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Deployable polyhedron mechanisms constructed by connecting spatial single-loop linkages of different types and/or in different sizes using S joints

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Abstract

This paper deals with the construction of deployable polyhedron mechanisms (DPMs) by connecting spatial single-loop linkages of different types and/or in different sizes. Linkages consisted of symmetric spatial triad units, including Bricard linkages, 8R (revolute joint) linkages and 10R linkages, are adopted as faces to form polyhedrons. These mechanisms exhibit single-DOF deployable motion and can be deployed through two ways. Several mechanisms based on 8R/10R linkages have multiple modes and can switch modes through transition configurations. Three types of DPMs are addressed: the first type is assembled using distinct types of linkages; the second type is constructed by connecting the same type of linkages in different sizes; the third type has multiple layers composed of the first two types.

Keywords

Deployable mechanisms, multiple motion modes, polyhedron mechanisms, single-loop linkage, variable-DOF

1. Introduction

A number of deployable mechanisms (DMs) have been proposed due to their ability to change the shapes and sizes when encountering different environments and tasks. For example, Chen [1-2] discussed a family of foldable closed-loop over-constrained spatial mechanisms, whose mobility analysis was discussed in literatures such as [3-4]. These mechanisms have one degree-of-freedom (DOF) and can spread onto a plane or fold into a bundle. By inserting planar mechanisms into the faces or edges of polyhedrons, or connecting deployable units, deployable polyhedron mechanisms (DPMs) are obtained in [5-10]. Other types of deployable mechanisms, such as those presented in [11,12], are out of the scope of this paper.

Meanwhile, reconfigurable deployable mechanisms were investigated during the past decades. A reconfigurable and deployable mechanism constructed using 1-DOF mechanisms and has four assembly modes was introduced for a canopy [13]. Using variable revolute (R) joint, a group of reconfigurable and deployable mechanisms was set forth by Wei [14]. In [15], a reconfigurable element was presented and used as modules to design mechanisms. The mechanisms can switch modes between the Hoberman sphere motion mode and the radially reciprocating motion mode. A 16-bar mechanism that can transform between 1-DOF spherical linkage mode and 1-DOF planar linkage mode by folding or spreading was proposed in [16]. Kong [17] analyzed a parallel mechanism (PM) with multiple motion modes based on deployable Bricard linkage upper platform. A reconfigurable mechanism which can be spread onto a plane and further folded to a bundle was designed in [18]. A family of foldable eight-bar linkages that have multiple modes is discussed in [19].

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In [20], we proposed a general method to design deployable PMs and loop coupled mechanisms (LCMs) using identical loops. Based on the work in [20], this paper will deal with the construction of DPMs with distinct types of loops or the same type of loops in different sizes. DPMs with multiple layers will also be presented. This paper together with reference [20] have presented a systematic approach to the construction of a family of deployable mechanisms by connecting single-loop mechanisms composed of symmetric spatial triads using S joints. The mechanisms obtained are overconstrained. They have multiple motion modes and can switch modes through transition positions.

This paper is organized as follows: Section 2 introduces the single-loop mechanisms and the DPMs constructed by two distinct types of linkages are presented in Section 3. DPMs based on the linkages with the same type but different parameters are provided in Section 4 and the multiple-layer DPMs are assembled in Section 5. Finally, conclusions are drawn.

2. The single-loop mechanisms

In this section, the loops will be analyzed. The loops are all composed of an even number of spatial triad units [21] shown in Fig. 1(a). The adjacent joints are perpendicular to each other. As described in [20], the construction method that will be proposed in this paper also applies to any loops with symmetry, only the orthogonal loops in which the spherical (S) joints are on the medians of the triangle links are used to illustrate the method.

The structures and kinematic analysis of the orthogonal Bricard linkage have been given in [20]. This section focuses on the analysis of the orthogonal 8R linkage [Fig. 1(b)]. The 8R linkage is composed of four triad units. The link length is \( l \) and the offset of the S joint is denoted by \( k \). As calculated in [22], the DOF of the 8R linkage is two. In a general position, the S joints of the 8R linkage are not on the same plane, as shown in Fig. 2(a). In order to construct polyhedron mechanisms using 8R linkages, the S joints should be coplanar. Now it will be verified that the DOF of the 8R linkage reduces to one when the S joints are constrained to be coplanar.

