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Investigation of hydrodynamic performance of an OWC (oscillating water column) wave energy device using a fully nonlinear HOBEM (higher-order boundary element method)

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Abstract: Based on a time-domain higher-order boundary element method (HOBEM), a two-dimensional (2D) fully nonlinear numerical wave flume (NWF) has been developed to investigate the hydrodynamic performance of a fixed oscillating water column (OWC) wave energy device. In the model, the incident wave is generated by the inner-domain sources such that the re-reflection at the input boundary can be avoided. A self-adaptive Gauss integral method is used in the case of the mismatch between meshes on free surface and body surface. A simplified pneumatic model is implemented to determine the air pressure imposed on the free surface inside the chamber. The proposed model is validated against the published experimental and numerical results for OWCs with an air chamber in the flat-bottom and slope-bottom flumes. Numerical tests are performed and show that the maximum air-pressure in the chamber does not occur at the same frequency as the maximum surface-elevation. For a fixed submergence depth of the OWC back wall, the peak efficiency tends toward a saturation status when the slope angle further increases to a certain value. The hydrodynamic efficiency is decreased from a critical value (wave slope $kA$, approximate 0.10 in the present study) to both stronger and weaker wave nonlinearity.

Key words: OWC; wave generation technique; HOBEM; Time-domain simulation; Hydrodynamic efficiency

1. Introduction

Because of its high energy flux density (10~40 times wind energy and 30~100 kW/m wave front

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[1]) and lower negative environmental impact, wave energy is considered to be one of the most promising form of clean renewable energy. Thousands of wave-energy converters have been invented, among which the oscillating water column (OWC) device is one of the most successful devices with regard to the design and principle of operation. OWC has been used extensively in many countries [2-6]. An OWC device consists a partially submerged chamber, inside which the water column oscillates up and down as induced by the incident wave, in turn compressing and expanding the air above the free surface. Such reciprocal airflows through a turbine at the opening duct of the air chamber can generate electricity. In case of a land-based OWC, the chamber is usually fixed, with the advantage of no mooring lines, wet power-transmission cables and an accessible coastal system. As the wave propagates toward the shoreline, however, the incident wave energy can be altered due to the effects of wave refraction, wave shoaling, wave breaking and bottom friction. Therefore, it is of practical importance to account for the effects of design parameters on the hydrodynamic performance of the OWC.

Previous theoretical and numerical studies of the hydrodynamic performance of OWC devices are largely based on linear wave theory. Evans [7] derived the wave-energy absorption efficiency of a fixed OWC device using the matched asymptotic expansions and ignoring the spatial variation of the free surface within the air chamber. In his study, Evans considered the chamber free surface as a rigid weightless piston and with a small width relative to the incident wavelength. Based on two-dimensional wave hydrodynamics and linearized duct dynamics, Lighthill [8] developed a technique to design a wave energy extraction device. Sarmento and Falcão [9] carried out an analytical study of the capture efficiency of a 2D OWC device for linear and nonlinear power take-off. They found that the maximum efficiency of the device with strongly nonlinear power take-off is only marginally inferior to that with linear take-off. Later, using the matched eigenfunction expansion technique and neglecting the viscous effect, Evans and Porter [10] derived the relationship between the efficiency of a 2D rectangular OWC device and the chamber width and the front-wall submergence. Nader et al. [11] adopted a finite element model to study scattered waves around single and multiple oscillating water column wave energy conversion devices based on linear wave theory. They found that the presence of neighboring OWCs has a significant influence on the power capture efficiency of individual devices. Wang et al. [12] performed a numerical study of the hydrodynamic performance of a shoreline-mounted OWC device using a
boundary element method based on the Wehausen and Laitone 3D shallow water Green’s function. The numerical computations were validated with physical experiments and included the topographical effects of bottom slope and water depth. Delaure and Lewis [13] employed a commercial 3D linear BEM model, WAMIT, to model a fixed OWC device, including dynamic chamber pressure. Due to the considerable complexity and nonlinear effects involved in the hydrodynamics of an OWC, recently research on this topic has resorted to a nonlinear numerical wave tank.

Numerical wave tanks can be categorized into two groups: viscous-flow models based on Navier-Stokes equations solver and potential flow models. The viscous models have the potential to provide better solutions at a cost of long computation time and higher requirements in computer resources. On the other hand, the potential flow models do not resolve the vortices generated by viscous effects, but can provide solutions in a reasonable time on standard computers. It was found that a linear dissipative term can be artificially incorporated into the free-surface boundary conditions to account for the energy dissipations due to viscosity [14-16], which is neglected in the conventional potential flow model.

