A numerical simulation of heat transfer in an enclosure with a non-linear heat source.

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\section*{Abstract}

Two dimensional simulations of the natural convection driven by absorption non-uniform concentrated solar radiation in a molten binary salt filled enclosure inclined at $0^{\circ} \leq \phi \leq 60^{\circ}$ is presented. The enclosure is volumetrically heated from the top boundary and accommodates a black rigid, heat-conducting plate of finite thickness at the lower boundary which aids generation of natural convection mixing at the lower boundary. The governing equations which accounts for the depth dependent absorption of radiation are solved using the finite element method. Numerical results reveal that increasing inclination angles decrease natural convection and higher Rayleigh promote natural convection.

\section{1. Introduction}

Interaction of heat transfer and fluid flows in inclined enclosures has attracted attention in academia and industry, owing to their significance in many systems such as heat exchangers, solar collectors and nuclear reactors heat exchangers, solar collectors and nuclear reactors \cite{1}. In applications,
such as crank thermo syphon type heat exchangers inclination found to be optimally enhance performance [2]. Several investigations for the effect of inclination angles on the stability of fluid layers, flow types and natural convection heat transfer and natural in domains driven by end to end temperature difference have been extensively studied, where the inclination angles were shown to influence the studied parameter [2–8]. Other investigations for the effects of inclination angles on radiation exchange between surfaces of an enclosure bounding a non-participating fluid [9], thermal radiation MHD phase change heat transfer [10] and entropy generation in porous media enclosures [11] are found.

In recent years the class of problems dealing with natural convection induced by absorption of radiation in fluid layers have generated great interest in many physical applications. For example in shallow natural water bodies,
natural convection driven by absorption of solar radiation is found to play an important role in driving the exchange flow and mixing which in turn significantly influence biological activity and water quality [12, 13]. Several experimental, numerical and theoretical investigations of the induced convective flows and heat transfer in rectangular [15–21], triangular [22–28] and parallelepiped [29–32] reservoirs are found in literature. Temperature fields are characterised by a nonlinear temperature stratification consisting of a stably stratified hot top layer owing to the direct absorption of solar radiation lying directly above an unstable thermal layer, due to solar radiation reaching the bottom surface absorbed and re-emitted, hence a potential source for a Rayleigh-Bernard (RB) type instability. The thermal instabilities primary form is characterised by rising plumes which extend from a thermal boundary layer and drives a convective flows and mixing. A distinct feature of the existence of a nonlinear temperature stratification is the physical limitation it imposes on the penetration length scale of the rising thermal plumes and the associated lower mixed layer thickness.

While some insight and understanding have been gained for the radiation induced natural convection in various systems, the effect of inclination angle on natural convection in enclosures in which the primary driving force is the volumetric absorption of thermal radiation appears to be lacking. Further, at the time of writing this paper, to the best of the authors knowledge, none of the existing literature for radiation on the driving mechanisms and flow features in enclosures subject to concentrated solar radiative heating have addressed the effect of inclination angles the nonlinear temperature stratification. Thus development of prediction models and design tools would
immensely contribute to providing understanding of the effect of inclination angle on radiation induced natural convection in enclosures. Understanding the heat transfer and fluid dynamics in volumetrically heated fluid layers is of greatest importance to inform efficient system design based on direct absorption of solar radiation concept [33–37]. Thus potentially resulting in improved system efficiencies at minimum costs.

The primary purpose of the present study is to provide numerical simulations for the temperature and flow fields in an volumetrically absorbing molten salt filled inclined enclosure and to investigate the effect of inclination angle on the non-linear temperature stratification profile and induced natural convection.

The present numerical analysis focuses on accounting for the depth dependent solar radiation absorption a discretised standard reference for solar radiation and absorption coefficients over wavelengths relevant to solar energy applications as well as accounting for the temperature dependence of the working fluid.