![Fig. 1 Construction of the orthogonal 8R linkage: (a) the triad unit; (b-c) the orthogonal 8R linkage](image1)

![Fig. 2 Variation of the orthogonal 8R linkage: (a) the general configuration; (b) the virtual-plane-constrained configuration](image2)

Let \( C \) and \( S \) stand, respectively, for the cosine and the sine of the angles. The transfer matrix \( \mathbf{T}_{i-1}^{i} \) from \( i-1 \)th local frame to \( i \)th local frame is described as [23].
\[
\begin{bmatrix}
C \theta_i & -S \theta_i & 0 & a_{i-1} \\
C a_{i-1} S \theta_i & C a_{i-1} C \theta_i & -S a_{i-1} & -d_i S a_{i-1} \\
S a_{i-1} S \theta_i & S a_{i-1} C \theta_i & C a_{i-1} & d_i C a_{i-1}
\end{bmatrix}
\]

where

\[
\begin{align*}
a_0 &= a_1 = a_2 = a_3 = a_4 = a_5 = a_6 = a_7 = l \\
a_0 &= a_2 = a_4 = a_6 = 270^\circ \\
\theta_1 &= \theta_5 = \theta \\
\theta_3 &= \theta_7 = \theta' \\
\theta_2 &= \theta_6 = \varphi_1 \\
\theta_4 &= \theta_8 = \varphi_2
\end{align*}
\]

In the deployable mode, suppose that \( \theta' = \theta \), for the sake of conciseness [Fig.1(c)]. \( \varphi_1 = \varphi_2 \) if the DOF of the virtual-plane-constrained 8R linkage is one, similar to the Bricard linkage. Otherwise, the DOF is still two. The position vectors of the centers of the S joints with respect to the local coordinate frames are

\[
\begin{cases}
\{l/2 & 0 & k\}^T & \text{for } i = 1, 3, 5 \text{ and } 7 \\
\{l/2 & k & 0\}^T & \text{for } i = 2, 4, 6 \text{ and } 8
\end{cases}
\]

Link 1 is fixed on the ground, and the global coordinate frame is attached to the centre of R1. The position vectors of the centres of the S joints in the global coordinate frame can be calculated as

\[
\begin{align*}
\{S_2\} &= \frac{1}{2} T_1^\mathbf{2} \{2 S_2\} 1 = \begin{cases}
\{l + l C \varphi_1/2 - k S \varphi_1 & 0 \}
\end{cases} \\
\{S_3\} &= \frac{1}{2} T_3^\mathbf{2} \{3 S_3\} 1 = \begin{cases}
(l + l C \varphi_1 - k S \varphi_1 + l C \theta C \varphi_1/2) & l \theta/2 \\
(k C \varphi_1 + l S \varphi_1 + l \theta S \varphi_1/2) & 0
\end{cases} \\
\{S_4\} &= \frac{1}{2} T_4^\mathbf{2} \{4 S_4\} 1 = \begin{cases}
\{S_{4x} & S \theta[(2 + C \varphi_2)(2 - 2 k S \varphi_2)]/2 & S_{4z} & 1\}
\end{cases}
\end{align*}
\]

where

\[
\begin{align*}
S_{4x} &= l - S \varphi_1(2 k C C \varphi_2 + l S \varphi_2)/2 + C \varphi_1[(l + l C \varphi_2/2 - k S \varphi_2)] \\
S_{4z} &= C \varphi_1(k C C \varphi_2 + l S \varphi_2)/2 + S \varphi_1[2 l + C \theta(l(2 + C \varphi_2) - 2 k S \varphi_2)/2 \\
\{S_5\} &= \frac{1}{2} T_5^\mathbf{2} \{5 S_5\} 1 = \begin{cases}
\{S_{5x} & S \theta[(2 + C \theta)(2 + C \varphi_2) - 2 k S \varphi_2])/2 & S_{5z} & 1\}
\end{cases}
\end{align*}
\]

