Characteristics of Wave Breaking and Blocking by Spatially Varying Opposing Currents

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Key Points:

- Current-induced breaker has a limiting wave steepness of \(ak = 0.3\), much smaller than that for a Stokes wave at deep water.
- Wave set-down and set-up exist by competing with the pressure gradient caused by the current-induced surface tilting.
- An undertow-like change to the current profile occurs due to wave breaking.
- Turbulence and vorticity generated by breaking wave are advected downstream by current, and interact with those generated by the following wave and current.
- Wave skewness and asymmetry experience drastic changes near breaking and blocking points.
Abstract

A Reynolds-Averaged Navier-Stokes (RANS) flow solver with a Volume of Fluid (VOF) surface capturing scheme is used to investigate wave breaking and blocking due to strong opposing currents. The Shear Stress Transport (SST) $k-\varepsilon$ model is modified in order to capture the turbulence properly. The unique capability of the RANS-VOF model allows us to reveal the distinct features of current-induced wave breaking and blocking. The limiting wave steepness at breaking onset is reduced considerably by the opposing current from the Stokes' limit for stationary waves. Although the wave height grows and decays faster before and after breaking under a more rapidly varying current, the maximum height at the breaking onset remains almost the same. The locations of incipient breaking are slightly shifted downsteam/upwave by the increased current gradient. It was found that the wave radiation stress alters the mean water level by competing with the pressure gradient arising from the current-induced surface tilting. The wave set-down and set-up is more pronounced for more intensive breakers occurring under a larger incident wave and current gradient. Our model results indicate an undertow-like change to the current profile due to wave breaking. The turbulence and vorticity generated by the current-induced breaking wave persist and are advected downstream and interact with those generated by the following wave and current.

1 Introduction

Wave breaking is a ubiquitous phenomenon that takes place at the ocean surface from the deep ocean to the surf zone near the coastline. It plays an important role in physical processes such as upper-ocean dynamics, air-sea interaction, coastal mixing and dispersion of nutrients and pollutants, nearshore circulation, and coastal morphodynamics. It is also important for practical applications such as ocean remote sensing and offshore and maritime operations. In the past decades, much of theoretical, experimental, numerical, and field studies have been dedicated to better understanding the wave breaking process. A number of review papers are available for breaking waves in deep and intermediate waters (Banner & Peregrine, 1993; Melville, 1996; Perlin et al., 2013) and in the surf zone (Peregrine, 1983; Battjes, 1988; Svendsen & Putrevu, 1996).

Wave breaking can be generated by a number of mechanisms including depth-induced, dispersive focusing, modulational instability, wind-wave, wave-current, and wave-structure interactions (Perlin et al., 2013). The current-induced breaking is one of the least understood breaking mechanisms. Waves propagating towards the mouth of an estuary or river often break during ebb tide when a strong opposing current is present and may cause navigation hazards (Zippel & Thomson, 2015, 2017). Observations at the Columbia River plume showed that the current-induced wave breaking is an important source of turbulence and enhanced mixing at the offshore front of the plume (Thomson et al., 2014). The interactions between waves and ebb tide or fluvial discharge at river mouth has significant implications on the morphodynamics and transport processes within river deltas and estuaries (Dodet et al., 2013; Olabarrieta et al., 2014; Anthony, 2015). Waves may be partially or completely blocked when encountering a strong opposing current in the open ocean and near the coast, causing extreme waves and marine hazards (Hjelmervik & Trulsen, 2009; Onorato et al., 2011; Toffoli et al., 2015; Ardhuin et al., 2017; Romero et al., 2017). It is well known that the spectral wind wave model SWAN (Booij et al., 1999) tends to overestimate wave heights in partially blocking currents with negative
gradients (Ris & Holthuijsen, 1996; Dodet et al., 2013; van der Westhuysen et al., 2012; Rapizo et al., 2017). The default dissipation calibrated for wind wave growth conditions is not suitable for steep waves approaching the blocking point. A new saturation-based whitecapping formulation was proposed by van der Westhuysen (2012) to enhance the current-induced wave dissipation in the far field (non-blocking or partial blocking conditions). Yao and Wu (2004) proposed a new spectral parameterization to characterize the dissipation of unsteady waves in the presence of currents. Better understanding of wave breaking and blocking by currents is necessary to improve the reliability of wave forecasts. The present study aims to study the geometric and hydrodynamic characteristics of wave breaking and blocking caused solely by strong opposing currents with varying magnitude in the wave direction.

Several laboratory experiments have been dedicated to wave blocking in spatially varying opposing currents. Lai et al. (1989) investigated the kinematics of wave-current interactions and confirmed that waves are blocked when the current velocity reaches one-quarter of the deep-water wave phase velocity. Chawla and Kirby (2002) proposed a modified bore model to quantify the wave dissipation in the presence of currents. It was found that the current-induced breaking is different from the depth-induced breaking in many ways, for example, the former is weak and unsaturated while the latter is intensive and saturated. Ma et al. (2010) observed a frequency downshift in both non-breaking and breaking waves in a spatially varying opposing current. In all these experiments, a variable cross-section is adopted to create a spatial gradient for the current with a constant discharge, by placing either a false bottom or side wall along part of the flume. As a result, even in the absence of current, the wave would shoal over a sloping bottom or steepen in a narrowing flume (see also Smith & Seabergh, 2001). It is therefore difficult to separate the effect of current from that of variable depth or channel width. In contrast, Suastika et al. (2000) designed a novel experiment by keeping the flume cross-section constant but the discharge varied along the flume, thereby created a longitudinal variation for the current without altering the water depth or the flume width. This unique setup allows us to study the physical processes associated with breaking waves induced solely by the opposing current.

Recent progress in breaking wave research has enabled us to identify the connections between crest geometry and energy dissipation (Perlin et al., 2013). One of the most used geometric properties of breaking waves is wave steepness. The limiting steepness at which incipient wave breaking occurs depends on the generation mechanism of breaking. Tian et al. (2012) identified breaking onset by visually locating vertical wave crest fronts, and found that dispersive wave energy focusing may cause waves to break at a smaller steepness than modulational instability. Wu and Yao (2004) found that strong opposing current causes partial wave blocking and significantly increases the limiting steepness of the dispersive focusing wave group. The vertical current shear strength also affects the limiting steepness of incipient breakers (Yao & Wu, 2005). Ma et al. (2013) observed, however, that the limiting steepness of a modulated wave group propagating against an opposing current was smaller than that in quiescent water. They also found that the opposing current has little effect on other geometric properties of extreme waves such as skewness and asymmetry.
Figure 1. Definitions of local wave parameters. Wave steepness $ak = \pi H/L$. Crest-front steepness is defined as $\varepsilon = h'/L'$ following Kjeldsen and Myrhaug (1979). MWL indicates mean water level.

Compared with wave steepness $ak$, the crest-front steepness $\varepsilon = h'/L'$ (Figure 1) is more appropriate to describe the local crest geometry and therefore a better indicator for breaking onset (Perlin et al., 2013). Note that for a second-order Stokes wave in deep water, the limiting wave steepness and crest-front steepness are 0.44 and 0.40, respectively. Kjeldsen and Myrhaug (1979) reported that the limiting crest-front steepness ranges between 0.32 and 0.78. Bonmarin (1989) found that the average crest-front steepness at breaking onset increases from 0.38 for spilling breakers to 0.61 for plunging breakers. Wu and Nepf (2002) reported the same limiting crest-front steepness for 2D spilling breakers. The crest-front steepness for 3D breaking waves increases slightly to 0.39 for spilling breakers with directional spreading, and to 0.41 for those with directional focusing (Wu and Nepf, 2002). Statistical analysis by Toffoli et al. (2010) for a large data set of laboratory and field measurements of the surface elevation, however, suggests that waves can attain a crest-front steepness of 0.55.

The Reynolds-Averaged Navier-Stokes (RANS) solver along with a Volume of Fluid (VOF) free surface capturing scheme and a turbulence closure model has become popular in studying breaking waves in the surf zone (Lin & Liu, 1998a, b; Mayer & Madson, 2000; Wang et al., 2009b; Bakhtyar et al., 2010; Pedrozo-Acuña et al., 2010; Jacobsen et al., 2012, 2014; Xie, 2013; Chella et al., 2016). Brown et al. (2016) evaluated the performance of five turbulence models in OpenFOAM for simulating the surf zone breakers and found that all five models reasonably predict, some even over-predict, the maximum wave height at the breaking onset. While these models could predict the wave setup reasonably well, they tend to over-predict the eddy viscosity and the vertical shear in undertow profiles (Derakhti et al., 2016). Lin and Liu (1998a) argued that the RANS-VOF model has difficulty resolving the production of turbulence in a rapidly distorted shear flow at the initial stage of wave breaking, partly due to the fact that these closure models were originally developed for quasi-steady not transient turbulent flows. To alleviate this problem, Chella et al. (2016) implemented additional turbulence damping scheme at the interface proposed by Naot and Rodi (1982). It is recommended that numerical simulation of wave breaking should be validated by available experiments due to the formidable complexity of the problem.

The large-eddy simulation (LES) models have this tendency as well in their applications in two-phase mixture based flow solvers, such as OpenFOAM’s interFoam solver, (Christensen,
2006; Zhou et al., 2017). For example, both the LES simulation of wave breaking over a barred beach by Zhou et al. (2017) and the RANS model by Jacobsen et al. (2014) over-predict the turbulence intensity near the breaking point. This is largely due to the fact that the two-phase flow solver does not satisfy the boundary conditions at the air-water interface well (Vukčević et al., 2017), therefore, generates spurious velocities in the air phase. The problem is exacerbated by the lack of specific interface boundary conditions for the turbulence closure models. Including density explicitly in the turbulence transport equations, as in Jacobsen et al. (2012) and Brown et al. (2016), may alleviate but not eliminate the problem. Neglecting turbulence exchange between air and water, Lin and Liu (1998a) implemented the zero-gradient boundary condition at the free surface as an approximation for one-phase closure models of breaking waves. Recognizing the similarity between the near-interface boundary layer and near-wall viscous sublayer, Lakehal and Liović (2011) introduced an exponential damping function to the eddy viscosity in the vicinity of the air-water interface. Zou and Chen (2017) found that in order for the two-equation turbulence closure models to produce a reasonable profile of wind blowing over a calm water surface, it is necessary to apply a near-wall-like damping function at the interface.