2. Geometry mathematical formulation and governing equations

A two dimensional inclined enclosure with H/D=1, shown in Fig 1 has been considered in this study. The domain vertical boundaries inclined at an angle denoted by \( \phi \) to the gravity vector are rigid and adiabatic, while the top wall is an open stress free boundary. The lower boundary is a black rigid plate of finite thickness (dx), whose sole purpose is to absorb all radiation transmitted to the lower surface, subsequently driving a natural convection [39–41] to heat and mix the molten salt. Molten KNO\( _3 \)-NaNO\( _3 \) salt is se-
lected as the working fluid owing to its long operational experience in thermal transport and storage handling. The salt confined within the domain walls is initially at rest and at a temperature $T_0$. At the time $t=0$, a non-uniform concentrated radiation flux is initiated and thereafter maintained on the top inclined surface (Fig 1). The directly deposited radiation into the fluid will heat the molten salt volumetrically by the direct absorption of solar radiation inside the fluid body and subsequently by natural convection from a lower absorber plate owing to the absorption of transmitted flux.

![Figure 1: Schematic of geometry and computational mesh of the problem H/D=1.](1a.pdf)

The temperature and flow fields within the enclosure are obtained by solving the two dimensional incompressible Navier-Stokes, continuity and
energy equations shown in equations (1)-(4).

\[ \nabla \cdot \vec{V} = 0 \quad (1) \]

\[ \rho \frac{\partial (\vec{V})}{\partial t} + \rho (\vec{V} \cdot \nabla) \cdot \vec{V} = -\nabla p + \nabla \cdot \mu \nabla V + g_x \beta \sin \theta \Delta T \quad (2) \]

\[ \rho \frac{\partial (\vec{V})}{\partial t} + \rho (\vec{V} \cdot \nabla) \cdot \vec{V} = -\nabla p + \nabla \cdot \mu \nabla V + g_z \beta \cos \theta \Delta T \quad (3) \]

\[ \rho C_p \left( \frac{\partial T}{\partial t} + \nabla \cdot VT \right) = \nabla \cdot (k \nabla T) + S \quad (4) \]

where S is the volumetric heating source term and g_x and g_y = 0, the driving force term components of gravity vector. \( \phi \) is the inclination angle measured in the anti-clockwise direction with respect to the vertical.

The volumetric heat generation source term S, is obtained by solving the radiative transfer equation given in equation (8) along with the boundary conditions given in equations (6), and (7) respectively.

\[ \frac{dI_{\lambda}(z)}{dz} + \alpha_{\lambda} I_{\lambda}(z) = 0, \text{ in } 0 < x < L, \quad (5) \]

\[ I(z) = C \eta (1 - r_a) I_{\lambda} e^{-\alpha_{\lambda} z} \text{ at } z = 0, \quad (6) \]

\[ I(z) = C \eta (1 - r_a) I_{\lambda} e^{-\alpha_{\lambda} H} \text{ at } z = H, \quad (7) \]

For simplification, omitting the mathematical details for derivation of the
RTE and local radiation flux for brevity and making the forward scattering approximation the volumetric heat generation, the total bottom heat flux \( q_{B,k} \) and spectral volumetric heat generation \( S \) are given in equations (8) and (11)

\[
\frac{dI_\lambda(z)}{dz} = \alpha_\lambda(C\eta(1 - r_a)I_\lambda e^{-\alpha_\lambda y}) \tag{8}
\]

\[
q_b = \frac{1}{\kappa} \sum_{\lambda=1}^{n} \left( C\eta(1 - r_a)I_\lambda e^{-\alpha_\lambda H} \right) Wm^{-2} \tag{9}
\]

\[
S_\lambda = C\eta(1 - r_a) \left( \alpha_\lambda I_\lambda e^{-\alpha_\lambda y} + \frac{1}{\kappa} \int_{0}^{D} (I_\lambda e^{-\alpha_\lambda H}) \right) Wm^{-3} \tag{10}
\]

Where the first and second terms on the RHS of equation (10) denotes the contributions of the direct absorption of the incident solar flux within the fluid body and the heat collected as heat generated at the lower boundary. The total volumetric heat generation, \( S \) (equation (11)) can be obtained by superimposition of the individual wavelengths [14].