The normal vector to the plane defined by joint centres of S1, S2 and S3 is obtained as

\[
N = (S_2 - S_1) \times (S_3 - S_2) = \begin{cases}
- l \theta \theta(2 - 1 + C \varphi_1)/4 \\
S \theta(l + l C \varphi_1 - 2 k S \varphi_1)/4
\end{cases}
\]

The equation of the plane is

\[
N_x(x - l/2) + N_y y + N_z(z - k) = 0
\]

The condition that the joint centre of S4 is on the plane defined by S1, S2 and S3 is

\[
N_x(S_{4x} - l/2) + N_y S_{4y} + N_z(S_{4z} - k) = - 1/2 l S \theta[l C \varphi_1/2 - 2 k S \varphi_1/2] S \varphi_1/G_s = [l C \varphi_1/2 - 2 k S \varphi_1/2] = 0
\]

which leads to

\[
\varphi_2 = \varphi_1 \text{ or } \varphi_2 = 2 \tan^{-1}(l/2k)
\]

When \( \varphi_2 = 2 \tan^{-1}(l/2k) \), S3 and S4 are concentric, S5 is on the plane requires S1 and S2 to be also concentric. In this case, \( \varphi_1 \) is also equal to \( 2 \tan^{-1}(l/2k) \) and the linkage is in spherical 4R
linkage mode [20]. Substituting the position vectors of S5, S6, S7 and S8 into Eq. (9), it is verified that all the S joints are on the same plane when \( \varphi_2 = \varphi_1 \). Hence, \( \varphi_2 = \varphi_1 \) is the unique solution.

The 8R linkage is a single-loop mechanism, the product of the transfer matrices equals the identity matrix, which means

\[
\frac{1}{3}T_3T_2T_1T_6T_5T_4T = I
\]  

(13)

Substituting Eqs. (1), (2) and (11) into Eq. (13), \( \varphi_1 (\varphi_1 = \varphi_2) \) can be represented by \( \theta \) as

\[
\varphi_2 = \varphi_1 = \arccos \left( \frac{1 - c\theta}{c^2 + 1} \right)
\]  

(14)

The results show that the virtual-plane-constrained 8R linkage has one DOF, the same as Bricard linkage. When four spheres are constrained on the same plane, assume that the center of S1 are fixed on the plane, S3 has one translational DOF on the plane and S5 and S7 have two translational DOFs on the plane [Fig. 3(a)], The DOF of the linkage can be calculated using the conventional formula for DOF (see [24, 25] for example)

\[
M = 6(q - p) + \sum_{i=1}^{p} f_i = 6(8 - 12) + 8 + 3 + 4 + 5 \times 2 = 1
\]  

(15)

where \( M \), \( q \), \( p \), and \( f_i \) represent the mobility, the number of links, the number of joints and the freedom of the \( i \)th joint.

If \( s \) more S joints are constrained to be on the plane [Fig. 3(b)], the DOF of the virtual-plane-constrained 8R linkage obtained using the conventional formula for DOF become

\[
M = 6(q - p) + \sum_{i=1}^{p} f_i = 6(8 - 12 - s) + 8 + 3 + 4 + 5 \times (2 + s) = 1 - s
\]  

(16)

However, the DOF of the linkage is in fact still one due to the symmetry characteristics of the linkage. Therefore, the virtual-plane-constrained 8R linkage with more than four S joints is overconstrained.

Fig. 3 Sketch of the virtual-plane-constrained orthogonal 8R linkage: (a) four spheres are constrained on the plane; (b) five spheres are constrained on the plane