Due to significant progress in the development of numerical wave tanks, numerical wave tanks have become an alternative for physical modeling of wave-structure interactions, such as the hydrodynamic performance of OWC devices. For example, Liu [17] applied a numerical wave tank based on the two-phase VOF model to investigate the operating performance of an OWC air chamber. Zhang et al. [18] developed a 2D two-phase numerical wave tank using a level-set immersed boundary method to study the flow field, surface elevation and air pressure in an OWC chamber in addition to the effects of the geometric parameters on the capture efficiency. Teixeira et al. [19] employed the two-step semi-implicit Taylor-Galerkin method to solve a Fluent model for simulating an OWC device and investigated the effects of the chamber geometry and the turbine characteristics in relation to the best device performance. Luo et al. [20, 21] used the commercial Fluent CFD software to explore the effect of wave nonlinearity on the capture efficiency of the fixed and heave-only floating OWC devices. López et al. [22] developed a 2D numerical model based on the RANS equations and the VOF surface capturing scheme (RANS–VOF) to study the optimum turbine-chamber coupling for a given OWC. Gkikas and Athanassoulis [23] modeled the pressure fluctuation inside the chamber of an OWC device under monochromatic wave excitation.
using a nonlinear system identification method. Koo and Kim [24] developed a 2D fully nonlinear idealized NWF to investigate a land-based OWC device using a lower-order boundary element method. Their NWF model includes both viscous and pneumatic-air effects is in good agreement with experiments. Later, Koo and Kim [25] extended their model to simulate the interaction of an irregular wave with a fixed OWC.

In a numerical wave tank, the re-reflection from the wave maker has to be minimized to maintain long-time simulations in cases with a surface piercing body, such as an OWC device. One method is to use a long wave tank and stop the computation when re-reflection waves reach the structure at the expense of large computational cost [20]. The second method is to adopt the absorbing wave maker theory to dissipate the reflection wave from the structure [19], which may not be effective for non-uniform structures. The third method is to include a damping layer in front of the wave maker to absorb the reflected waves, This method requires the incident wave to be known in advance [24]. The second and third methods assume that the incident and reflected waves can be separated in the wave train, which is difficult to achieve under some complicated circumstances. The fourth method is to adopt an inner source generating wave combined with a damping layer, such that the reflected wave can pass through the source surface and be absorbed in the damping-layer behind. This internal wave generation technique has been used in many applications [26-29]; however, to the authors’ knowledge, less work has been done on wave interactions with a land-based OWC device in cases of typically occurring long waves, which is different from the former three methods.

Unlike the constant panel method, the higher-order boundary element method (HOBEM) can model the corners or edges on the body surface exactly and possess higher accuracy and efficiency [30]. HOBEM has been widely used in many nonlinear water wave problem except for OWC modeling. The primary objective of this work was to combine fully nonlinear HOBEM with an inner source wave generation technique to investigate the effects of the chamber geometric parameters, bottom slope, as well as pneumatic damping coefficient and wave nonlinearity on the hydrodynamic efficiency of a fixed OWC device. This paper is organized as follows. Section 2 briefly describes the proposed numerical model. In Section 3, the present model is verified against the available experimental and numerical results. In Section 4, the results concerning the hydrodynamics and capture efficiency of the OWC device are presented and discussed. Finally, the conclusions are presented in Section 5.
2. Mathematical formulations

Fig. 1. Schematic diagram of a fixed OWC in a flume

2.1 Boundary integral equation

A schematic of the NWF with a fixed OWC located at the right hand end of the tank is shown in Fig. 1. A Cartesian coordinate system is defined with the origin in the plane of the undisturbed free surface \( z=0 \), with the \( z \)-axis positive upwards. As shown in Fig. 1, \( h \) denotes the static water depth, \( SD \) the immergence of the backward wall, \( LB \) the base length of the sea bottom slope, \( \theta \) the bottom slope, \( A \) the width of the air duct, \( D \) the immergence of the front wall, \( B \) the chamber breadth and \( C \) the front-wall thickness. The corresponding boundary value problem defining the fluid motion is set up as follows. Let \( t \) denote time and \( \eta \) the free surface elevation above the still water level. The waves are generated by controlling the volume flux density of a vertical source distribution inside the model boundary.

It is assumed that the fluid is incompressible, inviscid, and the flow irrotational. The fluid motion can therefore be described by a velocity potential \( \phi \) related to the velocity. Generally, the velocity potential satisfies the Laplace equation. However, due to the presence of the source distribution, the velocity potential satisfies the Poisson equation in the present study [27 28]:

\[
\nabla^2 \phi = q^*(x_s, z, t) 
\]

(1)

where \( \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \) is the 2D Laplacian operator, and \( q^*(x_s, z, t) \) is the pulsating volume flux density of the internal source distribution described below. Following Brorsen and Larsen [27], the incident wave is specified by writing the flux density as follows:

\[
q^*(x_s, z, t) = 2v\delta(x-x_s) 
\]

(2)

where \( v \) is the horizontal fluid speed corresponding to the wave to be generated; \( x_s \) is the horizontal
position of the vertical source, and \( \delta(x-x_0) \) is the Dirac delta function. In the present study, the horizontal velocity \( v \) is given by the following Stokes second-order analytical solution:

\[
v = \frac{A_1 g k \cosh(k(z+h))}{\omega} \cos(kx - \omega t) + \frac{3A_1^2 \omega k}{4 \sinh^2(kh)} \cosh(2kx - \omega t)
\]

where \( A_1 \) is the incident wave amplitude; \( g \) is the gravitational acceleration; angular frequency \( \omega \) and wave number \( k \) satisfy the dispersion relation as follows:

\[
\omega^2 = gk \tanh(kh)
\]

To prevent the impulse-like behavior of the wave maker and reduce the corresponding unnecessary transient waves, the input wave velocity at the source surface is treated as increasing gradually by timing a temporal ramp function, which initially satisfies a calm water condition and smoothly approaches unity as the simulation proceeds. The ramping functions is given by

\[
R_m = \begin{cases} 
\frac{1}{2} \left(1 - \cos \frac{\pi t}{T_m}\right), & t \leq T_m \\
1, & t \geq T_m 
\end{cases}
\]

where \( T_m \) is specified as the length of time for which the input wave is ramped. \( T_m \) is defined as two times wave period \((2T)\) in the present study.

