delta)}{2\mu_o \delta}$$

(D3)

Combining Eqn. (D2) & (D3) we can determine the value of the shift $\Delta$ at time $t_j$.

$$\Delta = \left| \ln \left( \frac{r_w^2 + 2\mu_o t}{r_w^2} \right) - \ln(t) \right|_{t=t_j}$$

$$\Delta = \ln \left( \frac{200\mu_o \delta}{r_w^2 (100 + \delta)} \right)$$

(D4)
From Eqn. (D2) we can safely assume that when $t \geq t_j$, 

$$\ln \left( \frac{r_w^2 + 2U_0 t}{r_w^2} \right) \cong \ln(t) + \ln \left( \frac{200U_0 \delta}{r_w^2(100+\delta)} \right)$$

(D5)

With the accuracy delta of 1%

Substitution Eqn. (D5) into Eqn. (D1) we obtain the Eqn. (D6) below, this equation is the equation which describes the linear portion of Eqn. (D1).

$$T_{wb}(t) = T_i(t) - \epsilon B \frac{\Psi Q_d}{2} \left[ \ln(t) + \ln \left( \frac{200U_0 \delta}{r_w^2(100+\delta)} \right) \right] - \eta^* B \frac{\Psi Q_d}{2} \left[ \gamma + \ln \left( \frac{\phi \mu r_T^2}{4\lambda k t} \right) \right] \quad t \geq t_j$$

(D6)

where $Q_d = \frac{\Gamma Q_{sc}}{k \psi \phi}$

This equation would be used in the following section to characterize a reservoir by using the transient temperature data.

$$T_{wb}(t) = T_i + \frac{B T Q_{sc}}{2kh} \left[ t^n - \epsilon \ln(t) - \frac{B T Q_{sc}}{2kh} \left[ t^{n*} \ln \left( \frac{\phi \mu c r_T^2}{4\lambda k t} \right) + \epsilon \ln \left( \frac{200U_0 \delta}{r_w^2(100+\delta)} \right) + \eta^* \gamma \right] \right]$$

(D7)

Where:

$$r_T = \sqrt{r_w^2 + 2U_0 t}; \quad \alpha = \frac{\phi \mu c}{4\lambda k}; \quad U_0 = C v(r, t) r; \quad v(r, t) = \frac{k d r}{\mu d t}$$

$$C = \frac{\rho C_p}{C_t}; \quad C_t = \rho \frac{C_p}{C_t} = \phi C_p (1 - \phi) C_p \rho r; \quad \epsilon = \frac{1 - \beta_T T_i}{C_p \rho};$$

$$\eta^* = \eta^* e^{-2\alpha U_0}; \quad \eta = \phi C \eta; \quad \eta^* = \beta_T T_i / C_p \rho$$

### Appendix E: Additional Case Studies

#### E.1 Estimating the Rate and Permeability*Thickness Values

These case studies use the data provided in Table B1 of Appendix B, with different surface production rate, permeability and thickness values input in the simulation. The values used in the simulation (Cases 2 and 3) are given in Table E1, while the result from the rate and permeability*thickness product estimation are given in Table E2, and also plotted in Figures 12(a), E1(a) & E1(b). Case 1 is the base case described in the paper, and is presented here for comparison.

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Case1</th>
<th>Case2</th>
<th>Case3</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gas flow rate at standard conditions</td>
<td>$Q_{sc}$</td>
<td>7.0216</td>
<td>7.0216</td>
<td>105.3</td>
<td>MMScf/day</td>
</tr>
<tr>
<td>Permeability</td>
<td>$k$</td>
<td>10</td>
<td>10</td>
<td>100</td>
<td>mD</td>
</tr>
<tr>
<td>Formation thickness</td>
<td>$h$</td>
<td>39</td>
<td>98</td>
<td>196</td>
<td>ft</td>
</tr>
</tbody>
</table>

Table E1: Case study description for rate and permeability thickness estimation
As can be seen from Tables E1 and E2, the $kh$ estimation error in these three cases is less than 4%.