3. **DPMs based on different types of loops**

In this section, DPMs based on different types of loops will be constructed, by connecting the virtual-plane-constrained Bricard linkages and 8R/10R linkages. When connecting additional loops, the number of redundant constraints increases but the mechanism still has 1-DOF [26]. As shown in Fig. 4, a 1-DOF rectangular pyramid mechanism is built by connecting four Bricard linkages and one 8R linkage. In the initial position, the Bricard linkages and the 8R linkage are in the shape of regular triangles and square respectively, and the joint axis of R1 [Fig. 4(a)] in the 8R linkage and R1' in the Bricard linkage are on the same plane. The mechanism can be deployed outward, which refers to the case that the distance between the joint centers of R1 and R1' increases [Fig. 4(b)], and inward, which refers to the case in which the distance between the joint centers of R1 and R1' decreases [Fig. 4(c)].
A triangular prism mechanism is constructed using two Bricard linkages and three 8R linkages, as shown in Fig. 5. The mechanism can also be deployed outward [Fig. 5(b)] and inward [Fig. 5(c)]. Apart from the 1-DOF deployable mode, the mechanism has an additional 2-DOF translation mode, in which the Bricard linkages are immobile and the 8R linkages move as planar 4R parallelogram linkages [Fig. 5(d)]. The mechanism in the translation mode is equal to the 14-bar mechanism in Fig. 5(e), in which the top link can translate along a spherical surface. It can switch modes from the deployable mode to the translation mode through transition configuration, which is referred to the initial position shown in Fig. 5(a). In this transition configuration, the Bricard linkages and 8R linkages are in the shape of regular triangles and squares respectively. The joint axis of R1’ [shown in Fig. 5(a)] in the upper Bricard linkage is collinear with the one in the lower Bricard linkage, and perpendicular to the joint axes of R1 and R7 of the 8R linkage.

A cuboctahedron mechanism is also constructed, using six 8R linkages and eight Bricard linkages, as shown in Fig. 6. The mechanism has 1-DOF when deployed and can switch to 3-DOF cuboctahedron mechanism [27] mode [Fig. 6(f)] through the transition configuration (initial position). The mechanism in the cuboctahedron mechanism mode is equal to the mechanism shown in Fig. 6(g). As observed from the 3D model in CAD, the radius of the sphere generated by the mechanism reaches 144.63mm when deployed outward [Fig. 6(d)] and can be 105.27mm when deployed inward [Fig. 6(e)]. The deploying ratio of the mechanism is \( r = \frac{\pi r_o^3}{\pi r_i^3} = \)
144.63^3/105.27^3 = 2.59. It is noted the deploying ratio will increase if the structure is optimized to avoid link interference. A 1-DOF icosidodecahedron mechanism is built based on twelve 10R linkages and twenty Bricard linkages, as shown in Fig. 7. The deploying ratio of the mechanism is \( r = \pi r_o^3 / \pi r_l^3 = 221.85^3 / 156.65^3 = 2.84. \)

Similarly, we can construct the following 15 DPMs (shown in Table 1): rhombicuboctahedron, truncated tetrahedron, truncated octahedron, truncated cube, snub cube, great rhombicuboctahedron, truncated icosahedron, truncated dodecahedron, rhombicosidodecahedron, snub dodecahedron, pentagonal pyramid, triangular cupola, square cupola, pentagonal cupola, and pentagonal rotunda. Like the DPMs proposed in [27], the above DPMs based on rhombicuboctahedron, truncated octahedron, great rhombicuboctahedron, rhombicosidodecahedron, triangular cupola, square cupola, pentagonal cupola have an extra motion mode with 6-DOF, 5-DOF, 5-DOF, 3-DOF, 1-DOF, 2-DOF and 3-DOF respectively.

Fig. 6 The cuboctahedron mechanism: (a) initial posture; (b, d) outward deploying; (c, e) inward deploying; (f) 3-DOF cuboctahedron mechanism mode; (g) the equilateral 3-DOF mechanism
Fig. 7 The icosidodecahedron mechanism: (a) initial posture; (b, d) outward deploying; (c, e) inward deploying

4. **Prism mechanisms based on loops with different sizes**

By connecting two single-loop linkages with the same type but different sizes, PMs are constructed. The deployable prism mechanism based on two Bricard linkages will be addressed as an example to illustrate the construction method. As shown in Fig. 8, the S joints of the bigger Bricard linkage are on the medians of the triangle links, while the S joints of the smaller one have offset from the medians. The distances between the medians of the triangles within the two loops along the direction of the joint axis of R2 (or R4, R6) and R1R3 (or R3R5, R5R1) are noted as D and T respectively [Fig. 8(a)]. D and T are calculated as

\[
D = l/\sqrt{3} - l'/\sqrt{3} \quad (17)
\]
\[
T = l/2 - l'/2 \quad (18)
\]

