Plasma-induced nonlinear optical phenomena in gas-filled hollow-core PCFs

Fabio Biancalana and Mohammed F. Saleh

Abstract. By using a new theoretical framework based on equations for the electric field envelope, we provide a complete theoretical explanation of several plasma-induced nonlinear optical phenomena, including the soliton self-frequency blueshift, recently observed experimentally in a gas-filled hollow-core PCF.

Keywords: Solitons, plasma physics, photonic crystal fibers, modulational instability, supercontinuum

PACS: 42.65.Re, 42.81.Dp, 42.65.Ky

Hollow-core photonic crystal fibre (HC-PCF) [1] with a kagomé-lattice cladding has recently been shown to be optimal for the investigation of broadband light-matter interactions between intense optical pulses and gaseous media. These fibers typically show transmission bands covering the visible and near-IR parts of the spectrum with relatively low loss and low group velocity dispersion (GVD), absence of surface modes, and high confinement of light in the core. Filled with a noble gas, they have recently been used in high-harmonic [2] and efficient deep UV generation from fs pump pulses at 800 nm [3]. It has been previously shown that the Raman threshold can be drastically reduced in a HC-PCF filled with a Raman-active gas (such as H2) [4]. The system can also be used for detailed experimental studies of, e.g., self-similar solutions of the sine-Gordon equation [5] and backward stimulated Raman scattering [6, 4].

In this work, based on Ref. [12], we have first developed a new model to study pulse propagation in gas-filled HC-PCFs in terms of the complex envelope of the pulse. By using this model, we show analytically for the first time that intra-pulse photoionization leads to a soliton self-frequency blue-shift.

Photoionization can take place by either tunneling or multiphoton processes. These regimes are characterized by the Keldysh parameter $p_K$ [11]. In the tunneling regime ($p_K \ll 1$) the time-averaged ionization rate $\mathcal{W}(I)$ is given by [13]

$$\mathcal{W}(I) = d (I_H/I)^{1/4} \exp[-b (I_H/I)^{1/2}],$$

where

\begin{align*}
d &\equiv 4 \delta_0 \left( \frac{3}{\pi} \right)^{1/2} \left( U_{th}/U_H \right)^{7/4}, \quad b \equiv 2/3 \left( U_{th}/U_H \right)^{3/2}, \\
\delta_0 &\equiv 4.1 \times 10^{16} \text{ Hz is the characteristic atomic frequency,} \\
U_I &\text{ is the ionization energy of the gas (} \sim 15.76 \text{ eV for argon),} \\
U_{th} &\approx 13.6 \text{ eV is the ionization energy of hydrogen}, \\
I_H &\equiv 3.6 \times 10^{16} \text{ W/cm}^2 \text{ and } I \text{ is the laser pulse intensity. For values of } I \text{ in the range of } 100 \text{ TW/cm}^2, \text{ the Keldysh parameter is } p_K \lesssim 1 \text{ for noble gases. However, experiments show that tunneling models provide excellent agreement with the experimental measurements even for } p_K \approx 1. \text{ As shown in Fig. (1b), Eq. (1) predicts an ionization rate that is exponential-like for pulse intensities above a threshold value. Loss due to absorption of photons in the plasma is proportional to the ionization rate. Hence, any pulse with } I \gg I_{th} \text{ will have its intensity strongly driven back to near the threshold value, resulting in a drastically reduced ionization loss. This allows us to use the first-order Taylor series to linearize the tunneling model just above } I = I_{th}, \text{ where the optical pulses can survive for relatively long time without appreciable attenuation. Expanding Eq. (1) in its linear regime around an arbitrary point } a = I_0/I_{th}, \text{ results in } \mathcal{W} \approx \sigma \Delta I \Theta(\Delta I), \text{ where } \Delta I \equiv I - I_{th}, \sigma = d e^{-3} (2x - 1)/(4a^{5/4} I_{th}), \text{ and } I_{th} = a I_{th}(2x - 5)/(2x - 1) \text{ is the threshold intensity, } x = b/\sqrt{\sigma}, \text{ and } a \text{ is chosen to reproduce}.
\end{align*}
the physically observed threshold intensity in the fiber of Fig. 1(a), $a \cong 2 \times 10^{-3}$. The purpose of the Heaviside function $\Theta$ is to set the ionization rate to zero below the threshold intensity. As shown in Fig. 1(b), the linearized model underestimates the ionization rate and thus the ionization loss, in comparison to the full model. This leads to a similar qualitative behavior between the two models even for $I > I_{th}$, since the ionization rate and the ionization loss are the key factors in the photoionization process. 