\[
S = \sum_{\lambda=1}^{n} S_\lambda = Wm^{-3} \tag{11}
\]

Thermophysical properties for molten KNO\(_3\)-NaNO\(_3\) salt are temperature dependent and are given in Table 1 [38]. The molten salt is also assumed to be Newtonian, absorbing and non scattering. The spectral irradiation is described based on the Air Mass (AM) 1.5D (SMARTS) model [42–45]. The reflection and transmission characteristics of the air-liquid are assumed for optically smooth surfaces given by Fresnel’s equations and Snell’s law.
Table 1: Thermophysical properties of molten KNO$_3$-NaNO$_3$ salt and properties of absorber plate material \[38, 50\]

<table>
<thead>
<tr>
<th>Property/units</th>
<th>KNO$_3$-NaNO$_3$</th>
<th>Copper</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$ (kgm$^{-3}$)</td>
<td>2090-0.636T</td>
<td>8960</td>
</tr>
<tr>
<td>$k$ (W (mK)$^{-1}$)</td>
<td>0.443+1.9 $\times$ 10$^{-4}$T</td>
<td>400</td>
</tr>
<tr>
<td>$C_p$ (J(kgK)$^{-1}$)</td>
<td>1443-0.172T (1396.044+0.172T)</td>
<td>385</td>
</tr>
<tr>
<td>$\mu$ (kg(ms)$^{-1}$)</td>
<td>22.714-0.12T+2.281$\times$10$^{-4}$T-1.474$\times$10$^{-7}$T</td>
<td>-</td>
</tr>
</tbody>
</table>

2.1. Initial and boundary conditions

The salt initially at a temperature above melting point temperature, $T(t=0) > 222^\circ$C is considered to be rest i.e. $(u,v(t=0)=0)$. Side walls are all rigid, non-slip and adiabatic, with velocity components and normal temperature gradients being zero, $(u=v=dT/dn=0)$. The top surface is taken to be stress free and adiabatic i.e $(du/dy=dv/dy=dT/dy=0)$, while the lower boundary is a rigid wall of finite thickness with no slip $(u=v=0)$. The perturbation temperature, $\Delta T$, developed on the lower boundary $(y=H)$, owing to the absorption of residual flux is given by equation (12)

$$\Delta T = \frac{\alpha q_b}{\kappa \rho C_p} \left( \frac{\kappa t}{\pi} \right)^{0.5}$$  

where, $q_b$ is the heat generated at the lower surface and is a function of I the solar Intensity and $\alpha$ is a spectral attenuation coefficient.

3. Numerical implementation

Finite Element Method (FEM) package, COMSOL Multiphysics [51] is used to solve the derived, time dependent system of non-linear partial differential equations (PDEs). Second-order elements for the velocity field and
linear elements for the pressure field, $(P_2 + P_1)$ discretisation is used [51]. Numerical solutions were obtained for a convergence criteria of $10^{-5}$.

3.1. Mesh refinement

Extensive mesh refinement studies were performed on calculated solutions to examine the dependence of numerical accuracy on mesh element sizes for the present problem; $H/D=1$, $\phi=0$ and spectral solar radiation at Air Mass 1.5D. Fig 2a depicts the variation of vertical temperature of the cross section at $x=0, y=0$ and at $t=5000s$ for mesh element sizes 20mm, 30mm and 50mm. It can be seen from Fig 2a that the vertical cross sectional temperature are similar and close at 20mm and 30mm Figure 2b presents the time history of the vertical velocity at a location above the lower boundary for mesh element sizes 20mm, 30mm and 50mm. Similarly, the values of the vertical velocity are almost the same for mesh element sizes, 20mm and 30mm as inferred from Fig 2b.

Time-step dependence studies for the calculated results were conducted using three time steps: 0.2, 0.5, and 1 s. As a compromise between the computational time and the accuracy a time step of 0.2 s is adopted in the numerical calculation and was found to be sufficient to resolve and frequency component in the simulation.