Recent development of the Eulerian-Eulerian two-fluid model also shows promise to resolve this issue. Ma et al. (2011) found that the presence of bubbles may suppress the turbulence under breaking waves, and concluded that excluding the bubble effects may have led to the overestimation of turbulence intensity by previous RANS closure models. The LES simulations by Derakhhi and Kirby (2014) showed that the turbulence in the breaking region was damped by the dispersed bubbles by approximately 20% for a large plunging breaker and by 50% for a spilling breaker. These studies suggest that incorporating more physics-based interface boundary conditions, such as those proposed by Brocchini and Peregrine (2001) and Brocchini (2002), would improve the prediction of the two-phase wave breaking process.

The current-induced breaker differs from the depth-limited surf zone breaker in the underlying mechanism for the wave to become increasingly nonlinear and steep. The free surface capturing scheme and the turbulence model capable to resolve the complex process of surf zone breakers are expected to capture the current-induced breakers as reasonably well. The RANS-VOF models have been applied to deepwater breaking waves (e.g., Lubin et al., 2006; Wang et al., 2009a; Iafrati, 2011; Xie, 2012), the intermediate depth waves breaking over a submerged object (Lupieri & Contento, 2015), but not current-induced breaking waves. Previous models of wave breaking cannot resolve the wave breaking process directly, e.g. the inviscid potential flow theory based model (Moreira & Peregrine, 2012; Moreira & Chacaltana, 2015), the surface-following Navier-Stokes equation based model (Mayer et al., 1998; Wu et al., 2010), the Boussinesq-type model (Chen et al., 1998; Zou et al., 2013), and the mild-slope equation based model (Kirby, 1984; Chen et al., 2005; Toledo et al., 2012; Touboul et al., 2016). The RANS-VOF model is a more suitable tool for elucidating the underling physics associated with wave breaking and blocking. Zou and Chen (2016, 2017) studied the wind and current effect on the evolution of a steepness-limited, plunging breaking wave group, using a RANS-VOF solver (Jacobsen et al. 2012) and a mixing-length turbulence model. The wave height evolution was well captured by the model (Zou & Chen 2016, 2017). It was found that the following and opposing winds shift the focus point downstream and upstream mainly due to the action of wind-driven current instead of direct wind forcing. For the purpose of the present 2-D modeling study, the LES models have no apparent advantage over their RANS counterparts adopted here (Jacobsen et al. 2014; Zhou et al., 2017; Zou & Chen, 2017), despite that LES models could capture the 3-D coherent turbulent structures after wave breaking. It is necessary to conduct
comprehensive experimental measurements of turbulence associated with current-induced breaking to examine the performance of these turbulence models in the future.

As indicated above, previous studies of current-induced wave breaking and blocking have mainly focused on the wave profile changes, breaking onset and wave dissipation. The objective of this paper is to investigate wave breaking and blocking processes solely due to strong opposing currents with variable strength in the wave direction with special attentions to mean water level, circulation pattern, turbulence and vorticity. The SST $k-\omega$ turbulence model by Mentor (1994) is extended by modifying the production term to avoid excessive turbulence production in the interior flow (Mayer & Madsen, 2000). A novel numerical wave-current flume is developed to study the wave breaking and blocking solely due to current, by excluding the effect of varying water depth and flume width as in Suastika et al.’s (2000) experiment.

Following the introduction in Section 1, the model is described in Section 2, and validated in Section 3 against the measurements from a novel experiment on wave blocking. In Section 4, the model simulation is used to examine the geometric and hydrodynamic characteristics of the current-induced wave breaking, i.e., the crest geometry at breaking onset, kinematic breaking criterion, wave set-down and set-up, energy dissipation, turbulence and vorticity distribution, and current profile changes due to breaking. The effects of horizontal gradient of current on these processes are also investigated. Conclusions are drawn in Section 5.

2 Methodology

2.1 RANS-VOF free surface flow solver

The RANS-VOF model solves the Reynolds-Averaged Navier-Stokes (RANS) equations for the mean flow field, with the fluctuating components incorporated by a turbulence closure model. The present study adopts the RANS-VOF flow solver in the open source CFD toolbox OpenFOAM, which uses a collocated unstructured grid, finite volume method, and the VOF surface capturing method (Hirt & Nichols, 1981; Jasak, 1996; Scardovelli & Zaleski, 1999; Rusche, 2002; Weller, 2005; Sussman et al. 2007; Lv et al., 2010, 2012).

The model solves the pressure and velocity fields from the continuity and momentum equations for an incompressible fluid

$$ \nabla \cdot \mathbf{U} = 0 $$

$$ \frac{\partial \rho \mathbf{U}}{\partial t} + \nabla \cdot (\rho \mathbf{U} \mathbf{U}) - \nabla \cdot (\mu_{\text{eff}} \nabla \mathbf{U}) = -\nabla p^* - \mathbf{g} \cdot \mathbf{X} \rho + \nabla \mathbf{U} \cdot \nabla \mathbf{U} \cdot \mu_{\text{eff}} \quad (2) $$

where $\mathbf{U}$ is the velocity vector, $\rho$ the fluid density, $p^* = p - \rho \mathbf{g} \cdot \mathbf{X}$ the pseudo-dynamic pressure, $\mathbf{g}$ the gravitational acceleration, $\mathbf{X}$ the position vector, and $\mu_{\text{eff}} = \mu + \rho \nu_t$ the effective dynamic viscosity, which takes into account of molecular dynamic viscosity $\mu$ and turbulent eddy viscosity $\nu_t$.

The free surface is captured by solving the VOF advection equation (Weller, 2005)

$$ \frac{\partial \alpha}{\partial t} + \nabla \cdot (\mathbf{U} \alpha) + \nabla \cdot [\mathbf{U} \alpha (1 - \alpha)] = 0 \quad (3) $$
where $\alpha$ is an indicator function to mark the location of the air-water interface, and $\mathbf{U}$ is a velocity field introduced to compress the interface. $\alpha = 1$ if the cell is full of water, $\alpha = 0$ if the cell is full of air, and $0 < \alpha < 1$ if the cell is a mixture of the two fluids. An extra convective term, $\nabla \cdot [\mathbf{U}, \alpha (1 - \alpha)]$, is added to the classic VOF transport equation to limit the smearing of the interface. More details about the VOF method can be found in Rusche (2002).

In the present study, an extended version of the OpenFOAM’s RANS-VOF solver, the waves2Foam package (Jacobsen et al., 2012), is employed to investigate the phenomenon of wave blocking. The package includes wave generation and absorption using the relaxation zone technique, which requires longer computational domains (1~2 wave lengths) to minimize the wave reflection. Alternatively, Higuera et al. (2013) presented an active absorbing boundary condition to cope with the wave reflection without incurring extra CPUs. However, this approach does not include currents and assumes shallow water waves, so that it is not applicable to waves in the intermediate or deeper water. This study examines the blocking of waves in relatively deep water, which will propagate out of the computational domain if not completely blocked by the opposing currents. Therefore, the relaxation zone technique is adopted here since it works adequately even in the presence of currents.

### 2.2 Governing equations for porous media flow

A perforated false bottom is often used in experiments to generate currents. Instead of simulating explicitly the perforated false bottom, we model it as a thin layer of porous media continuum, through which the water flows out of the bottom smoothly. The continuity and momentum and VOF equations for the fluid, Eqs. (1)~(3), need to be modified before their applications to flow in a porous media. The Darcy-Forchheimer approximation is used to model the flow resistance due to the presence of porous media (Whitaker, 1996). The momentum equation including the effect of porous media is given by (Higuera et al., 2014)

$$
\frac{1 + c}{n} \frac{\partial \rho \mathbf{U}}{\partial t} + \frac{1}{n} \nabla \cdot \left( \rho \mathbf{U} \mathbf{U} \right) - \frac{1}{n} \nabla \cdot (\mu_{se} \nabla \mathbf{U}) = -\nabla p^* - \mathbf{g} \cdot \nabla \rho + \frac{1}{n} \nabla \mathbf{U} \cdot \nabla \mu_{se} - \mathbf{F}_p \tag{4}
$$

where $n$ is the porosity defined as the volume ratio of voids over a control volume, $\mathbf{U}$ is now the volume-averaged Darcy velocity related to pore (intrinsic) velocity, $\mathbf{U}^* = \mathbf{U} / n$, $c$ is the coefficient accounting for the added mass effect, and $\mathbf{F}_p$ is the resisting force due to the presence of the porous media

$$
\mathbf{F}_p = a \mathbf{U} + b \rho |\mathbf{U}| \mathbf{U} \tag{5}
$$

The resistance coefficients, $a$ and $b$ in Eq. (5), are respectively due to the linear and nonlinear quadratic friction. They have been parameterized depending on the fluid viscosity, porosity and mean nominal diameter of the porous media, and the Keulegan-Carpenter number for oscillatory flow (Van Gent, 1995).