Unlike the PM constructed by nR orthogonal linkages with the same size using n S joints in [20], the loop can only be connected to the other loop with different size by n/2 S joints. As shown in Fig. 8(a), only three S joints are used to connect the two Bricard linkages with different sizes. The ratio of one side length and the adjacent side length of the semi-regular hexagons defined by the S joints of the two Bricard linkages [the red hexagon and green hexagon respectively in Fig. 8(b)] respectively are distinct. The bigger loop rotates when deploying, if one link of the smaller Bricard linkage is fixed.
Table 1 DPMs and their DOF in motion modes except the deployable mode

<table>
<thead>
<tr>
<th>DPM</th>
<th>DOF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rhombicuboctahedron</td>
<td>6-DOF</td>
</tr>
<tr>
<td>Truncated tetrahedron</td>
<td>N/A</td>
</tr>
<tr>
<td>Truncated octahedron</td>
<td>5-DOF</td>
</tr>
<tr>
<td>Truncated cube</td>
<td>N/A</td>
</tr>
<tr>
<td>Snub cube</td>
<td>N/A</td>
</tr>
<tr>
<td>Great rhombicuboctahedron</td>
<td>5-DOF</td>
</tr>
<tr>
<td>Truncated icosahedron</td>
<td>N/A</td>
</tr>
<tr>
<td>Truncated dodecahedron</td>
<td>N/A</td>
</tr>
<tr>
<td>Rhombicosidodecahedron</td>
<td>3-DOF</td>
</tr>
<tr>
<td>Snub dodecahedron</td>
<td>N/A</td>
</tr>
<tr>
<td>Pentagonal pyramid</td>
<td>N/A</td>
</tr>
<tr>
<td>Triangular cupola</td>
<td>1-DOF</td>
</tr>
<tr>
<td>Square cupola</td>
<td>2-DOF</td>
</tr>
<tr>
<td>Pentagonal cupola</td>
<td>3-DOF</td>
</tr>
<tr>
<td>Pentagonal rotunda</td>
<td>N/A</td>
</tr>
</tbody>
</table>

a. The DOFs are those of the mechanisms in the polyhedron mechanism modes in which the two links of the triads are on the same plane.

Fig. 8 The relationship between the two Bricard linkages with different sizes: (a) the positions of the S joints; (b) semi-regular hexagons defined by the S joints

Now it will be verified that the three spheres of the bigger Bricard linkage and those of the smaller Bricard linkage form two regular triangles respectively during the deploying process (Fig. 9). The position vectors of the spheres of the bigger Bricard linkage have been given in [20], and those of the smaller Bricard linkage are to be calculated.
The joint centre of the first S joint of the smaller Bricard linkage is obtained as

\[
S'_1 = \left\{ \frac{l'}{2} - T, -D, k \right\}^T
\]  

(19)

The position vectors of the other two S joints are calculated as

\[
\begin{align*}
\{S'_2\} &= \frac{1}{2} T^2 T \left\{ \{S'_1\} \right\} = \{S'_{2x}, (l' - l)C\theta'/\sqrt{3} - (l - 2l')S\theta'/2, S'_{2x} \}^T, \\
\end{align*}
\]

(20)

where

\[
S'_{2x} = -3l'C\theta' + 3(l - 2l')C^2\theta'/2 + 3l'b - 3kab - \sqrt{3}(l - l')C\theta'S\theta'/3b
\]

\[
a = \frac{2l\theta' + 1}{2l\theta' + 1}
\]

\[
b = 1 + C\theta'
\]

\[
\begin{align*}
\{S'_3\} &= \frac{1}{2} T^2 T^2 T^2 \left\{ \{S'_1\} \right\} = \{S'_{3x}, S'_{3y} \}^T, \frac{1}{2}a + k(l/c - 1) - 1^T
\end{align*}
\]

(21)