The mesh element size of 20mm gives an error in the average temperature of 0.31% compared to the solution obtained at the finest mesh element size considered. Fig 1 presents a unstructured mesh used in the simulation.
Figure 2: Mesh convergence plot of numerical solutions for enclosure for $H/D=1$, AM1.5D and $\phi=0^\circ$: (a) Vertical temperature profile at the mid section at $t=5000s$. (b) Time history for the vertical velocity within the boundary layer.
3.2. Validation

The model was validated by comparing simulation results with published data of Coates and Patterson [21, 20], which deals with the unsteady natural convection in an enclosure subject to solar radiation. Full description of the experimental study is presented in Coates and Patterson [21].

The experimental and modelled maximum velocity against the square root of the dimensionless time are compared in Fig 3 and it can be seen that agreement between the results is good. Less than 5% deviation between experimental and numerical results was obtained thus, giving confidence in the numerical model.

![Graph](image)

Figure 3: Maximum velocity plotted against the square root for time of experiments [21] and numerical simulations.
4. Results

Fig 4 a-d presents temperature plots in the enclosure for no inclination angle ($\phi=0$) illuminated directly from a top surface at various time steps. The spectral heat source is described spectral solar radiation at Air Mass 1.5D and wavelength dependent absorption coefficients.

In Fig 4a, at $t=50s$, after the solar radiation is initiated and maintained, the temperature profile shows the early heating of the top fluid layer along the top surface and non uniformly downwards through the fluid depth towards the bottom surface. During this time step the non absorbed solar radiation transmitted to the lower surface heats the lower surface, thereby subsequently developing a thermal boundary layer adjacent the lower surface. At the lower boundary, the presence of near parallel isotherm indicates no fluid motion and a primary conductive heat transfer regime. For continued heating, at heating time, $t=130s$ (Fig 4b), the temperature field shows the development of the stratification of the fluid layer. The plot reveal a non linear temperature field, with zones of decreasing downward temperatures. In the lower fluid, thermal plumes which to emerge out of the thermal boundary are evident. The occurance of the thermal plumes indicates the introduction of thermal instability. In the top fluid layer, the near parallel isotherm indicates no fluid motion and a conductive heat transfer regime. However fluid flow with low velocities are inferred from the isotherms. In Fig 4c, at a later heating time, $t=1200s$ a non-linear temperature profile and thermal stratification of the fluid layer are further establish. At this timestep it easily be seen that the hot fluid layer advances downwards into the fluid layer and towards the bottom boundary. In the lower fluid column, thermal plumes extends from
the thermal boundary into the fluid layer where the height of the thermal plumes are seen to be confined within the lower fluid layer. This is consistent with observations of Hattori et al. [29].

Figure 4: Normalised transient surface temperature plots at (a) $t=50s$ (b) $t=130s$ (c) $t=1300s$ (d) $t=5000s$

At $t=5000s$ (Fig 4d), for no heat losses, a fully stratified non linear temperature fluid is obtained. A previously observed convective mixing from the thermal plume activity is seen to diminish. Isotherms are seen to fill the entire enclosure illustrating no flow and conductive heat transfer regime. The stabilizing effects of the surface layer due to the direct absorption of the
incident solar radiation of shorter wavelengths in the fluid and is attributed to the density gradient being parallel but in the same direction as the gravity vector. The destabilizing effect which sets up an unstable Rayleigh Bernard type instability and subsequent the convection [29].

4.1. Time history for the fluid velocities and heat transfer

Fig 5a shows time history of the fluid vertical velocity taken at a point $x=D/4,y=-0.98H$. The plot is illustrative of the stages of flow development occurring within the boundary layer. Three stages of flow development can be easily be seen which are extensively discussed in Amber and O’Donovan [53] and for brevity will be briefly discussed below:

I. The early flow regime which occurs for $t<120s$. In the initial regime, the flow that occurs before the the initial peak, increases smoothly from rest and generally has low velocities as shown in by the insert in Fig 5a. The heat transfer from the lower surface at this heating stage is primarily by conduction.