The VOF advection equation accounting for the effect of porosity is given by

$$
\frac{\partial \alpha}{\partial t} + \frac{1}{n} \nabla \cdot (\mathbf{U} \alpha) + \frac{1}{n} \nabla \cdot [\mathbf{U}, \alpha (1 - \alpha)] = 0 \tag{6}
$$

where the correction factor $1/n$ ensures that the indicator function $\alpha$ is bounded between 0 and 1 even in the presence of porous media.
The above governing equations (4) to (6) have been incorporated independently by Higuera et al. (2014) and Jensen et al. (2014) into the OpenFOAM-based RANS-VOF solver. The implementation of porous media flow model by Higuera et al. (2014) does not interfere with the mesh generation. Thus after generating the mesh, the user still has the freedom to designate areas occupied by porous media. It is straightforward to assign different properties, such as porosities and resistance coefficients, to different layers of porous media. We adopt the approach by Higuera et al. (2014) in this study, and implemented this porous media flow treatment and incorporated it in the waves2Foam package (Jacobsen et al., 2012), along with the modified SST $\omega$ closure model introduced in the next section.

### 2.3 Turbulence modeling

One of the turbulence models embedded in OpenFOAM and waves2Foam, the Shear-Stress Transport (SST) $k-\omega$ model, is improved and used in this study due to its merits in flow with adverse pressure gradients and separations. Menter (1994) introduced this two-equation turbulence model by combining the best features of the $k-\varepsilon$ model in the free shear flow and the $k-\omega$ model in the inner part of the boundary layer. The model solves the transport equations for the turbulent kinetic energy, $k$, and the specific dissipation, $\omega$,

$$\frac{\partial \rho k}{\partial t} + \nabla \cdot (\rho \mathbf{U} k) = \tilde{P}_i - \beta' \rho e k + \nabla \cdot \left( \left[ \mu + \sigma_1 \mu_1 \right] \nabla k \right)$$

(7)

$$\frac{\partial \rho \omega}{\partial t} + \nabla \cdot (\rho \mathbf{U} \omega) = \frac{u_\lambda}{\nu} \tilde{P}_i - \beta' \rho e \omega + \nabla \cdot \left( \left[ \mu + \sigma_2 \mu_2 \right] \nabla \omega \right) + 2(1 - F_i) \rho \sigma_w \frac{1}{\omega} \nabla k \cdot \nabla \omega$$

(8)

where a production limiter is used to prevent the build-up of turbulence in stagnant regions,

$$\tilde{P}_i = \min(P_i, 10 \beta' \rho e \omega)$$

(9)

The blending function in Eq. (8), $F_i$, is equal to zero outside the boundary layer (the $k-\varepsilon$ model), and switches to one inside the boundary layer (the $k-\omega$ model).

In standard formulations the production, $P_i$, is based on the strain rate tensor,

$$S = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$$

(10)

Using linear stability analysis, Mayer and Madsen (2000) demonstrated that this formulation generates turbulent kinetic energy in a potential flow or the interior part of the flow just outside of the boundary layer. Jacobsen et al. (2012) reported that waves at seaward side of the breaking point were dissipated due to the nonphysical non-zero shear in the interior part of the flow. We observed the same phenomenon from the model run over tens of wave periods, even when the near-wall-like turbulence damping was applied at the interface (Naot & Rodi, 1982; Chella et al., 2016). The unphysical wave dissipation was more significant for short waves than for long waves. Mayer and Madsen (2000) suggested that using a rotation-based production term would eliminate the problem of spurious turbulence generation. In this study, following Mayer and Madsen (2000), the production term is formulated as

$$P_i = \rho \nu \Omega$$

(11)
where $\Omega = \sqrt{2W_i W_j}$ is the magnitude of the rotation tensor

$$W_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i} \right). \quad (12)$$

The turbulent eddy viscosity is given by

$$\nu_t = \frac{a_k}{\max(a, \omega, SF_i)} \quad (13)$$

where $S = \sqrt{2S_i S_j}$ is the magnitude of strain rate, and $F_i$ is the second blending function. More detailed information about Menter’s (1994) Shear-Stress Transport (SST) $k - \omega$ model including the closure constants, $\sigma_i$, $\sigma_k$, $\beta$, $\gamma$, and the blending functions, $F_i$ and $F_j$, can be found in Appendix A.

### 2.4 Initial and boundary conditions

The first step to model wave blocking is to generate a fully developed, spatially varying opposing current field, against which the wave will propagate. The initial condition for the current-only flume is a given current field which typically does not satisfy mass conservation at each longitudinal cross-section of the flume, which leads to free surface disturbances. To damp these undesired disturbances, two relaxation zones are adopted at the left and right boundaries of the numerical flume (see Figure 2). Once the opposing current field reaches an equilibrium state, waves are generated by specifying the water particle velocities at the wavemaker boundary.

The inflow boundary condition for the numerical wave-current flume is case dependent. If the water current into the flume is withdrawn out of the flume fully via the bottom (see Figure 2), it is unnecessary to superimpose additional current velocity at the wavemaker. Otherwise, the current velocity is imposed at the wavemaker in addition to the wave velocity. No-slip condition is imposed on the solid boundary at the bottom. Open air boundary is applied to the atmosphere above the free surface.

For the turbulence field, the law of the wall is applied near the solid boundary to the turbulent kinetic energy $k$, specific dissipation $\omega$, and eddy viscosity $\nu_t$. A small amount of turbulent kinetic energy is seeded in both the initial and inflow boundary conditions, $k = 0.5(\delta C)^5$ where $C$ is wave celerity and $\delta = 2.5\times10^{-3}$. The corresponding initial and inflow conditions for the specific dissipation are determined from Eq. (13) and a small eddy viscosity ratio $\xi = 10$ so that $\nu_t = \xi(\mu/\rho)$ (Lin & Liu, 1998a).

### 3 Model validation

The model was validated in Appendix B for a deep water steepness-limited, dispersive focusing breaking wave group without wind and current effect, by comparisons with the model by Zou and Chen (2016, 2017) and the experiment Case DF2 in Table 1 of Tian and Choi (2013). The model has also been verified with the theory of wave action conservation (Bretherton & Garrett, 1968) for a linear wave propagating against an opposing current, in which case the spatially varying current is achieved by placing a submerged bar over an otherwise flat bottom.

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Readers are referred to Chen (2017) for detailed descriptions of the model setup and verification. As described in the introduction, this setup would lead to the depth-induced changes of wave wavelength and amplitude and therefore wave breaking if the incident wave is large enough. As the objective of the present study is to elucidate the current effect on wave breaking, a different approach is adopted in this study to generate the spatially varying opposing current without changing the water depth. By excluding the depth-induced shoaling effect in the former studies (Chen et al., 1998; Mayer et al., 1998; Zou et al., 2013), we can investigate wave breaking due to the blocking current separately. We will next describe the novel experiment designed by Suastika et al. (2000) to study wave blocking, then present our model setup and predictions compared with the experimental measurements. As the paper focuses on gravity waves instead of capillary waves, all the cases were run without switching on the surface tension effect.

3.1 Physical experiment

The 40 m long flume was equipped with a wave generator at one end and permeable wave damping materials at the opposite end where the water flows into the flume with controlled discharge (Suastika et al., 2000; Suastika, 2004). At a 12 m long measurement section in the middle of the flume, the current discharge from right to left was gradually withdrawn through a perforated false bottom and is brought to zero before reaching the left end of the flume. To better control the streamwise current discharge variations, the 12 m long measurement section was divided equally into six compartments. Downstream from the measurement section a stagnation region exists where the cross-sectional averaged current velocity is zero. It is in this region that waves were generated by a piston-type wavemaker with a second-order wave solution input and automatic reflection absorption.

![Computational domain setup to simulate the novel experimental study on wave blocking (Suastika et al., 2000). Current is introduced into the flume through the right boundary. As water flow is gradually withdrawn over the bottom portion $x = 11 \sim 23$ m, the depth-averaged current velocity decreases in magnitude downstream and is brought to zero at and beyond $x = 11$ m. Waves are generated in the stagnant region without current effect on the left boundary.](image)

Table 1. Wave and current conditions for the 16 wave blocking case studies by the present model.

<table>
<thead>
<tr>
<th>Case</th>
<th>Period $T$ (s)</th>
<th>Height $H$ (cm)</th>
<th>$kH/2$</th>
<th>$kh$</th>
<th>Current (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1*</td>
<td>1.1</td>
<td>4.6</td>
<td>0.080</td>
<td>1.91</td>
<td>$0$, $x &lt; 11$</td>
</tr>
</tbody>
</table>
| 2*   | 1.2           | 4.4            | 0.066  | 1.65 | $U_c(x) = \begin{cases} 
0, & x < 11 \\
-0.046(x-11), & 11 \leq x < 23 \\
-0.55, & x \geq 23.
\end{cases}$ |
### 3.2 Numerical flume

Figure 2 shows the present 2D model setup for the experiment on wave blocking. The same Cartesian coordinate system is used as in the experiment. The x-axis is positive in the incident wave direction, and the z-axis positive upwards with \( z = 0 \) located at the still water level. The 12 m long section with variable discharge spans from \( x = 11 \) m to \( x = 23 \) m. The maximum flow discharge in the flume is \( Q_m = 0.12 \text{ m}^3/\text{s} \), the flume width \( b = 0.4 \text{ m} \), and the still water depth \( h = 0.55 \text{ m} \). Thus the depth-averaged current velocity in the constant discharge region is given by

\[
U_c = \frac{Q_m}{bh} = -0.55 \text{ m/s}.
\]

Second-order waves are generated by the wavemaker at the left end of the flume, and a uniform current profile \( U_c = -0.55 \text{ m/s} \) is applied at the inflow boundary on the right. Two relaxation zones are used to damp the initial free surface disturbances generated by the current. They also serve to absorb the reflected and outgoing waves, respectively, at the wavemaker and current inflow boundaries. To account for the perforated false bottom in the experiment, we designate a 12 m long porous media region covering 3 layers of grids at the flume bottom. A constant velocity of 0.025 m/s is applied over the 12 m long bottom section, \( x = 11 \text{~} 23 \text{ m} \), ensuring that the mass inside the flume is conserved.