On the free surface, both the fully nonlinear kinematic and dynamic boundary conditions are satisfied. In the present work, the mixed Eulerian-Lagrangian method is used to describe the time-dependent free surface with moving nodes in both horizontal and vertical directions. A damping layer with coefficient \( \mu_1 \) at the inlet of the numerical flume is added to absorb the reflected wave from the OWC device. To incorporate the viscous effect due to vortices or flow separation at the chamber skirt, a constant damping layer with coefficient \( \mu_2 \) is implemented at the free surface in the chamber. Then, the free surface boundary conditions can be written as follows:

outside of the chamber \( (6-1) \)

\[
\begin{align*}
\frac{dx(x,t)}{dt} &= \nabla \phi - \mu_1(x)(X - X_0) \\
\frac{d\phi}{dt} &= -g \eta + \frac{1}{2} |\nabla \phi|^2 - \mu_1(x) \phi
\end{align*}
\]

inside of the chamber \( (6-2) \)

\[
\begin{align*}
\frac{dx(x,t)}{dt} &= \nabla \phi \\
\frac{d\phi}{dt} &= -g \eta + \frac{1}{2} |\nabla \phi|^2 - \frac{\rho}{\rho} - \mu_2 \frac{\partial \phi}{\partial n}
\end{align*}
\]

where air pressure \( P \) on the free surface is set to be zero (atmospheric pressure) outside of the chamber, whereas the pneumatic pressure is imposed inside the chamber. \( X_0=(x_0, 0) \) denotes the initial static position of the fluid particle. The material derivative is defined as \( d/dt = \partial/\partial t + \nabla \phi \cdot \nabla \).
Coefficient $\mu_2$ is determined by numerical tests and comparison with experiments. Damping coefficient $\mu_1(x)$ is defined as follows:

$$\mu_1(x) = \begin{cases} \omega \left(\frac{x-x_1}{L}\right)^2, & x_1 - L < x < x_1 \\ 0, & \text{else} \end{cases}$$

(7)

where $x_1$ is the starting positions of damping zone, and $L$ is the length of the damping zone and given one incident wavelength in the present study.

The boundaries at the bottom, backward wall and front wall are considered impermeable. Therefore, the zero normal velocity condition is imposed as follows:

$$\frac{\partial \phi}{\partial n} = 0$$

(8)

where $n$ is the outward normal vector of the solid boundary.

To solve the above boundary value problem in the time domain, the initial conditions are required as follows:

$$\phi|_{t=0} = \eta|_{t=0} = 0$$

(9)

By applying Green’s second identity to the fluid domain $\Omega$, the boundary value problem presented in the previous section can be converted in the usual manner into the following boundary integral equation:

$$a(p_s)\phi(p_s) = \int (\phi(p_s) \frac{\partial G(p_s, p_f)}{\partial n} - G(p_s, p_f) \frac{\partial \phi(p_f)}{\partial n}) d\Gamma + \int q \cdot G(p_s, p_f) d\Omega$$

(10)

where $\Gamma$ represents the entire computational boundary. $P_s$ and $P_f$ are the source point $(x_0, z_0)$ and the field point $(x, z)$, respectively, and $a$ is the solid angle coefficient determined by the surface geometry of a source point position. $G$ is a simple Green function and can be written as

$$G(p, q) = \frac{1}{2\pi} \ln r, \text{ where } r = \sqrt{(x-x_0)^2 + (z-z_0)^2}.$$

The above boundary integral equation for $(\phi, \frac{\partial \phi}{\partial n})$ is solved by a boundary element method using a set of collocation nodes on the boundary and higher-order elements (i.e., three-node line elements used to discretized the entire curved boundary surface) to interpolate among collocation points. Both the boundary geometry and the physical variables are interpolated based on the nodal values and the quadratic shape functions within the boundary elements, i.e., the elements are isoparametric [31].
where $\zeta$ denotes local intrinsic coordinate, ranging in the range of [-1.0, 1.0]. $[x, z]$, $\phi_k$, $(\partial \phi / \partial n)_k$ and $f_i$ are the coordinates, potentials, normal derivatives of the potential and the shape functions corresponding to the $k$-th node in the local system; $m$ is the total number of nodes in the element (i.e., $m=3$). The quadratic shape functions $f_i(\zeta)$ can be expressed as follows,

$$f_i(\zeta) = \frac{\zeta(\zeta - 1)}{2}$$
$$f_2(\zeta) = (1 + \zeta)(1 - \zeta)$$
$$f_3(\zeta) = \frac{\zeta(1 + \zeta)}{2}$$