II. The transitional regime occurring for $120s < t < 4200s$. In the transitional regimes, increased fluid velocities with sharp irregular fluctuations of varying amplitudes can be seen. This velocity fluctuations coincides with the occurrence and disappearance of thermal plumes during heating times. A fully developed unsteady flow with complex and irregular fluid circulation characterises this regime. The heat transfer from the lower surface at this heating stage is primarily by convection.

III. The quasi steady flow occurring for $t > 4200s$. the magnitudes and frequency of oscillation, are observed to be reduced.
Fig 5b, presents time history for the averaged heat transfer from the lower surface corresponding to Fig 5a. Similarly, the plots show three regimes of heat transfer; conductive heat transfer in the early stage, convective heat transfer in the transitional stage and quasi steady state heat transfer in the quasi steady state regime. The heat transfer in Fig 5b is consistent and can easily be matched with the major fluid developments and features identified from the surface plots (Fig 3) and flow velocity time history (Fig 5a).

![Figure 5: Time history of (a) Velocity and (b) heat transfer from a point on the lower boundary at point x,y (H/4, -0.98H)](image-url)
5. Effect of inclination angle

From the previous section it can be seen the volumetric heating of the fluid layer generated a non-linear temperature stratification characterised by two distinct layers. The effect of the non linear temperature on the profile on the physical limitation on the thermal plumes and convective mixing depth has been demonstrated.

The effect of geometry inclination angle on the observed temperature field and heat transfer can be of interest. Fig 6a-e presents the isotherms and corresponding velocity contours for the enclosure in Fig 3 for angles of inclination, $\phi = 0^\circ, 15^\circ, 30^\circ, 45^\circ$ and $60^\circ$ at $t=1250s$, a time within the transitional regime where thermal plumes are evident and convective heat transfer is highest.

Fig 6 a show the temperature field within the enclosure for $\phi=0^\circ$ at $t=1250s$. The features at this inclination has been discussed in the previous section and extensively in [53]. In Fig 6b, for an inclination angle of $\phi=15^\circ$, a non uniform temperature profile is seen consisting of a hot stratified layer above a convection layer. From visual inspection the hot stratified layer thickness smaller compared to that obtained at $0^\circ$. Thermal plumes are seen to penetrate much higher in to the lower fluid layer the flow can be and remains confined within it.

For higher inclination angles $\phi=30^\circ$ (Fig 6c), $45^\circ$ (Fig 6d) and $60^\circ$ (Fig 6e), observations reveal that (i) natural convection from the lower surface decreases (ii) thermal plumes are diminished (iii) the mixing layer depth decreases.
Figure 6: Transient surface temperature plots for $\phi$ values: (a) $0^\circ$ (b) $15^\circ$ (c) $30^\circ$ (d) $45^\circ$ (e) $60^\circ$. 
On the basis of a non-dimensional stratification parameter defined as the ratio of the time averaged temperature difference in the mixing depth and the difference of the time average temperature difference in the surface layer, it was found that as the angle of inclination increased from 0° to 60° thermal stratification is seen to increase by 29, 41, 54 and 67%.

Fig 7a, presents a plot of the velocity time history in the frequency domain for the flow in the transitional regime for 800s \( \leq t \leq 2000s \). The plot shows characteristics of an unsteady flow reveals three peaks with significant amplitude seen at \( f_1=0.006 \), \( f_2=0.02 \) and \( f_3=0.034 \) for inclination angle \( \phi=0^\circ \) (Fig 7a). Although other frequencies and amplitudes are visible they are of an order smaller than the amplitude of the main frequencies identified. In Fig 7b, inclination angle \( \phi=15^\circ \), various peaks in the frequency may be identified, however three peaks of relatively equal amplitudes can seen at \( f_1=0.0145 \), \( f_2=0.018 \) and \( f_3=0.035 \). The difference in amplitude between these main frequencies and other frequencies is reduced.