The computational domain spans 22 m in the horizontal direction and 0.7 m in the vertical. The base mesh has a uniform grid of 0.02 m in both directions. In the vicinity of the free surface, the base mesh is consecutively refined once over the region \(-0.21 < z < 0.13 \text{ m} \), and twice over \(-0.09 < z < 0.11 \text{ m} \). The finest grid size at the free surface is 0.005 m in both directions. The final mesh totals about 0.2 million cells. To avoid the possible premature wave breaking due to large grid Aspect Ratio (Jacobsen et al. 2012), \( AR = \Delta x/\Delta z = 1 \) was kept for all the cells. Although as discussed in Lubin et al. (2006) and Wang et al. (2009a), grid convergence analysis is quite difficult and questionable for this kind of flow characterized by unsteady air/water interface breaking, another set of finer grid with higher resolutions (0.0025 m) at the free surface was used for case 1 and 2, which leads to a sharper interface and more detailed interface changes in the wave breaking process. But the overall large scale dynamics of wave
breakings is not significantly affected by the finer grid after incipient wave breaking (Wang et al., 2009a).

The model is run first to generate a steady current field and then to generate waves on top of the current. The numerical flume is initialized with a spatially varying current field according to

\[
U_c(x) = \begin{cases}
0, & x < 11 \\
-0.046(x - 11), & 11 \leq x < 23 \\
-0.55, & x \geq 23.
\end{cases}
\] (14)

The horizontal shear of the longitudinally varied current is \( \partial U_c / \partial x = -0.046 \text{ s}^{-1} \), which is consistent with the experimental design in Suastika (2004). A steady current field is achieved after running the model for about 200 s. When imposing a monochromatic wave with period \( T = 1.1 \text{ s} \) on this current, it requires at least 70 periods for the waves to reach a quasi-steady state.

The effect of current gradient on wave breaking and blocking is studied by employing a second current field with twice the horizontal gradient, \( \partial U_c / \partial x = -0.092 \text{ s}^{-1} \), as listed in Table 1,

\[
U_c(x) = \begin{cases}
0, & x < 14 \\
-0.092(x - 14), & 14 \leq x < 20 \\
-0.55, & x \geq 20.
\end{cases}
\] (15)

These results will be reported in the next section. Note that the two current profiles, Eqs. (14) and (15), have the same horizontal velocity at the middle point, \( x = 17 \text{ m} \), of their respective porous bottom.
3.3 Current profiles

Figure 3a shows the predicted longitudinal distribution of the depth-averaged current velocity. The theoretical distribution is calculated by dividing the local discharge, $Q(x)$, over the cross-sectional area, $bh$,

$$U_c(x) = \frac{Q(x)}{bh}. \quad (16)$$

As a result of the uniform withdrawal through the 12 m long porous bottom, the local discharge decreases linearly downstream from $Q_m = 0.12$ m$^3$/s at $x = 23$ m to zero at $x = 11$ m. The experimental data were collected in the central plane of the measurement section. It is seen that both the prediction and the measurement show a nearly constant gradient, which is consistent with the theory (Eq. 16). The magnitudes of both the predicted and measured longitudinal velocity are larger than the theoretical value, which is expected since the theory ignores the boundary layer effects. We notice, however, that the measured velocity has even larger magnitude than the prediction. The lateral boundary layers at the flume side walls, which are neglected by the present 2D model, are partly responsible for this difference. The way the water flow was withdrawn in the physical flume through the perforated false bottom may also contribute to the discrepancy. Lateral profiles measured across half the flume width showed that the longitudinal velocity decreases linearly from the center of the flume to the side wall (Suastika, 2004). Placing a suction pipe in the center of the flume naturally makes the water flow faster at the pipe’s immediate vicinity.
Figure 3b shows the predicted vertical current profiles at four cross-sections $x = 17, 18, 19,$ and $20$ m, and the measured profiles at $x = 17, 18,$ and $19$ m. It is noticed that both sets of current profiles are approximately uniform in the middle part of the water column, and decrease slightly in magnitude towards the water surface. The flow discharge is decreasing towards the wavemaker because of the gradual withdrawal of water through the flume bottom. Therefore, the flow speed at the surface is expected to decelerate and slight vertical shear appears. Consistent with the trend of depth-averaged current velocity (Figure 3a), the predicted current profiles have smaller magnitude, and seem to lag horizontally the measured profiles by 1 m. The predicted vertical current profiles agree well with the measurements if the latter are shifted 1 m upstream, i.e. from $x = 19$ m to 20 m. This discrepancy observed in the current profiles is taken into account in the model-data comparisons of wave amplitude evolutions in the next sub-section.

3.4 Wave profile evolution

Monochromatic waves with target wave height $H = 5$ cm, 7 cm and period $T = 1.1$ s, 1.2 s for cases 1, 2, 5, 7 given in Table 1 are generated on top of the developed current field. The minimum value of current velocity to block the wave predicted by the theory in Appendix A is $U_c = -0.43$ m/s for a 1.1 s wave, and $U_c = -0.47$ m/s for a 1.2 s wave, both of which are smaller in magnitude than the maximum depth-averaged current velocity at the constant discharge region, $U_c = -0.59$ m/s. Therefore, wave blocking is expected to occur for both waves.

Figure 4 shows the snapshots of wave profiles and dynamic pressure distribution for waves with a wave period of $T = 1.1$ and 1.2 s propagating against the opposing current field. As the current velocity increases in magnitude from left to right, the waves propagating toward the right become increasingly steeper because of the shortening wavelength and amplifying wave height. The wave crest is sharpened and the trough flattened (Figure 4). The increased crest-trough asymmetry in the surface elevation towards the breaking point signifies larger wave skewness and more contributions from higher harmonics and increased nonlinearity (Elgar et al., 1990; Peng et al., 2009; Zou & Peng, 2011).
Figure 4. Wave profiles and dynamic pressure (color bar) distribution at \( t = 81 \) s. (a) Case 1: \( T = 1.1 \) s, target \( H = 5 \) cm; (b) Case 2: \( T = 1.2 \) s, target \( H = 5 \) cm. Units in Pa for dynamic pressure, \( p' \), in Eq. (2). For both cases \( U_c = -0.55 \) m/s, horizontal current gradient \(-0.046 \text{ s}^{-1} \) over \( x = 11\sim23 \) m.

The crest-trough asymmetry in Figure 4 can be quantified by the wave skewness, which represents the lack of symmetry of the wave profile relative to the horizontal axis. The non-dimensional value of the skewness can be obtained from the third moment of surface elevation normalized by the second moment of surface elevation to the power of 1.5,

\[
Sk = \frac{\langle (\eta - \langle \eta \rangle)^3 \rangle}{\langle (\eta - \langle \eta \rangle)^2 \rangle^{3/2}}.
\]  

(17)

Near the breaking point, the wave starts to pitch forward and the wave asymmetry (relative to the vertical axis) becomes negative (Figure 5b). These behaviors are similar to waves propagating over a beach where decreasing water depth has the same effect on the wavelength and propagating speed as increasing the opposing current in the wave direction (Elgar et al., 1990; Wang et al., 2009). Both wave skewness and asymmetry are important nonlinear features of a breaking wave (Babanin et al., 2007). Doering and Bowen (1995), Peng et al. (2009) and Zou and Peng (2011) established an empirical formulae relating wave skewness and asymmetry to local Ursell number for waves shoaling and breaking on a natural beach and over a low-crested structure. Wave orbital velocity and acceleration skewness near breaking point has been linked to sediment transport (Hoefel & Elgar 2003) and is correlated with local wave profile and may be further influenced by bottom slope and bottom friction (Zou & Hay 2003a, Zou et al 2003b).

Slight wave breaking is observed, and as a result wave height decays. Both waves are completely blocked some distance away from the incipient breaking location, with the 1.2 s wave being blocked further away from the wavemaker than the 1.1 s wave. We note that the dynamic pressure (excess in pressure with respect to the hydrostatic) for the 1.2 s wave penetrates deeper into the water column than that for the 1.1 s wave, since the 1.2 s wave has longer wave length than the latter even in the presence of the same current field. The pressure field beyond the blocking point is purely hydrostatic.
Figure 5. Spatial evolution of wave skewness (a) and asymmetry (b) for Case 1: \( T = 1.1 \) s, target \( H = 5 \) cm and Case 2: \( T = 1.2 \) s, target \( H = 5 \) cm in Figure 4.

Figure 6 shows the primary wave amplitude evolution along the flume for the 4 validation cases in Table 1. Compared with the experimental data, the present model captures the amplifying effect of the increasing opposing current on the wave amplitude, and the complete blocking of waves in regions with strong current. As shown by the vertical current profile comparisons in Figure 3b, the experimental data is shifted 1 m away from the wavemaker toward the right boundary. The blocking point for the 1.2 s wave is located further away from the wavemaker since its group velocity and thus blocking current velocity are larger than the counterparts for the 1.1 s wave. The maximum wave amplitude is predicted well for 1.1 s wave, but the predicted breaking location was about 1 m upstream from the observation. This is likely due to the fact that the predicted current profile is shifted by about 1 m relative to the observation as indicated in Figure 3. This led to under- and over-predicted wave amplitude before and after the breaking point. Overall, the model-data comparison is improved for 1.2 s wave since the longer wave is less sensitive to current effect.