One can prove from first principles that propagation of light in a HC-PCF filled with an ionized Raman-active gas can be then described by the following coupled equations [12]:

$$
\begin{align*}
\left[ i \partial_z + \hat{D}(i \partial_t) + a \right] |\Psi|(|\Psi|^2 - \frac{\alpha_2}{2\gamma_0 c^2} + i\alpha) \right] |\Psi| = 0, \\
\partial_t n_e = \left[ \hat{\sigma}/A_{eff} \right] (n_T - n_e) |\Psi|^2 \Theta (|\Psi|^2)
\end{align*}
$$

(2)

where $\Psi(z,t)$ is the electric field envelope, $z$ is the longitudinal coordinate along the fiber, $t$ is the time in a reference frame moving with the pulse group velocity, $\hat{D}(i \partial_t) \equiv \sum_{m>2} \beta_m (i \partial_t)^m/m!$ is the full dispersion operator, $\beta_m$ is the $m$-th order dispersion coefficient calculated at an arbitrary reference frequency $\omega_0$, $\gamma_k$ is the Kerr nonlinear coefficient of the gas, $R(t) = (1 - \rho) \delta(t)$ is the normalized Kerr and Raman response function of the gas, $\delta(t)$ is the Dirac delta function, $\rho$ is the relative strength of the non-instantaneous Raman nonlinearity, $h(t)$ is the causal Raman response function of the gas, the symbol $\otimes$ denotes the time convolution, $c$ is the speed of light, $k_0 = \omega_0/c$, $\omega_0$ is the pulse central frequency, $\omega_k = \left[ e^2 n_e / (e_0 m_e) \right]^{1/2}$ is the plasma frequency associated with an electron density $n_e(t)$, $e$ and $m_e$ are the electron charge and mass, and $e_0$ is the vacuum permittivity, $\alpha = \alpha_1 + \alpha_2$ is the total loss coefficient, $\alpha_1$ is the fiber loss, $\alpha_2 = (A_{eff} U_l) / (2 |\Psi|^2) \partial_t n_e$ is the ionization-induced loss term, $A_{eff}$ is the effective mode area, $\Delta |\Psi|^2 = |\Psi|^2 - |\Psi|^2 = I_{th} A_{eff}$, $|\Psi|^2 = I_{th} A_{eff}$, and $n_T$ is the total number density of ionizable atoms in the fiber, associated with the maximum plasma frequency $\omega_p = \left[ e^2 n_T / (e_0 m_e) \right]^{1/2}$. In these coupled equations, the recombination process is neglected since the pulse duration (of the order of tens of fs) is always shorter than the recombination time. If $|\Psi|^2$ is measured in W, $\hat{\sigma}/A_{eff} \equiv \gamma$ has the dimensions of W$^{-1}$m$^{-1}$. This is the nonlinearity associated with plasma formation in the fiber. According to recent experimental measurements, $\gamma_k$ depends linearly on the gas pressure. These coupled equations (2) are the most important contribution of the present work.