Fig 7c-e, corresponds to frequency domain plots for 30°, 45° and 60°. Generally a decrease in amplitude, and oscillations frequency occurs with increasing inclination angles, with the lowest amplitudes and fluctuations evident at \( \phi=60^\circ \) (Fig 7e). The main frequencies are generally seen to shift towards higher frequencies of reduced amplitudes with increasing inclination angle which is consistent with observations from flow transitions identified previously (Fig 7).
Figure 7: Flow behaviour in the frequency domain for φ values: (a) 0° (b) 15° (c) 30° (d) 45° (e) 60°.
5.1. Effect of Rayleigh number and inclination angle

In Fig 8, a plot showing the effect of increasing Rayleigh number \(10^8 \leq Ra \leq 10^{11}\) and inclination angle \(0^\circ \leq \phi \leq 60^\circ\) on the time averaged heat transfer, from the lower boundary is presented. The Rayleigh number used here is defined as \(Ra = \rho^2 C_p g \beta q H^4 / \mu k^2\). From this plot it can be seen that the time averaged Nusselt number (Nu) is seen to increase with increasing in Rayleigh number and decrease with increasing inclination angles.

![Figure 8: Heat transfer coefficient (Nu) variation with inclination angle \(\phi\).](image)

6. Conclusion

Two dimensional simulations for the transient heat transfer in an enclosure (H/D=1), directly illuminated by non-uniform concentrated radiation is presented for inclination angles \(0^\circ \leq \phi \leq 60^\circ\) and \(10^9 \leq Ra \leq 10^{11}\). The
model accounts for the spectral depth dependent volumetric absorption of the heat source based on a standard reference. A non-linear temperature profile with a hot top surface layer above a convection layer was found. The temperature field was found to be influenced by the inclination angle. Natural convection was found to decrease with increasing inclination angle as the heating effect of the lower boundary becomes less significant and increased worth increasing Rayleigh number.

References


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7. Tables

Table 1 Thermophysical properties of molten KNO$_3$-NaNO$_3$ salt and properties of absorber plate material [38, 50]
8. Figure Captions

Figure 1: Schematic of geometry and computational mesh of the problem H/D=1.

Figure 2: Mesh convergence plot at φ=0°

Figure 3: Mesh convergence plot at AM1.5D and φ=0°

Figure 4: Maximum velocity plotted against the square root for time of experiments [21] and numerical simulations.

Figure 5: Normalised transient surface temperature plots at (a) t=50s (b) t=130s (c) t=1300s (d) t=5000s.

Figure 6: Time history of (a) Velocity and (b) heat transfer from a point on the lower boundary at point x,y (H/4, -0.98H).

Figure 7: Transient surface temperature plots at (a) φ= 0° (b) 15° (c) 30° (d) 45° (e) 60°.

Figure 8: Flow behaviour in the frequency domain (a) φ= 0° (b) 15° (c) 30° (d) 45° (e) 60°.

Figure 9: Heat transfer coefficient (Nu) variation with inclination angle φ.
Figure 9: Schematic of geometry and computational mesh of the problem H/D=1.
Figure 10: Mesh convergence plot of numerical solutions for enclosure for $H/D=1$, AM1.5D and $\phi=0^\circ$ (a) Vertical temperature (b) Time history for the vertical velocity within.
Figure 11: Maximum velocity plotted against the square root for time of experiments [21] and numerical simulations.
Figure 12: Normalised transient surface temperature plots at (a) $t=50s$ (b) $t=130s$ (c) $t=1300s$ (d) $t=5000s$
Figure 13: Time history of (a) Velocity and (b) heat transfer from a point on the lower boundary at point x,y (H/4, -0.98H)
Figure 14: Transient surface temperature plots at (a) $\phi = 0^\circ$ (b) $15^\circ$ (c) $30^\circ$ (d) $45^\circ$ (e) $60^\circ$. 
Figure 15: Flow behaviour in the frequency domain (a) $\phi = 0^\circ$ (b) $15^\circ$ (c) $30^\circ$ (d) $45^\circ$ (e) $60^\circ$. 
Figure 16: Heat transfer coefficient (Nu) variation with inclination angle $\phi$. 