The observed current profile in the 3-D physical flume was not reproduced by the 2-D numerical flume exactly, especially in the middle of the flume where the current is withdrawn downwards through a perforated false bottom by suction pipes shown in Figure 2. As measured in Suastika (2004), there was a significant lateral variation of the current velocity in the physical flume. The unknown secondary circulating currents near the wavemaker, \( x = 10-13 \) m, may also contribute to the discrepancy in Figure 6. The measurements showed a sudden increase of wave amplitude when the wave encounters the opposing current. The measured current in Figure 3a is under-predicted by present 2-D model which neglects the lateral boundary layers. The under-prediction of current velocity may cause the wave to grow slower than the measurement.
The streamline curvature in this region is the largest (see Figure 2 for an illustration of the streamline) because the suction at the bottom of the flume changes the flow direction by nearly 90 degrees. The turbulence is suppressed by the convex curvature similar to that on a convex surface. The conventional eddy-viscosity type models, such as SST $k-\omega$ turbulence model adopted in this study, cannot capture this effect (Hellsten, 1998; Smirnov & Menter, 2009). Therefore, the eddy viscosity may be over-predicted (cf. Figure 12) and cause the wave to grow slower than the measurement. Once measurements of turbulence in this problem become available in the future, the rotation-curvature correction proposed by Smirnov and Menter (2009) may be incorporated to address this issue.

Figure 6. Predicted (solid and dashed lines) and observed (symbols) primary wave amplitude evolution by Suastika (2004) in the presence of spatially varying opposing current. (a) $T = 1.1$ s, Case 1, $H = 5$ cm (solid) and Case 5, $H = 7$ cm (dashed); (b) $T = 1.2$ s, Case 2, $H = 5$ cm (solid) and Case 7, $H = 7$ cm (dashed). For all four cases, $U_c = 0.55$ m/s, horizontal current gradient $-0.046$ s$^{-1}$ over $x = 11\text{~m}$.

Around the breaking point, the model reasonably predicts the amplitude of the 1.1 s wave while over-predicts that of the 1.2 s wave. 3-D effect becomes significant near the breaking point which is not resolved by the present 2-D model. The 3-D effect on the current field may cause spatial oscillation of the instantaneous blocking point, complicate the wave breaking, and therefore affect the wave amplitude evolution along the flume (Wu & Nepf, 2002; Perlin et al., 2013).

Another possible factor responsible for the discrepancy is the presence of the perforated false bottom in the experiment, which consisted of two perforated plates with different porosities. Suastika (2004) observed significant wave damping even in the absence of current and ascribed this mainly to the perforated false bottom. In the present study we do not model the two plates explicitly, and simply designate a thin layer of grids at the flume bottom as a porous media continuum. Some preliminary tests were conducted to assess the wave dissipation in the
absence of current. We note that employing porous media to represent the perforated false bottom, as expected, results in wave damping. The damping coefficient is, however, about one-third of that from the measurement. Attempts were also made by adjusting the porosity and resistance coefficients of the porous media in order to attain the same level of wave damping as in the experiment. However, no significant difference was seen in the damping coefficient. It seems challenging to achieve the same level of damping by modeling just a thin layer of porous media at the flume bottom. The under-prediction of wave damping in the absence of current may contribute to the larger wave height predicted in the presence of current (Figure 6b).

4 Characteristics of current-induced wave breaking

In this section the geometric properties of current-induced breaking waves, such as the limiting wave steepness and breaking crest asymmetry, are first quantified. The kinematic breaking criterion is assessed by comparing the horizontal particle velocity at the crest with the crest propagation speed. Similar to shoaling waves in the surf zone, wave set-down and set-up caused by the current-induced shoaling and breaking process are analyzed. The wave energy dissipation around the breaking/blocking point is studied by examining the wave height evolution along the flume. The turbulence and vorticity distribution for the current-induced wave breaking is also investigated. The current profiles are observed to change as a result of wave-current interaction. The effect of horizontal current gradient on these quantities is also examined.

4.1 Crest geometry at breaking onset

The limiting wave steepness associated with incipient wave breaking is defined as

\[ ak = \frac{H k}{2} = \frac{\pi H}{L}, \quad (18) \]

where \( a, H, k, \) and \( L \) are, respectively, the local wave amplitude, height, wave number, and wave length at the breaking onset (Figure 1). The wave height is determined by the elevations of the crest and the adjacent preceding trough. The wave length is defined as the distance between the two troughs adjacent to the incipient breaking crest. We note that in the presence of a spatially varying current, the wave length varies along the flume. Therefore, the wave length may be more accurately determined by enclosing the crest in the middle of two troughs than by calculating the distance between two zero up-crossings or down-crossings. In addition, the presence of current field results in a departure of the mean surface elevation from the still water level. Using two troughs to determine the wave length avoids the difficulty of defining zero-crossings from the spatial wave profiles.

For all the model runs listed in Table 1, the limiting wave steepness, \( ak \), ranges from 0.29 to 0.32, with an average of \( ak = 0.3 \), far less than the limiting value of 0.44 for a deep water Stokes wave. The wave steepness at breaking onset seems insensitive to the initial wave height, period, and horizontal current gradient. It appears that the steepening effect of the opposing current decreases considerably the limiting steepness. While some RANS-VOF models under-predict the wave height at the breaking onset, the present model slightly over-predicts the wave height (see model-data comparisons in Figure 6b) for the reasons described in section 3.4. The predicted limiting wave steepness, \( ak = 0.3 \), for current-induced wave breaking is consistent with the parameterizations proposed by Chawla and Kirby (2002) and Suastika and Battjes (2009) based on experiments. Both authors adopted \( ak = 0.3 \) to indicate the breaking onset in their empirical bulk dissipation formulas for current-limited wave breaking.

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Besides determining the wave length directly from the spatial wave profiles, we calculated the wave length, \( L = CT \), from the phase velocity, \( C \), determined from the temporal surface elevations. The phase velocity calculated around the breaking crest is 0.91 m/s for 1.1 s wave (case 1), and 0.94 m/s for 1.2 s wave (case 2). This reduces the limiting steepness to 0.26~0.27 for the two waves. The reason for the discrepancy is that using the temporal surface elevations in the vicinity of incipient breaking results in a propagation velocity for the crest, which is not necessarily equal to the phase velocity (Shemer and Liberzon, 2014).

To calculate the crest-front steepness, the mean water level needs to be determined first. A number of densely-spaced probes (0.01 m) were placed around the breaking crest and the temporal changes of surface elevations recorded. The mean surface elevations at each probe were then deducted from the spatial wave profiles. The average value of the crest-front steepness for the two cases in Figure 4 is \( \varepsilon = 0.39 \), which is close to 0.38, measured at the onset of 2D spilling breakers (Wu & Nepf, 2002). The latter studied wave breaking induced by the dispersive focusing mechanism in the absence of current. The similar values of the crest-front steepness between the two suggest that the current-induced wave breaking may share some characteristics of spilling breakers, such as wave breaking strength. We note that if the mean water level is assumed to be the same as the still water level, the crest-front steepness would be slightly higher, \( \varepsilon = 0.42 \).

As observed in Figure 5, the wave skewness experiences a rapid increase before breaking, while the wave asymmetry sees only a minor change. The latter is typical for a steep wave propagating over a flat bottom. In the presence of an opposing current, the wave steepens due to the shortening wave length and the amplifying wave height. The faster increase of skewness in combination with little change of asymmetry might explain why the crest-front steepness remains the same with and without current, while the limiting wave steepness is significantly reduced by an opposing current. It follows that the crest-front steepness is a better indicator for breaking onset than the wave steepness, as the former is related more directly to the crest geometry.

4.2 Kinematic breaking criterion

The kinematic breaking criterion involves determining the horizontal water particle velocity at the crest and the crest propagation velocity (Shemer & Liberzon, 2014). Figure 7 shows four snapshots of horizontal velocity distribution around the incipient breaking crest. It is seen that the maximum horizontal velocity appears at the crest tip (Figure 7a-b). Wave breaking is initiated at the crest’s forward face when the particle velocity becomes larger at a later instant, \( t = 82.30 \) s (Figure 7c).
Figure 7. Spatial distribution of horizontal velocity around the incipient breaking crest (a zoom in view near the free surface and breaking points) for case 1, target $H = 5 \text{ cm}$, $T = 1.1 \text{ s}$, $U_c = 0.55 \text{ m/s}$, horizontal current gradient $-0.046 \text{ s}^{-1}$ at time instant of (a) 82.10 s; (b) 82.20 s; (c) 82.30 s; (d) 82.60 s.

Two independent methods are applied to determine the actual crest propagation velocity. The first method estimates the rate of crest displacement from the spatial wave profiles. The estimated crest velocity from the spatial wave profiles in Figure 7a-b, is $0.92 \text{ m/s}$. The second method uses the temporal surface elevations recorded at wave probes around the breaking point. The instants of occurrence of maximum surface elevations are identified, and the averaged time lag determined. The crest velocity calculated from the second method is $0.91 \text{ m/s}$, which is close to that determined from spatial wave profiles.

The maximum horizontal velocity at breaking onset (Figure 7b) is about $0.94 \text{ m/s}$, slightly larger than the crest velocity determined from the two methods above. This confirms the kinematic criterion for the inception of wave breaking in the presence of a strong opposing current, and complements the experimental evidence of Shemer and Liberzon (2014) who confirmed the kinematic criterion for a spilling breaker in the absence of current. It is plausible
to assume that beyond the incipient breaking, the accumulation of mass at the crest leads to the formation of a bulge on the forward face of the crest (Duncan et al., 1999). Similarly, we note that for current-induced wave breaking, a bulge forms on the crest’s forward face (Figure 7d). The bulge persists on the front face and it then breaks down into turbulence. This is consistent with the visual observation of whitcaps in waves breaking on and then blocked by currents (Suastika, 2004).

Figure 8. Spatial distribution of (a-A) depth-averaged current, (b-B) wave crest and trough envelopes (crosses) and mean water level (dash-dotted lines), and (c-C) mean water level with (dash-dotted lines) and without (dashed lines) waves, cases 6 and 14: $H = 0.11$ m, $T = 1.1$ s, $U_c = 0.55$ m/s. Left column (a-c): horizontal current gradient $-0.046$ s$^{-1}$; right column (A-C): horizontal current gradient $-0.092$ s$^{-1}$.