Then Eq. (10) in discretized form can be expressed as follows

$$\alpha(p_s)\phi(p_r) = -\sum_{j=1}^{N_0} \left[ \int_{-1}^{1} \frac{\partial G(p_s, p_j(\zeta))}{\partial n} \phi(p_j(\zeta)) \frac{\partial \phi(p_j(\zeta))}{\partial n} G(p_s, p_j(\zeta)) |J(\zeta)| d(\zeta) \right]$$
$$+ \sum_{j=1}^{N_0} \int_{-1}^{1} q^*(\zeta) G(p_s, p_j(\zeta)) |J(\zeta)| d(\zeta)$$

where $N$ and $N_0$ are the number of the discretized elements on the surface (free surface, body surfaces, backward and front walls) and wave generation boundary, respectively; $J(\zeta)$ is the Jacobian matrix relating the physical coordinates to the local intrinsic coordinates within an element. Four-point Gauss quadrature method is adopted to integrate Eq. (13) numerically if $p_s \neq p_f$. When $p_s = p_f$, field point $p_f$ is then linked to the nodal points of the element which is subdivided. A polar coordinate transformation is adopted to calculate the integral within each sub-element. By making use of the shape functions, the derivatives of the velocity potential on the free surface in the free surface boundary conditions Eq. (6) can be obtained from follows

$$\left[ \begin{array}{c} \frac{\partial \phi}{\partial x} \\ \frac{\partial \phi}{\partial z} \end{array} \right] = \left[ \begin{array}{cc} \frac{\partial x}{\partial \zeta} & \frac{\partial z}{\partial \zeta} \\ n_x & n_z \end{array} \right]^{-1} \left[ \begin{array}{c} \frac{\partial \phi}{\partial \zeta} \\ \frac{\partial \phi}{\partial n} \end{array} \right]$$

where \( \mathbf{n} = (n_x, n_z) \) is unit vector normal to the boundary surface.

Finally, the algebraic system is assembled by moving the unknowns to the left-hand side of Eq. (13) and keeping the specified terms on the right-hand side. The Gauss elimination method is used to solve this equation.

After solving the boundary value problem and obtaining the fluid velocities and normal vector on the free surface, the free surface boundary conditions in Eq.(6), considered as ordinary differential equations for variables \((\phi, x, \eta)\), are advanced in time. For this purpose, a fourth-order Runge-Kutta (RK4) scheme is adopted (See detail in [32]). Thus all the boundary surfaces are re-gridded and updated at each time step. Based on the horizontal coordinates of new nodes obtained by the mesh generation, the spatial position and the potential can be calculated by interpolation using Eq.(11). To find which old element the new node belongs to, the following criterion is used

\[
S_0 - \sum_{i=1}^{M_1} S_i \to 0, \quad (15)
\]

where \(S_0\) is the length of the old element, \(S_i\) is the length of a sub-element consisting of an end node of the old element and the considered new node, and \(M_1\) is the number of sub-elements surrounding the node \((M_1=2\) in the present study).

In the process of mesh division, the spatial step is varied with the wave length on the free surface, but fixed on the body surface. As both long wave and short wave are considered in the present study, it could lead to a mismatch between meshes on the free surface and body surface. To overcome this problem, a self-adaptive Gauss integral method is adopted to divide the larger element into finer meshes as shown in Fig.2. The subdivision is stopped until the sub-element length is equal to or smaller than the neighboring one. Then the integral in the larger element is transformed into that in more sub-elements with more Gauss points. The physical and geometrical variables of new sub-elements can be calculated by interpolation using Eq. (11) within the old element.
2.2 Pneumatic model

The air and liquid in the interior of the chamber are strongly coupled because the air pressure affect the flow through the dynamical free-surface boundary condition and the free-surface variation in turn modified the air pressure. Following Koo and Kim [24], we calculates the air pressure $P$ inside the OWC chamber in Eq. (6) using the continuity equation of air flow during the volume change of the chamber and assuming a linear relationship between the chamber pressure and the air duct velocity. This pneumatic model has been successfully applied to Wells-turbine cases in both experimental and numerical analysis [19 20 33]. Thus, the air pressure in the chamber can be expressed as follows

$$P(t) = C_{dm} \cdot U_d(t)$$  \hspace{1cm} (16)

where $C_{dm}$ is the linear pneumatic damping coefficient and $U_d(t)$ is the air velocity in the duct. Then the air flow velocity $U_d(t)$ can be expressed as follows:

$$U_d(t) = \frac{\Delta V}{A \Delta t}$$  \hspace{1cm} (17)

where $\Delta V = V_{t+\Delta t} - V_t$ represents the change of air volume in the chamber at each time step $\Delta t$. $A$ is the sectional area of the air duct as shown in Fig.1. Substituting Eq. (17) into Eq. (16) gives the time varying pressure in the chamber

$$P(t) = C_{dm} \cdot \frac{1}{A} \frac{\Delta V}{\Delta t}$$  \hspace{1cm} (18)

Neglecting the spring-like effects of air compressibility inside the OWC chamber [24], the change of air volume can be easily obtained from the variation of integration of the free surface across the chamber between $t$ and $t+\Delta t$.

2.3 OWC hydrodynamic efficiency

In this study, the hydrodynamic efficiency of the OWC device is obtained using the relation between the pneumatic power and the power of the corresponding incident wave. The average pneumatic power, i.e., absorbed from the waves by the OWC, during a wave period $T$, can be expressed as the time-average of the product of the flow rate by the air pressure variation:
\[ P_{\text{OWC}} = \frac{1}{T} \int_{-T}^{T} Q(T)(P - P_0)dt = \frac{1}{T} \int_{-T}^{T} B \dot{z}(t)(P - P_0)dt = \frac{1}{T} \int_{-T}^{T} C_{\text{dyn}} U_d(t)A U_d(t)dt \]  

(19)

where the flow rate \( Q = B \dot{z}(t) = AU_d(t) \). \( \dot{z}(t) \) is the time-varying relative mean vertical velocity between the OWC and the free surface. As a 2D numerical simulation is performed in the present study, the transverse wave tank width is assumed to be 1.