The centres of the three spheres form a triangle, whose squared side lengths are

\[
|S'_2 - S'_1|^2 = |S'_3 - S'_2|^2 = |S'_1 - S'_3|^2 = \left[ C\theta'(D - D + S\theta'\left(\frac{l}{2} - l'\right)) \right]^2
\]

\[
+ \left\{ \\frac{l'}{2} + T - ka - \frac{c\theta'}{2b} \left( l' - \frac{l}{2} \right) C\theta' + 2(l' + S\theta'D) \right\}^2
\]

\[
+ \left[ k - l'a + \frac{kC\theta'}{b} + S\theta'ad + C\theta' a\left(\frac{l}{2} - l'\right) \right]^2
\]

(22)

Equation (22) indicates that the triangle formed by the three spheres of the smaller Bricard linkage is a regular triangle. Based on the results in [20], the squared side lengths of the triangle defined by the spheres of the bigger Bricard linkage are yielded as

\[
|S'_2 - S'_1|^2 = |S'_3 - S'_2|^2 = |S'_1 - S'_3|^2
\]

\[
= \left\{ -6kl\sqrt{3} - 9l\theta'(\sqrt{3} + 4k^2 + l^2[3 + C\theta(3 + C\theta)]/[2(1 + C\theta)] \right\}
\]

(23)

where

\[
d = 1 + 2C\theta
\]

The triangle defined by the three spheres of the bigger Bricard linkage also keeps as a regular triangle. There always exist \(\theta\) and \(\theta'\) to equalise the two side lengths. Let \(l = 0.05\text{m}, k = 0.02\text{m}\) and \(l' = 0.03\text{m}\), \(|S'_2 - S'_1|^2 = |S'_2 - S'_3|^2 = 0.0044\text{m}^2\) in the initial position, when \(\theta = 120^\circ\).

Based on the results, a prism mechanism is constructed using the two Bricard linkages with different sizes, as shown in Fig. 10. The mechanism can be deployed outward [Fig. 10(b)] and inward [Fig. 10(c)].
A prototype of the prism mechanism is fabricated to verify the feasibility of the mechanism (Fig. 11). The links are 3D printed with magnet disc inside, and the S joints are designed using steel balls.

Similarly, a prism mechanism based on two 8R linkages with different sizes is built, as shown in Fig. 12. Unlike the PM in [20], the mechanism has no planar 4R linkage mode or spherical 4R linkage mode. The distances between the medians of the triangles within the two loops along the direction of R4R8 and R1R7 are noted as D and T respectively. D and T (shown in Fig. 13) are calculated as

\[ D = l - l' \]  
\[ T = l/2 - l'/2 \]
5. Multiple-layer DPMs

In this section, multiple-layer DPMs are constructed, based on the DPMs proposed above and in [20]. Two cases are considered, including extending DMs along axial direction and radial direction respectively.

5.1 Axial direction

By overlaying the DPMs, the mechanisms are extended along axial direction. The 1-DOF \( n \)-layer DM in Fig. 14 is connected using \( n \) Bricard linkage-based PMs proposed in [20]. Let \( R1’, R2’, R3’, R4’, R5’ \) and \( R6’ \) of the first PM be coincident with \( R1, R2, R3, R4, R5 \) and \( R6 \) of the second PM respectively, a double layers DM is obtained. Using the construction method, multiple-layer DM can be constructed. The mechanisms can be deployed outward [Fig. 14(c)] and inward [Fig. 14(d)]. It can be applied to sunshield [28], or other aerospace mechanisms [29-30].

A three-layer prototype is 3D printed with rigid links and flexible joints. The compliant prototype has higher folding ratio than the rigid mechanism in Fig. 14. The mechanism is in the shape of a prism in its stable position [refers to the initial state in Fig. 15(a)] and can be folded into four layers through two approaches [Fig. 15(b) and Fig. 15(c) respectively]. The flexible joints also provide a method to develop origami mechanisms into think panel foldable mechanisms. It is noted that the mechanisms with offset rigid joints (as adopted in [19, 31-32]) can also be folded into several layers. The folding ratio can be expressed as

\[
r = \frac{2kn}{(n + 1)t}
\]

where \( n, k \) and \( t \) represent the number of the Bricard-based PMs that used to construct the mechanism, the offset of the S joints and the thickness of the triangle links respectively.