4.3 Wave set-down and set-up

Figure 8 shows the spatial distribution of depth-averaged current, wave crest and trough envelopes, and mean water level with and without waves. In the absence of waves, the water level departs from the still water level as a result of the non-uniform current field. The Bernoulli’s principle states that the water level rises when the current decelerates towards the wavemaker (Figure 8c-C). In the presence of waves, the excess flux of momentum, the so-called radiation stress, and its gradient due to the non-uniform wave field, competes with the pressure gradient resulting from the current-induced surface tilting.

As the wave propagates into the opposing current field with linearly increasing strength (Figure 8a-A), the wave height increases as a result of shoaling (Figure 8b-B). Before reaching the breaking point, the radiation stress also increases steadily, which would naturally cause a lowering of the mean water level, wave set-down, if there is no external current field. But since a non-uniform current field is introduced into the flume and the mean water level is already tilted, the pressure gradient created by the current-induced surface tilting competes with the radiation stress gradient pointing in the opposite direction towards the wavemaker. The interaction between the two forcing will strike a balance as to the final configuration of the mean water level. As seen in Figure 8c-C, the current-induced surface tilting close to the wavemaker is reduced by the wave radiation stress. Because of the larger current gradient and hence faster
growth of wave height, the resulting larger radiation stress gradient deflects more the mean water level near the breaking point (Figure 8C).

Beyond the breaking point, wave energy is dissipated and radiation stress decreases. The mean water level rises as a result. This wave set-up is clearly observed in Figure 8c-C, where a larger current gradient causes a larger and more rapid set-up. There is no more set-up beyond the blocking point. Note that the location where wave set-up starts does not coincide with the location of the breaking point; it occurs some distance downwave from the incipient breaking (see also Figure 9). This is consistent with experimental observations of wave set-up in the surf zone (Bowen et al., 1968; Svendsen, 1984; Battjes, 1988). Svendsen (1984) found that the radiation stress in the transition region after initial breaking stayed nearly constant even with a 30%~ 40% decrease in wave height. It was argued in Battjes (1988) that while a rapid decay of wave height after the initiation of breaking indicates dissipation of the wave energy, it occurs on a shorter time scale than that of the total kinetic energy of the ordered, large-scale motion and the total convective momentum flux. Therefore, the initiation of breaking is not accompanied by an immediate change in the mean horizontal pressure gradient. The same argument may apply for the current-induced breakers.

Figures 9 and 10 show, respectively, the effects of incident wave height and period on the spatial distribution of wave crest elevation and mean water level in the presence of waves and currents. As seen in Figure 9b,d, with increase of the wave height, the wave radiation stress plays an increasingly important role in balancing the current-induced surface tilting. As the wave energy is dissipated over a longer distance, i.e. wider breaker zone, the wave setup increases steadily over a longer distance beyond the breaking point.

Given the same initial wave height and current field, the waves are less likely to be blocked when the wave period increases from 1.1 s to 1.4 s. Thus the wave dissipation rate decreases. As a result of the slower decay of wave height, both the amount and the slope of the wave set-up beyond the breaking point become smaller (Figure 10c-d). The wave set-up disappears when the wave with \( T = 1.4 \) s managed to propagate through the current with negligible dissipation. Since the wave heights after incipient breaking exhibit a less abrupt variation with the increase of the wave period, the radiation stress plays a weaker role in striking a balance with the current-induced surface tilting. This may explain the model predictions close to the breaking point, \( x = 22.5 \) m in Figure 10b and \( x = 20 \) m in Figure 10d, which show less departure of the mean water level from that induced by the non-uniform blocking current.
4.4 Wave energy dissipation

In the presence of strong opposing currents, most waves steepen and break at or before the blocking point. Even those propagating through the current field lose a considerable amount of energy due to wave breaking. It has been shown that compared with depth-induced breakers, current-induced wave breaking is unsaturated and weak in strength (Chawla & Kirby, 2002). The standard bore model accommodating the shallow water waves’ dissipation may not be applicable to the current-induced breakers occurring in relatively deep waters. The characteristics of the wave height evolution in the vicinity of the breaking/blocking points are examined. The decay rate of the wave height relates to the breaker strength and energy dissipation rate.
Figure 11. Wave height evolution along the flume for different incident wave height, wave period and current gradients for $U_c = -0.55$ m/s. (a, b) $T = 1.1$ s and (c, d) $T = 1.2$ s and $H = 5, 8$ and 12 cm; (e, f) $H = 5$ cm and $T = 1.1, 1.2, 1.3, 1.4$ s. Left column: horizontal current gradient $-0.046$ s$^{-1}$; right column: horizontal current gradient $-0.092$ s$^{-1}$.

Figure 11 shows the wave height evolution along the flume for different wave and current conditions. As seen in Section 3.2, a depth-averaged current with $U_c = -0.55$ m/s blocks waves with $T = 1.1$ s and 1.2 s (Figure 11a-d). As the initial wave height increases, waves break early with the incipient breaking occurring more downstream (closer to the wavemaker), due to the early reach of the limiting steepness, $ak$, for big waves (see definition in Section 4.1). It is observed that the breaking points are more scattered for a slowly varying current (current 1) than for a rapidly varying current (current 2). For the same initial wave height and period, the maximum wave height at the breaking onset is approximately the same, irrespective of the magnitude of the current gradient.

The rate of wave height decay beyond the breaking point varies depending on the wave and current conditions. Under a small current gradient, the wave heights for $T = 1.1$ s decay approximately in a linear manner and at the same rate (Figure 11a). While this is also the case for the small waves in Figure 11c, there is some variation of decay rate for the 1.2 s wave with a larger initial wave height, $H = 12$ cm (Figure 11c). This variation of the decay rate is more
pronounced for waves under a more rapidly varying blocking current (Figure 11b,d). For waves with the same initial wave height (Figure 11e,f), the dissipation decreases with increase of the wave period, since longer waves possess a larger group velocity to propagate through the same blocking current field. Although the wave height grows faster under a rapidly varying current, the maximum height at the breaking onset remains the same. The locations of incipient breaking are slightly shifted depending on the gradient of the underlying current.

For completely blocked waves in Figure 11d, three stages of wave height decay could be identified: beyond breaking onset, prior to complete blocking, and continuous breaking in between. The decay rate is approximately constant in each stage. It is seen that at the third stage, the wave height decreases sharply to zero prior to the blocking point. For waves that are not blocked ($T = 1.3$ s in Figure 11e-f), the third stage of wave height decay is no longer observed. The rate of wave height decay is the largest at the breaking onset; it then gradually decreases to zero. In other words, the energy dissipation rate is not constant beyond the breaking onset, similar to the surf zone breakers (e.g., Ting & Kirby, 1994). These observations are consistent with the experimental measurements for both blocked and non-blocked waves (Figs. 8 and 9 in Chawla & Kirby, 2002). The actual dissipation rate depends on the specific wave and current conditions beyond the breaking onset. Therefore, wave models designed to dissipate energy continuously at a constant rate, once the breaking criterion is satisfied, may not predict well the wave height evolution in the presence of strong currents. Furthermore, a stopping criterion for wave dissipation to terminate is required since wave breaking ceases once a stable wave height is attained as seen in Figure 11e-f. This was also deemed necessary for surf zone breakers reentering deeper water where the bottom becomes horizontal past the crest of a submerged bar or the downward slope shoreward of the bar crest (Dally et al., 1985; Battjes, 1988).

4.5 Turbulence and vorticity distribution

The spatial distribution of turbulence and vorticity associated with wave breaking and blocking is examined in this section. We chose to use the RANS model since it has been used extensively to study the surf zone breakers which have a lot in common with the current-induced breaker in this study. The turbulence predictions by the model was validated in Appendix C for an isolated spilling breaker in a transient focused wave group. Brown et al. (2016) evaluated five turbulence models in OpenFOAM applied to spilling/plunging breakers in the surf zone. It was demonstrated that most turbulence models could predict the general features of turbulence transport observed in the experiment (Ting & Kirby, 1994), although some fine tuning or further research are required to obtain better quantitative agreement. The current-induced breaker differs from the depth-limited surf zone breaker in the way the wave becomes too steep to remain stable. But this model should be, in principle, adequate as a first attempt to investigate the current-induced breaker. The findings of the numerical simulations need to be confirmed by future laboratory experiment and field measurements of turbulence.
Figure 12. Time evolution of current-induced breaker generated (a-d) turbulent kinetic energy (TKE), (e-h) mean flow vorticity, (i-l) eddy viscosity, and (m-p) specific dissipation near the breaking and blocking region for case 14 ($H = 0.11 \text{ m}$, $T = 1.1 \text{ s}$, $U_c = -0.55 \text{ m/s}$, horizontal gradient $-0.092 \text{ s}^{-1}$ over $x = 14\sim20 \text{ m}$).
Figure 12 shows a time sequence of the spatial distribution of turbulent kinetic energy, mean vorticity, and eddy viscosity near the breaking/blocking region of the current-induced breaker. As seen in Figure 12a-d, the turbulent kinetic energy appears in the breaking wave crest front, which is the source region for turbulence generation. This is the “roller” region characteristic of spilling breakers. It exists in the upper level of the breaking crest front, and is defined as the aerated area of recirculating flow in the front of the turbulent bore (Battjes, 1988).