From linear wave theory, the average power per unit width in the incident wave is expressed as follows:

\[ P_{\text{inc}} = \frac{1}{2} \rho g A_i^2 C_g \]  

(20)

where \( A_i \) is incident wave amplitude, and \( C_g \) is the group velocity of the incident wave packet defined by \( C_g = d\omega / dk \). For the plane progressive waves of small amplitude, the energy flux rate can also be expressed as follows:

\[ P_{\text{inc}} = \frac{1}{4} \rho g A_i^2 \frac{\omega}{k} (1 + \frac{2kh}{\sinh 2kh}) \]  

(21)

Thus, the hydrodynamic efficiency is given by

\[ \xi = \frac{P_{\text{owc}}}{P_{\text{inc}}} \]  

(22)

The efficiency \( \xi \) is in the range of \((0, 1)\), in which \( \xi = 1 \) means that the OWC device effectively captures all of the incident wave energy, which is not feasible, due to the radiated wave generated by the oscillatory motion of the interior free-surface, the scattering waves by the device and various viscous damping.

### 3. Model validation

#### 3.1 An OWC over a flat-bottom

To validate the present model, the developed numerical wave flume (NWF) was used to investigate the experimental example of an OWC device described by Morris-Thomas et al. [34] in a flat-bottom flume. The still water depth is 0.92 m. Three experimental cases are considered and the key geometric parameters of the air chamber are given in Table 1, including chamber width, thickness and immersion depth of the front wall, and the air duct width, and also shown in Fig. 1.

The incident wave amplitude \( A_i = 0.04 \) m is used unless specified otherwise. After performing numerical test, the damping coefficient \( \mu_2 \) is neglected and the pneumatic damping coefficient
$C_{dm}=3.5$ are adopted. The length of the numerical flume is chosen as five times wave length $(5.0\lambda)$ and the damping layer at the inlet of the flume is $1.0\lambda$.

| Table 1. Geometry parameters of the air chamber (m) |
|-------|-------|-------|-------|
| Case | $B$ | $C$ | $D$ | $A$ |
| 1    | 0.64 | 0.04 | 0.15 | 0.005 |
| 2    | 0.64 | 0.04 | 0.23 | 0.005 |
| 3    | 0.64 | 0.08 | 0.15 | 0.005 |

As the air and liquid are coupled inside the chamber, numerical tests were performed to assess the convergence of the adaptive spatial and temporal discretization for Case 1 with wave period $T=3.24$ s. Fig. 3 depicts the time history of the free-surface elevation at the center of the chamber and the air pressure in the chamber, obtained using a fine mesh (30 meshes per wave length, i.e., $\Delta x=\lambda/30$), an intermediate mesh (20 meshes per wave length, i.e., $\Delta x=\lambda/20$) and a coarse mesh (10 meshes per wave length, i.e., $\Delta x=\lambda/10$). In all cases, 10 meshes were distributed on the front-wall surfaces, and 20 meshes were used across the depth of the numerical wave flume. The results obtained using the fine and intermediate meshes are identical, indicating that mesh convergence was achieved using the intermediate mesh. Similar tests for three time steps ($T/40$, $T/60$ and $T/80$) using the intermediate mesh indicate that $\Delta t=T/60$ is sufficient.

![Comparison of wave elevation at the chamber center and the air pressure in the chamber](image)

(a) Surface elevation at the chamber center  (b) Air pressure in the chamber

The efficiency of the proposed numerical model in wave generation and re-reflection...
minimization is shown in Fig 4. Fig. 4 shows the time series of the surface elevation at the chamber center and the air pressure in the chamber for case 1 with wave period \( T = 3.81 \) s, \( A_i = 0.031 \) m and the comparisons between the present internal source wave generation method and a non-absorbing external wavemaker method. From the figures, it can be observed that the results from these two wave generation methods match well with each other before \( t = 46 \) s. After that, the curves from the present internal source wave generation method still remains the same, but those by the non-absorbing external wavemaker method is contaminated by the re-reflection wave from the OWC device. In addition, both the surface elevation and the air pressure in the chamber are amplified and phase-shifted by the re-reflection. For a long time simulation with surface piercing bodies, inner source generation of waves should be used since the non-absorbing external wavemaker method cause errors especially for the long wave length cases.