Equation (26) shows that the folding ratio increases with the increase of \( k \) and decrease of \( t \). For
the prototype shown in Fig. 15, we have \( n = 3, k = 0.02m, t = 0.005m \), and its folding ratio is 6.

![Fig. 15](image)

(a) (b) (c)

Fig. 15 The prototype of the multiple-layer DM constructed using Bricard linkages: (a) initial posture; (b) outward deploying; (c) inward deploying

A 1-DOF three-layer DM based on Bricard linkages with different sizes (Fig. 16) is also constructed, using two DMs in different sizes and one variation of PM in [20]. The mechanism has higher stowed-to-deployed ratio than the mechanism in Fig. 14. It is measured in the CAD software that the height of the DM is 143.87mm in the initial state [Fig. 16(b)] and can reach to 74.99mm [Fig. 16(d)] when deployed inward. The deploying ratio is 1.92.

![Fig. 16](image)

(a) (b) (c) (d)

Fig. 16 The multiple-layer DM constructed using Bricard linkages with different sizes: (a) the construction method; (b) initial posture; (c) outward deploying; (d) inward deploying

By joining \( n \) DMs in Fig. 5 and \( n-1 \) PM in [20] along axial direction, multiple-layer DMs are obtained. The DM has one DOF when deployed and the DOF is \( 2n \) in the translation mode. For example, the two-layer DM in Fig. 17 has four DOF in the translation mode.
Fig. 17 The multiple-layer DM constructed using Bricard linkages and 8R linkages: (a) the construction method; (b) initial posture; (c) outward deploying; (d) inward deploying; (e) 4-DOF translation mode

Fig. 18 The double-layer deployable unit constructed using Bricard linkages: (a) initial posture; (b) outward deploying; (c) inward deploying

Fig. 19 The deforming process: (a) prism mechanism configuration; (b-e) deforming process; (f) double-layer configuration
5.2 Radial direction

Extending the mechanisms along radial direction refers to constructing polyhedrons whose faces have double layers. First, 1-DOF double-layer unit is designed, as shown in Fig. 18. The unit is obtained by deforming the prism mechanism in Fig. 10, through the deforming process presented in Fig. 19. The bigger Bricard linkage deploys while rotating, when fixing one link of the smaller Bricard linkage. In the initial state, R1//R1’ (R1//R1’ are parallel) and R2 and R2’ are collinear [Fig. 18(a)].

A double-layer prism mechanism is obtained by connecting two double-layer units, as shown in Fig. 20. The two units are symmetric about the mirror plane defined by the S joints. The PM has one DOF and can be deployed outward [Fig. 20(b)] and inward [Fig. 20(c)]. The interference problem of the S joints can be solved by using steel balls and links with magnet disc.

Fig. 20 The double-layer prism mechanism constructed using Bricard linkages: (a) initial posture; (b) outward deploying; (c) inward deploying

Fig. 21 Variation of the double-layer deployable unit

In order to construct DPMs using the double-layer units, three additional S joints are inserted on the bigger Bricard linkage, as shown in Fig. 21. Based on the modified units, 1-DOF double-layer tetrahedron DM and octahedron DM are obtained, as shown in Figs. 22 and 23 respectively.

DPMs constructed using 8R/10R-based double-layer units can also be built, which can be deployed but have no planar or spherical 4R/5R linkage mode.
Fig. 23 The double-layer octahedron DM constructed using Bricard linkages: (a) initial posture; (b) outward deploying; (c) inward deploying

6. Conclusions

A construction method for designing DPMs has been addressed. Spatial single-loop linkages, such as Bricard linkages, 8R linkages and 10R linkages composed of several symmetric spatial triad units are connected using S joints. The DPMs have only one DOF when deployed and several mechanisms involving 8R/10R/12R linkages have multiple modes. The DPMs with different types of loops, the same type of loops but different sizes and multiple-layer DPMs have been discussed respectively.

This paper and reference [20] have presented a systematic approach to the construction of a family of deployable mechanisms by connecting single-loop mechanisms composed of symmetric spatial triads using S joints. The single-layer DPMs have potential applications for decorations and the multiple-layer DPMs can be used in applications that requires large folding ratio, such as sunshield or other aerospace mechanisms [28-30].

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References