As the broken wave passes by, the turbulence generated is left behind and transported backward to the rear face of the crest and downward to the interior region. With the turbulent bore propagating forward, strong turbulence is continuously generated at the bore front until the blocking point (Figure 12d). One peculiar feature for the current-induced breaker is that the strong turbulence is generated with an accompanying opposing current. As a consequence, the turbulence generated in the vicinity of the blocking point is instantly advected downstream by the current, i.e., up-wave toward the wavemaker. We notice that the turbulence advected downstream from the blocking point interacts with the newly generated turbulence by the following wave. The resulting turbulence then spreads out by advection toward the wavemaker and by diffusion downward into the interior region.

The vorticity field after wave breaking onset is highly 3-D including a complex evolution of the coherent vortex structures such as reversed horseshoe vortices and obliquely descending eddies (Nadaoka et al., 1989; Watanabe et al., 2005; Farahani & Dalrymple, 2014). As noted in Lin and Liu (1998b), the “oblique vortices” observed in laboratories are essentially the special forms of turbulence in breaking waves, and they become visible because of the presence of air bubbles during the breaking process. On the other hand, the “horizontal vortices” have strong coherent structures which is mainly due to the mean flow motions, which are largely two-dimensional in most laboratory experiments. Thus 2-D RANS-VOF model is adequate to derive the mean vorticity from the predicted 2-D mean flow fields. This practice has been adopted by a number of researchers for breaking waves in the shallow and deep water (e.g., Iafrati, 2011; Xie, 2013; Deike et al., 2015; Pizzo et al., 2016).

Similar to the turbulent kinetic energy, the vorticity is generated in the breaking wave crest front (Figure 12e-f). Before the wave breaks, the vorticity field is rather uniform in the water, with only a small value present due to the current. When the wave breaks, a region of negative vorticity appears in a thin layer beneath the free surface. The negative vorticity, $O(10) \text{s}^{-1}$, then strengthens and increases in span, spreading out backward and downward to the trough region. The negative vorticity pattern is indicative of the early stages of a shear layer (Qiao & Duncan, 2001). As the turbulent bore propagates forward, the vorticity is continuously generated at the bore front, and is diffused slightly into the interior region. Since there is strong opposing current ahead, the vorticity cannot be transported beyond the blocking point. Rather, it is constantly advected toward the region where incipient wave breaking occurs. At some point, the vorticity advected downstream by the current and the newly generated vorticity by the next wave interact with each other at the free surface, where some mean vorticity may be destroyed in this interaction process. Thus for current-induced breakers, both vorticity generation and destruction occur at the free surface. This is different from surf zone breakers, where the breaking wave generated vorticity could easily reach the bottom in shallow waters and form a complicated vorticity pattern due to the combined effects of breaking wave and bottom turbulent boundary layers (Lin & Liu, 1998b).
The eddy viscosity is another important parameter measuring the mixing rate of momentum, solutes, and sediments. It is seen that the distribution pattern of eddy viscosity in Figure 12i-l is in general similar to that of turbulent kinetic energy. Note that because of the length limit of the computational domain, the current has not yet developed its fully turbulent profile. This explains why the value of eddy viscosity beyond the wave blocking point is negligibly small compared with that generated by the breaking wave. In reality, a fully developed turbulent current has a parabolic profile for the eddy viscosity across the water depth. The magnitude of the eddy viscosity at mid-water column could reach $O(1e-3) \text{ m}^2/\text{s}$ for the present current condition, which is comparable to or even larger than those generated by the breaking waves.

The specific dissipation, $\omega$, indicates the rate at which the turbulent kinetic energy is converted into the internal thermal energy. As seen in Figure 12m-p, its distribution pattern is largely similar to that of vorticity (Figure 12e-h). The dissipation rate is the largest close to the free surface, where the wave crest breaks and experiences rapid topology changes while propagating forward. The advection of the specific dissipation by the underlying current is not as pronounced as that of the turbulence energy (Figure 12a-d). Referring to the definition in Eq. (13), eddy viscosity is the ratio of turbulent kinetic energy and specific dissipation. Figure 12 suggests that the turbulent kinetic energy is advected further downstream than the dissipation. This results in the maximum eddy viscosity occurring further downstream from the breaking wave crest, which is the source region of breaker induced turbulence. Further downstream the breaking region, $x < 19 \text{ m}$ (Figure 12i-l), the turbulent kinetic energy $k$ decreases despite the possible turbulence production arising from the underlying current shear. But since the turbulence length scale $l$ increases considerably downward into the interior region, the eddy viscosity, $\nu_t \sim l\sqrt{k}$, becomes significant in the middle water column, similar to that in an open channel flow.
Figure 13. (a) Spatial distribution of mean vertical current profiles in the presence of waves (case 14; $H = 0.11$ m). Wave crest/trough envelopes (upper and lower dashed lines) and mean water level (dotted line) are superimposed. (b) Non-zero return flow flux due to mass transport of waves, Eq. (19), accumulated upstream from the blocking point $x = 22.5$ m. (c) Vertical current profiles with (dashed, dotted, and solid lines) and without (dash-dotted line) waves. For all cases in (a)-(c), wave period $T = 1.1$ s, $U_c = -0.55$ m/s, horizontal gradient $-0.092$ s$^{-1}$ over $x = 14$~20 m.

4.6 Changes of current profiles

Figure 13a shows the spatial evolution of mean vertical current profiles in the presence of waves with $H = 0.11$ m and $T = 1.1$ s. The mean current field is time-averaged using model results of the last 5 wave periods. As the wave shoals, a positive Eulerian current in the direction of wave propagation is generated between the crest and the trough envelope (higher and lower dashed lines) associated with mass transport in the waves. After the wave breaks, the positive Eulerian current is mainly concentrated above the mean water level (MWL, dotted line), while the superimposition of the Eulerian current with the opposing blocking current underneath, results in a small negative current between the MWL and the trough near the breaking region ($x = 19$~22 m). It is noted that there appear some disturbances at the edge of the porous bottom $x = 20$ m, and secondary recirculating flows near the water surface between $x = 14$~16 m.
The non-zero mass flux transported between the crest and the trough disappears when the wave is fully blocked at $x = 22.5$ m. All the mass transport accumulated along the wave shoaling and breaking process is diverted underneath as a return flow that is superimposed on the original blocking current field. The total mass conservation in the numerical flume is confirmed by simple calculations. The depth-integrated return flow flux at each cross-section can thus be calculated according to

$$q(x) = \int_{-\infty}^{\bar{\eta}} u \, dz - \int_{0}^{\infty} u \, dz$$

(19)

where $u$ is the mean horizontal velocity, $\bar{\eta}$ is the mean crest elevation, and $z_0$ is the $z$-coordinate where the mean horizontal velocity turns positive. Figure 13b shows the spatial distribution of the depth-integrated return flow flux as a result of the Stokes drift. Going downstream from the blocking point, the discharge of the return flow increases steadily. The maximum increase of the return flow occurs in the transition region ($x = 20\sim22$ m for $H = 0.11$ m) after initial breaking. But given the relatively small amount of the return flow (depth-averaged current 0.02 m/s), it is believed that the wave induced mass transport has a minor effect on the overall current profiles in this problem.

Figure 13c shows the comparison of vertical current profiles with and without waves. It is observed that at each cross-section, the presence of waves mainly alters the current profile beneath the surface, which in turn alters the lower portion of the current profile by the principle of mass conservation. Given the spatial sequence of current profile changes, it is reasonable to assume that the changes originate from the vicinity of the blocking point, and propagate downstream, i.e., up-wave towards the wavemaker. This is consistent with the downstream advection of the breaker-generated turbulence as observed in Section 4.5. The larger the initial wave height, the more intense the turbulence, and thus the more changes to the current profile.

5 Conclusions

The distinct characteristics of wave breaking and blocking due to spatially varying opposing currents are investigated by a Reynolds-Averaged Navier-Stokes (RANS) solver along with the VOF surface capturing method. The SST $k-\omega$ turbulence model is adopted with the production term modified to avoid excessive turbulence production in the interior part of the flow. The model is validated with a novel experiment where the spatially varying current was generated by withdrawing water over a large portion of a flat bottom in order to exclude the effect of variable water depth in previous studies. The characteristics of the current-induced wave breaking, such as the geometric properties of the breaking crest, kinematic breaking criterion, wave set-up and set-down, energy dissipation, turbulence and vorticity generation, and undertow-like current profile changes are then examined using the RANS-VOF model.

In the presence of an opposing current with increasing strength in the wave direction, the wave shortens, steepens, and then breaks when the water particle velocity at the crest exceeds the propagation speed of the crest, thus confirming the kinematic breaking criteria for current-induced breakers. The two-phase flow solver does not satisfy the boundary conditions at the air-water interface well (Vukčević et al., 2017), therefore, generates spurious velocities at the air-water interface which may affect the accuracy of the velocity prediction at the free surface near the breaking point. This problem may be alleviated using the ghost fluid method proposed by Vukčević et al. (2017) to discretize the free surface boundary conditions at the interface in the
future work. It was found that the current-induced wave breaking shares some similarities with the spilling breakers. When a wave is about to break, a bulge forms on the front face of the crest and remains on that face, confined to a smaller area, and then breaks down into turbulence. This is consistent with the visual observation of whitecaps during wave breaking on opposing currents (Suastika, 2004).

The geometric properties of wave crest at breaking onset are identified by examining the spatial and temporal variation of surface elevations. Only spilling breakers were observed in this study. The limiting wave steepness predicted by the model, $ak = 0.30$, is considerably smaller than that for a Stokes wave in deep water without current blocking, i.e., $ak = 0.44$. It is, however, consistent with the breaking onset criterion used by Chawla and Kirby (2002) and Suastika and Battjes (2009) to quantify the energy dissipation in their bore models for current-induced breaking. The limiting skewness for current-induced breaker, however, is found to show less departure from that for a Stokes wave, which is consistent with Ma et al.’s (2013) finding from their study of opposing current effect on a modulated wave group. The crest-front steepness at breaking onset is comparable with those for depth-limited spilling breakers in the surf zone.