![Surface elevation and Air pressure](image)

**Fig. 4** Time series of surface elevation at the chamber center and air pressure in the chamber

Fig. 5 shows the hydrodynamic efficiency \( \xi \) versus the dimensionless infinite water-depth wave number \( Kh = k h_{\text{ref}} \) (i.e., \( K = \omega^2 / g \)) for cases 1-3 in Table 1. The comparisons among the linear analytical solutions [10], the experimental data [34], the numerical results by a two-phase flow Navier-Stokes solver [19] and the present numerical model are also given in the figures. It can be observed that the analytical solutions over-predict the hydrodynamic efficiency since it neglects the wave nonlinearity and the viscous dissipation, but the resonant frequencies predicted by the analytical method and the proposed numerical model agree well with each other. In addition, the general shapes of the hydrodynamic efficiency curves agree satisfactorily with each other. Overall, the present model compares well with the experimental data, and outperforms the two phase flow
model at high-frequency. Near the resonant frequency region (approximately $1.2 < Kh < 1.7$), the present model is in better agreement with the experimental data than the two phase flow model [19]. The latter overpredicts the peak efficiency, especially in case 1. In particular, an accurate hydrodynamic efficiency is predicted by the present model in which the re-reflection waves are eliminated by using internal source wave generation, but the efficiency estimated by the non-absorbing external wavemaker method may be severely contaminated by the re-reflection phenomena as shown in Fig. 4. For example, for test cases of $Kh=0.35$, 0.67 and 1.10 in Fig. 5(a), the predicted hydrodynamic efficiency $\xi$ increases from 0.19, 0.33 and 0.58 to 0.41, 0.63 and 0.76, respectively, in the presence of re-reflection waves. The wave re-reflection effect on hydrodynamic efficiency is larger for longer waves, since wave re-reflection is more problematic in a relatively short flume compared to the wave length.

![Graphs showing hydrodynamic efficiency versus dimensionless wave number in comparison with the theory by Evans and Porter (1995) [10], the observations by Morris et al (2007) [34] and the two phase flow CFD model results by Zhang et al (2007) [19].]
3.2 An OWC over a sloping bottom

Model is compared with the experimental results of Liu [17] in a slope-bottom flume to test its robustness. In this experiment, the still water depth is \( h = 0.7 \) m; the air chamber width is \( B = 0.263 \) m; the width of the air duct is \( A = 0.006 \) m; the thickness of the front wall is \( C = 0.044 \) m; the slope angle of and base length of the sea bottom slope are fixed at \( \theta = 26^\circ \) and \( LB = 1.006 \) m, respectively, as shown in Fig. 1. The incident wave amplitude is \( A_i = 2.2 \) cm. The daft of the front wall is chosen as \( D = 0.109 \) m and 0.153 m. The damping coefficient \( \mu_2 = 0.167 \) and the linear pneumatic damping coefficient \( C_{dm} = 6.69 \) are used for \( D = 0.109 \) m, and \( \mu_2 = 0.21 \) when \( D = 0.153 \) m, and Fig. 6 shows the time series of the surface elevation at the chamber center with wave period \( T = 1.225 \) s and draft of the front wall \( D = 0.109 \) m. It is shown that predicted surface elevation is in very good agreement with the experiment in the presence of sloping bottom. Fig. 7 gives the relative wave amplitude at the chamber center against the dimensionless wave length \( \lambda / B \) with draft of the front wall \( D = 0.153 \) m, together with the results of the numerical calculations of a VOF model [17] and the physical model [17]. It can be seen when the dimensionless wave length is small, both numerical calculations agree well with the experimental data. When the dimensionless wave height increases, the experimental data lies in between two numerical results that deviate from each other.

![Time series of surface elevation at the OWC chamber center](image1)

![Relative wave amplitude in the chamber as a function of relative wavelength \( \lambda / B \)](image2)

4. Numerical results

In this section, the capability of the present model for OWC simulation is tested against the experiment by Morris-Thomas et al. [34]. The surface elevation, the air pressure in the chamber, and the effects of the geometrical parameters, the slope angles, pneumatic damping coefficient and
wave nonlinearity on the hydrodynamics efficiency are investigated.

Fig. 8 shows snapshots of the wave profiles at $t=20T$ and $24T$, in which the total length of the numerical flume is is given by $L=3\lambda+B$, and the damping layer is one wave length ($\lambda$) thick at the inlet of the flume. The solid vertical line in the figure denotes the front wall of the OWC. There is a good match between the two wave profiles, including the reflected wave in front of the OWC front wall and the wave in the chamber, indicating numerical stability. In addition, the reflected wave from the device is well absorbed in the damping zone at the inlet of the flume. It can also be observed that the wave shape is periodic and sinuidoidal at the seaward side of the front wall at $Kh=0.35$ in Fig. 8 (c); however, as $Kh$ increases to 1.83 and 2.93, due to stronger nonlinrity, the wave shapes are not sinusoidal and the secondary harmonic wave crests occur in Fig. 8 (a) and (b).
Fig. 8 Snapshot of wave elevation along the wave flume at $t=20T$ and $24T$

Fig. 9 shows the time series of the relative surface elevation at the center of the chamber and the air pressure in the chamber for case 1 with different $Kh$. From Fig. 9(a), the stationary wave is formed and the relative surface elevation at the center of the chamber increases with the decrease of $Kh$. The maximum wave amplitude in the chamber can reach 2.1 times the incident wave amplitude at $Kh=0.35$. This is because the incident waves are largely transmitted through the front wall and into the chamber for the longer waves. Moreover, the waves in the chamber can be further enlarged due to the sloshing effects. Fig. 9(b) indicates that the air pressure in the chamber become maximum at $Kh=1.83$. This means that the rate of air volume changing in the chamber is the largest for $Kh=1.83$ which is near the resonant frequency at $Kh=1.78$. From Fig. 9(a), it can be calculated that the rate of surface variation per wave period $(\eta_{\text{max}}-\eta_{\text{min}})/T=1.23$, 1.83 and 1.07 for $Kh=2.93$, 1.83, 0.35, respectively. It is also These factors explained the largest air pressure at $Kh=1.83$. Fig. 9 also indicates a phase shift between the surface elevation and the pressure, which contributes to the average pneumatic power calculated by Eq. (19).
Fig. 9 Time series of (a) the surface elevation at the chamber center and (b) the air pressure in the chamber.