### E.2 Estimating Parameters of the Near-Wellbore Damage Zone

These case studies use the data provided in Table B1 of Appendix B, while a near-wellbore zone of lower permeability is included in the simulation model. The damage zone radius is denoted as $r_d$ while the permeability of the damage zone is $k_{skin}$. The values for the two simulated cases (Cases 2 and 3) are given in Table E3, while the result from the near-wellbore
analyses is given in Table E4, and also plotted in Figures 12(b), E2(a) & E2(b). Case 1 is the
base case described in the paper, and is presented here for comparison.

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Case1</th>
<th>Case2</th>
<th>Case3</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius of damage zone</td>
<td>( r_d )</td>
<td>3.28</td>
<td>1.64</td>
<td>3.28</td>
<td>ft</td>
</tr>
<tr>
<td>Permeability of damage zone</td>
<td>( k )</td>
<td>5</td>
<td>5</td>
<td>3</td>
<td>mD</td>
</tr>
<tr>
<td>Formation thickness</td>
<td>( h )</td>
<td>39</td>
<td>39</td>
<td>39</td>
<td>ft</td>
</tr>
</tbody>
</table>

Table E3: Case study description for near-wellbore analysis

![Transient Temperature showing Transition Time](image1)

![Transient Temperature showing Transition Time](image2)

Figure. E2: Transient temperature signal and the slopes corresponding to the damage zone and the clean
formation in Case 2 (left) and Case 3 (right)

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Case1</th>
<th>Case2</th>
<th>Case3</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transition time</td>
<td>( t_d )</td>
<td>3.886</td>
<td>1.16</td>
<td>4.697</td>
<td>hr</td>
</tr>
<tr>
<td>Pressure at transition time</td>
<td>( P_{td} )</td>
<td>1650.45</td>
<td>1718.72</td>
<td>1504.49</td>
<td>psi</td>
</tr>
<tr>
<td>Gas viscosity at average condition (i.e. ( T_i, P_{avg} ))</td>
<td>( \mu_{avg} )</td>
<td>0.01465</td>
<td>0.01474</td>
<td>0.01446</td>
<td>cP</td>
</tr>
<tr>
<td>Gas density at average condition (i.e. ( T_i, P_{avg} ))</td>
<td>( \rho_{avg} )</td>
<td>5.3929</td>
<td>5.4988</td>
<td>5.1664</td>
<td>lb/ft³</td>
</tr>
<tr>
<td>Gas specific heat capacity at average condition (i.e. ( T_i, P_{avg} ))</td>
<td>( C_{p,avg} )</td>
<td>726.73</td>
<td>729.95</td>
<td>719.85</td>
<td>Btu/(lbm°F)</td>
</tr>
<tr>
<td>Gas thermal expansion coefficient at average condition (i.e. ( T_i, P_{avg} ))</td>
<td>( \beta_{T,avg} )</td>
<td>0.009189</td>
<td>0.009233</td>
<td>0.009096</td>
<td>°F</td>
</tr>
<tr>
<td>Slope of pressure pseudo-pressure relationship</td>
<td>( B )</td>
<td>5.0×10⁻¹³</td>
<td>5.0×10⁻¹³</td>
<td>5.0×10⁻¹³</td>
<td>s</td>
</tr>
<tr>
<td>Semilog slope of linear portion</td>
<td>( a_{skin} )</td>
<td>-1.5754</td>
<td>-0.7653</td>
<td>2.7054</td>
<td>°F/ln(sec)</td>
</tr>
<tr>
<td>Estimated, damage zone radius</td>
<td>( r_d )</td>
<td>3.08</td>
<td>1.73</td>
<td>3.32</td>
<td>ft</td>
</tr>
<tr>
<td>Estimated, damage zone permeability thickness</td>
<td>( k h_{skin} )</td>
<td>202.41</td>
<td>228.93</td>
<td>120.78</td>
<td>mD·ft</td>
</tr>
</tbody>
</table>

Table E4: Case study results for near-wellbore analysis
As can be seen from Tables E3 and E4, the damage zone $kh$ estimation error in these three cases is less than 15%.