For the same horizontal current gradient, large waves tend to break earlier further downstream of the current since they reach the limiting wave steepness earlier. For the same initial wave height and period, the maximum wave height attained at the breaking onset is approximately the same regardless the magnitude of the current gradient. The decay rate of wave height beyond the breaking point is dependent on the wave, current and current gradient conditions. The wave height decays almost linearly and at the same rate for small waves in a slowly varying current. However, the decay rate varies considerably for large waves under a more rapidly varying current. For completely blocked waves, an approximately piecewise linear, three-stage wave height decay are identified: (1) beyond breaking onset; (2) prior to blocking; and (3) continuous breaking in between. For waves that are not blocked, the third stage of wave height decay disappears; the wave height shows an initial exponential decay and then remains constant some distance away from the breaking point.

Our model results of spatial distribution of turbulence energy and vorticity generated in the breaking waves indicate that the current-induced breaker shares many features typical of spilling breakers not induced by current blocking. However, our study also reveals some features unique for current-induced breaking. One conspicuous feature for current-induced spilling breaker is that the turbulence and vorticity is continuously generated at the breaking wave crest front with a strong opposing current flowing underneath and beyond the blocking point. Therefore, the turbulence and vorticity generated by the propagating broken wave crest are instantly advected downstream/upwave to the region where the incipient wave breaking occurs. That turbulence and vorticity would then interact with those newly generated by the following wave and current, resulting in more complex turbulence and vorticity patterns than those in spilling breakers without current blocking.

Similar to surf zone breakers, wave set-down appears as the waves shoal and steepen over the opposing current, and wave set-up appears a short distance shoreward of the breaking onset location. A new feature for these current-induced breakers is that the spatially varying opposing current causes a surface tilting of the mean water level even in the absence of waves. It is through this current-induced surface tilting that the wave radiation stress gradient exerts its influence on the mean water level. The larger the initial wave height and the larger the magnitude of the current horizontal gradient, the larger set-down and set-up are generated by the
wave radiation stress gradient. Longer waves with larger wave periods lead to smaller wave set-up, since it is less likely for these waves to experience breaking and energy loss while propagating through the same current field.

It was also observed that an undertow-like change to the current profile as a result of current-induced breaking. The presence of waves mainly alters the current profile just beneath the surface, which in turn alters the lower portion of the current profile by the principle of mass conservation. Consistent with the downstream advection of the breaker-generated turbulence, the changes to the current profile downstream the breaking/blocking points penetrate deeper into the water column. Larger initial wave height or current gradient leads to more intensive the turbulence and the more changes to the current profile.

Current-induced wave breaking is believed to be an important source of turbulence at the offshore front of a river plume encountering the ocean, which may contribute to plume mixing through the downward transport of wave-driven TKE (Thomson et al., 2014). As a first attempt to investigate the current-induced breaker by a RANS-VOF model, this study reveals many interesting features unique for this type of steepness-limited breaker. It would be worthwhile future work to conduct comprehensive experimental measurements of turbulence associated with current-induced breaking to confirm the findings of the present numerical simulations.

Acknowledgments

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Appendix A: Menter’s (1994) Shear Stress Transport (SST) $k-\omega$ turbulence model

Menter (1994) proposed a Shear-Stress Transport (SST) $k-\omega$ model for the turbulent kinetic energy, $k$, and the specific dissipation, $\omega$,

\[
\frac{\partial \rho k}{\partial t} + \nabla \cdot (\rho \mathbf{U} k) = \tilde{P}_i - \beta' \rho k + \nabla \cdot [\mu + \sigma_{\mu} \mu] \nabla k \quad (A1)
\]

\[
\frac{\partial \rho \omega}{\partial t} + \nabla \cdot (\rho \mathbf{U} \omega) = \frac{\omega}{\nu_t} \tilde{P}_i - \beta \rho \omega^2 + \nabla \cdot [\mu + \sigma_{\omega} \mu] \nabla \omega] + 2(1-F_r)\rho \sigma_{\omega} \frac{1}{\omega} \nabla k \cdot \nabla \omega \quad (A2)
\]

which are Eqs. (7-8) in Section 2.3.

The two blending functions are defined as
\[ F_1 = \tanh(\arg^*) \]

\[ \arg^* = \min \left\{ \min \left[ \max \left( \frac{\sqrt{k}}{\beta \sigma}, \frac{500\nu}{\sigma^2} \right), \frac{4\rho \sigma_{w} k}{CD_{wa} y^2} \right] \right\}, 10 \} \]  

\[ CD_{wa} = \max \left( 2\rho \sigma_{w} \frac{1}{\omega} \nabla k \cdot \nabla \omega, 10^{-10} \right) \]  

\[ F_2 = \tanh(\arg^*_2) \]

\[ \arg^*_2 = \min \left[ \max \left( \frac{2\sqrt{\beta}}{\sigma}, \frac{500\nu}{\sigma^2} \right), 100 \right] \]  

where \( y \) is the distance of a given grid point to the nearest wall.

Each of the constants, \( \sigma, \sigma_w, \beta, \gamma \), in Eqs. (A1) and (A2) are blended by

\[ \phi = F_1 \phi + (1 - F_1) \phi_2 \]  

where

\[ \sigma_{x1} = 0.85, \sigma_{x2} = 0.5, \beta_1 = 0.075, \gamma_1 = 0.5532 \]

\[ \sigma_{x2} = 1.0, \sigma_{w1} = 0.856, \beta_2 = 0.0828, \gamma_2 = 0.4403 \]  

\[ \beta^* = 0.09, a = 0.31 \]

Appendix B: Validation of steepness-limited deep water breaking wave simulation by the present RANS-VOF model

Figure B1 shows the comparisons of the evolution of surface elevation for a steepness-limited, dispersive focusing breaking wave group by the present RANS-VOF model, the model by Zou and Chen (2016, 2017) and the experiment Case DF2 in Table 1 of Tian and Choi (2013). The surface elevation measured at \( x = 2.84 \text{ m} \) by Tian and Choi (2013) was used to drive the numerical model. A plunging breaker was observed around \( x = 6.3 \text{ m} \) and starting from \( t = 22.5 \text{ s} \). It is evident from Figure B1 that the present RANS-VOF model with the modified SST \( k - \omega \) model described in section 2.3 captures the observed surface elevation well and is in good agreement with model results of Zou and Chen (2016, 2017) both upstream and downstream from the breaking region.
Figure B1. Comparison of predicted surface elevation time evolution for a steepness-limited, dispersive focusing breaking wave group (DF2 in Table 1 of Tian & Choi, 2013) by the present RANS-VOF model (solid lines) and the Smagorinsky subgrid-scale stress model by Zou and Chen (2016, 2017) (dotted lines), and the experiments by Tian and Choi (2013) (dashed lines) at 4 gauges (a) $x=2.84$ m, (b) $x=5.13$ m, (c) $x=7.04$ m, and (d) $x=9.07$ m from the physical wavemaker. See Figure 1 in Zou and Chen (2017) for experimental and numerical flume set-up.

Appendix C: Validation of turbulence predictions for a spilling breaker in a transient focused wave group

Figure C1 shows the comparisons of the predicted surface elevation and turbulent kinetic energy for a spilling breaker by the present 2D RANS-VOF model and the experiment by Pujol and Nepf (2012). A transient wave group with a frequency bandwidth of 0.789 Hz and a center frequency of 1.08 Hz was generated in a 0.3 m deep flume using the same dispersive wave focusing technique as in Appendix B. Because of the slightly different wavemaker boundary
conditions in model and experiment as well as wave nonlinearity, the predicted breaking location and timing deviate slightly from the flume experiment. The predicted surface elevation is in good agreement with the observation. The breaking intensity, jet penetration depth, and breaking duration and length affect the spatial and temporal evolutions of the turbulent kinetic energy (TKE). As demonstrated in Figure C1, despite the highly transient nature of the isolated breaking event, the predicted TKE has the same order of magnitude as the measurement. The observed oscillation in TKE with time are also reasonably captured by the model.

However, we caution that this type of unsteady breaking is different from the quasi-steady breaking in the surf zone or over a spatially varying opposing current studied in this paper. There is normally only one breaking crest for this type of breaking in a transient focused wave group. The turbulent region generated by this breaker is of finite length and propagates downstream slowly (Derakhti & Kirby, 2014). After the passing of the breaking crest, the turbulence concentrated near the free surface is diffused downward and advected back and forth by the following small waves or the residual currents in the wave group. For quasi-steady wave breaking, wave breaks repetitively creating a large region where the turbulence generated successive breaking wave crests reinforced each other. In this sense, the current-induced breaker is more similar to the surf zone breaker than the isolated deep water breaker in a dispersive focusing wave group.

Figure C1. Comparison of predicted surface elevation (a,b) and turbulent kinetic energy (c,d) near free surface by the present RANS-VOF model and the experimental measurements (Pujol & Nepf, 2012) for a dispersive focusing spilling breaking wave group at distance (a,c) $x-x_b = 0.65$ m and (b,d) $x-x_b = 0.95$ m relative to the breaking location $x_b$. 
References


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Table 1. Wave and current conditions for the 16 wave blocking case studies by the present model.

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*Note: The wave and current parameters for cases 1, 2, 5, and 7 are chosen to be the same as the experiments by Suastika (2004) to validate the present model.

\[ U_c(x) = \begin{cases} 
0, & x < 11 \\
-0.046(x - 11), & 11 \leq x < 23 \\
-0.55, & x \geq 23.
\end{cases} \]  

Eq. (14)

\[ U_c(x) = \begin{cases} 
0, & x < 14 \\
-0.092(x - 14), & 14 \leq x < 20 \\
-0.55, & x \geq 20.
\end{cases} \]  

Eq. (15)