Fig. 10 shows the distribution of the relative maximum wave crest ($\eta_{\text{crest}}/A_i$) along the flume nearby the chamber with $Kh=2.93$, 1.83 and 0.35, in which the horizontal coordinate ‘0’ denotes the position of the back wall of the chamber. As shown in Fig. 10, for a smaller $Kh$ (a), the wavelength is shorter and the nonlinearity is stronger, therefore the wave reflection is amplified but the transmission is reduced. In contrary, the transmission is amplified but the reflection is reduced for the longer wave as shown in Fig. 10 (c). In addition, with the decreasing of relative chamber width $B/\lambda$ from Fig. 10(a) to (c), the fluctuation of the wave surface in the chamber is decreased to almost zero.
Keeping the parameters $B=0.64$ m, $C=0.04$ m, $A=0.005$ m, $\theta=0$, $C_{dm}=3.5$ and $A_i=0.04$ m constant, the influence of the front wall submergence depth on the hydrodynamic efficiency $\zeta$ is shown in Fig. 11. The hydrodynamic efficiency $\zeta$ remains almost the same for different draft of the front wall in the low frequency region, but it decreases with increasing draft $D$ in the high-frequency region. This is because the relative draft $D/\lambda$ is the determining factor in this problem. In the low frequency long wave region, the draft of the front wall is small enough (for example, $0.014\leq D/\lambda \leq 0.02$ for $Kh=0.25$ and $0.03\leq D/\lambda \leq 0.05$ for $Kh=1.1$) such that the variation of the $D$ does not affect the efficiency. Conversely, in the high-frequency short wave region, the draft of the front wall is not small relative to the wave length, so the variation of $D$ does affect the efficiency. Large $D$ has to broaden the bandwidth of the efficiency peak and thus increasing the total area under the $\zeta$ curves. Furthermore, the longer the front wall draft leads to the smaller resonant frequency. The optimal efficiency is achieved around $Kh=1.55(D/h=0.250)$, $1.65(D/h=0.207)$ and
1.72\((D/h=0.163)\). This characteristic is thought to be caused by an increased mass of water column within the chamber and the smaller resonant frequency for the relatively large \(K_h\) [35].

Fig. 11 Comparison of hydrodynamic efficiency for different draft of front wall

Fig. 12 shows the capture efficiency \(\zeta\) of OWC obtained for front-wall thicknesses of \(C/h=0.044\), 0.088 and 0.109 with \(D/h=0.250\) and the other parameters kept the same as those in Fig. 8. It can be observed that the increase of thickness \(C\) results in a relatively small natural frequency and low hydrodynamic efficiency. The optimal efficiency of 0.74 and 0.73 occurs at \(K_h=1.72(C/h=0.044)\) and \(1.65(C/h=0.088)\), respectively. This phenomenon of the thickness of the front wall has little effect on the peak hydrodynamic efficiency was also found experimentally by Morris-Thomas et al. [35] and numerically by Zhang et al. [18].

Fig. 12 Comparison of hydrodynamic efficiency for different thickness of front wall

Keeping the parameters constant as with case 1 in table 1, Fig. 13 illustrates the efficiency \(\zeta\) of OWC obtained for relative chamber width of \(B/h=0.870\), 0.696 and 0.544. As the chamber width \(B\) increases, the efficiency \(\zeta\) curve shifts to smaller frequency. The hydrodynamic efficiency \(\zeta\)
increases with increasing $B$ in the low-frequency region, vice versa in the high-frequency region. Both the resonant frequency and the peak efficiency $\zeta$ decrease with the increasing $B$, and occurs at $Kh=1.82$ ($B/h=0.544$), $1.72$ ($B/h=0.696$) and $1.54$ ($B/h=0.870$) with the hydrodynamic efficiencies of 0.76, 0.74 and 0.69, respectively. The shift of the resonant frequency to a smaller $Kh$ for a larger width $B$ is in accordance with the approximated nature frequency formula by Veer and Thorlen [36] for the water mass oscillating in a chamber as follows:

$$\omega_h = \frac{1}{\sqrt{(D + 0.41B^{0.5})/g}}$$  \hspace{1cm} (23)