In order to extract useful analytical information from Eqs. (2), further simplifications are necessary. For pulses with maximum intensities just above the ionization threshold (which we dub floating pulses), the ionization loss is not large and can be neglected to a first approximation. For such pulses, only a small portion of energy above the threshold intensity contributes to the creation of free electrons. Furthermore, for floating pulses one can remove the $\Theta$-function from the equations, provided that the cross-section $\hat{\sigma}$ is replaced by a properly reduced $\hat{\sigma}'$ that takes into account the overestimation of...
the ionization rate. Introducing the following rescalings and redefinitions: ξ ≡ z/z₀, τ ≡ t/t₀, Ψ₀ ≡ [κz₀]−1/2, ψ ≡ Ψ/Ψ₀, r(τ) ≡ R(t)/t₀, φ ≡ 1/2κz₀[ωb/ω₀]², φ₁ ≡ 1/2κz₀[ωr/ω₀]², and σ ≡ ησ₀/κz₀, where z₀ ≡ t₀²/|β₂(ω₀)| is the second-order dispersion length at the reference frequency ω₀, and t₀ is the input pulse duration. Hence, the two coupled equations for floating pulses can be replaced by

\[
\left[ i\partial_\xi + \hat{D}(i\partial_\tau) + r(\tau) \right] \psi(\tau) - \phi = 0 \quad (3)
\]

The effect of the Raman and ionization perturbations on the soliton dynamics in HC-PCFs can be studied using Eqs. (3). The second equation can be solved analytically, φ(τ) = φ₁ \{ 1 - e^{-\sigma r(\tau)} \}, with the initial condition φ(−∞) = 0, corresponding to the absence of any plasma before the pulse arrives. For a small ionization cross-section, φ(τ) ≃ \eta \int_{-\infty}^{\tau} |\psi(\tau')|^2 d\tau', where \eta ≡ \sigma \phi₁. Moreover, in the long-pulse limit |\psi(\tau − τ')|^2 ≃ |\psi(\tau)|^2 − \partial_\tau |\psi(\tau)|^2. This allows the two coupled equations to be reduced to a single partial integro-differential equation:

\[
i\partial_{\xi} |\psi| + \hat{D}(i\partial_{\tau}) |\psi| - \tau_\text{R} |\psi| \partial_\tau |\psi|^2 - \eta \psi \int_{-\infty}^{\tau} |\psi|^2 d\tau' = 0 \quad (4)
\]

where \tau_\text{R} ≡ \int_{0}^{\tau} r(\tau') d\tau'. This equation shows clearly that the effect of ionization is exactly opposite to that of the Raman effect: the fourth term in Eq. (4) involves a derivative of the field intensity, while the fifth term involves an integral of the same quantity. One can then conjecture that the last term will lead to a soliton self-frequency blue-shift due to ionization, instead of a red-shift due to Raman self-scattering [14]. This clearly shows that, in the range of validity of perturbation theory (i.e., for floating solitons), photoionization leads to a soliton self-frequency blue-shift [12]. This blue-shift is accompanied by a constant acceleration of the pulse in the time domain—opposite to the Raman effect, which produces pulse deceleration [14]. This blue-shift (distinct from the effects discussed in Ref. [7]) is limited only by ionization loss, which slowly decreases the pulse intensity until it falls below the threshold value.

In the presence of ionization-induced losses above the threshold intensity, Eqs. (2) must be numerically solved to study the full dynamics of floating pulses. Figures 1(c,d) show the temporal and spectral evolution of a high-order input soliton, closely following the results reported in [10]. When the intensity of the energetic pulse exceeds the threshold value as a result of self-compression, a fundamental soliton is ejected from the main pulse and continues to blue-shift until ionization loss reduces its amplitude below the threshold value. At longer distances, another compression occurs and a second soliton is generated. The use of a kagomé-style HC-PCF is essential to observe the soliton blue-shift, since conventional photonic-bandgap fibers have much stronger dispersion variations, which would quickly destabilize any possible solitary wave as in [9].

In conclusion, a direct photoionization process can act on solitons by constantly blue-shifting their central frequencies when the intensity of solitons is slightly above the photoionization threshold, thus representing the exact counterpart of the Raman self-frequency red-shift. This spectral transformation is limited only by the ionization losses. The new theoretical model, presented by Eqs. (2), is suitable for analytical calculations, and has led us to predict a number of other new phenomena such as long-range non-local correlation forces, and spectral transformation between red- and blue-shift in Raman-active gases, which we shall present at the conference. Our results reveal new physics and offer novel opportunities for the manipulation and control of the soliton dynamics inside these versatile optical waveguides.

REFERENCES