Fig. 13 Comparison of hydrodynamic efficiency for different chamber width $B$

Fig. 14 shows the influence of bottom slope of $\theta=16^\circ$, $26^\circ$, $36^\circ$ and $46^\circ$ on hydrodynamic efficiency $\zeta$ with $SD=0.5$ m and the other parameters kept the same as those of case 1. These four bottom slopes correspond to the base length $LB=1.74$ m, 1.03 m, 0.69 m and 0.48 m, respectively, in which the former three bases are extended to outside of the chamber. The hydrodynamic efficiency $\zeta$ increases with decreasing slope angle in the low-frequency region, and vice versa in the high-frequency region. This is because the relative water depth change compared to wavelength is smaller in the low-frequency region and larger in the high frequency region. At the small slope angle, the hydrodynamic efficiency $\zeta$ increases with slope angle. After the slope angle increase to a certain value, the base length $LB$ gradually approaching zero, hydrodynamic efficiency $\zeta$ is no longer affected by the sloping angle.
The pneumatic damping coefficient $C_{dm}$ is a key controlling factor of hydrodynamic efficiency $\xi$. It depends on the turbine performance in the air duct. Fig 15 shows the hydrodynamic efficiency $\xi$ of case 1 against the pneumatic damping coefficient $C_{dm}$ for different incident wave periods. As seen from the figure, there is an optimal pneumatic damping coefficient for different incident wave periods. For long waves, the optimal coefficient $\xi$ is larger and hydrodynamic efficiency is not sensitive to the change of pneumatic damping coefficien. It is the opposite for the short waves. This result is consistent with the work of Cargo et al. [37], in which they also found that the optimum pneumatic damping coefficient is highly dependent on wave period but not on wave height.

Fig. 14 Comparison of hydrodynamic efficiency for different bottom slope

Fig. 15 Hydrodynamic efficiency versus pneumatic damping coefficients

To study the effect of the wave nonlinearity on the hydrodynamic efficiency $\xi$, Fig. 16 gives the variation of hydrodynamic efficiency $\xi$ with the incident wave amplitude at $Kh=1.8$ and 3.5 of case
Moreover, the quadratic fitting curves are added to each data-set to interpret the model results. Similar trend is obtained at these two frequencies, i.e., the peak efficiency $\zeta$ increases with the incident wave amplitude to a maximum and then decreases thereafter. The wave amplitude at which maximum efficiency occurs is $K_h=1.8$ in Fig. 16 (a) than at $K_h=3.5$ in Fig. 16 (b).

An explanation as for such phenomena may be found in the experimental work of Grue [38] who analyzed the nonlinear wave components induced by the wave scattering a horizontal circular cylinder and a rectangular step. Grue [38] demonstrated that the behavior of higher-order scattering waves is characterized by two different intrinsic components, i.e., bounded waves and free waves. The higher harmonic free waves possess phases different from the incident wave and may reduce the whole free surface displacement, whereas the phases of the higher harmonic bounded waves are locked to the incident wave, and may enhance the free surface displacement. At the initial stage of the increasing wave amplitude, the higher harmonic bounded waves increase in magnitude faster than the free waves. This leads to the efficiency $\zeta$ increasing with $A_i$. Further increasing $A_i$ enhanced the higher harmonic free wave more than the bounded wave, and the primary wave amplitude is reduced due to its energy loss to the higher harmonic free wave components. This leads to decreasing efficiency $\zeta$ with $A_i$. Luo et al. [24] also explored the relation between capture efficiency and incident wave amplitude using a CFD model at the optimal wave period ($T=3.25$ s, $h/\lambda=0.424$). They found that the capture efficiency decreases with increasing incident wave amplitude, which consistent with the result in Fig. 16(b), i.e., near a deep water condition, the hydrodynamic efficiency decreases with increasing incident wave amplitude.

![Graph](image)

(a) $K_h=1.8$  
(b) $K_h=3.5$

Fig. 16 Hydrodynamic efficiency vs. incident wave amplitude
5. Conclusions

Based on potential theory and the mixed Eulerian-Lagrangian technique, a two-dimensional time domain fully nonlinear NWF was developed using the HOBEM and was used to investigate a land-based OWC device with a compressed air chamber. An internal wavemaker was used to generate the incident waves in order to avoid the re-reflection phenomena at the inlet boundary so that a long time simulation can be performed in a relatively short wave flume. The air effect due to the time-varying pressure inside the chamber is incorporated by adding a pressure term to the dynamic free-surface boundary condition. The power take-off is accounted for by assuming a linear relationship between the chamber pressure and the air velocity at the duct. The hydrodynamic performance of an OWC device is examined with and without a bottom slope.

We found that the maximum surface-elevation in the chamber does not occur at the resonant frequency of the chamber but at a lower frequency at which wave transmission into the chamber is larger. The maximum air-pressure in the chamber does not occur at the same frequency as the maximum surface elevation, but at the resonant frequency of the chamber at which the air volume flux is the greatest.

The geometric parameters of the air chamber have significant influence on the hydrodynamic efficiency, therefore, an important design parameters for OWC. The shorter draft of the front wall leads to slightly larger peak hydrodynamic efficiency at a higher incident wave frequency. Because the resonant frequency of water motion in the chamber decreases with the draft of the front wall. The resonant frequency and peak hydrodynamic efficiency decrease slightly with increasing front-wall thickness. The hydrodynamic efficiency increase with the chamber width at the low incident wave frequency vice versa at the high incident wave frequency. Also the peak efficiency occurs at a higher frequency and increases with decreasing the chamber width.

For a fixed submergence depth of the back wall, the hydrodynamic efficiency increases with the bottom slope in the high frequency region, but the opposite is true in the low-frequency region. This may be caused by the downshift of resonant frequency due to decrease water depth in the chamber. the enlarged wave length outside the chamber and the sloshing effect reduction within the chamber from a larger shoaling effect, respectively. effect of bottom slope however starts to taper off
after the bottom slope reach a certain value. The optimal pneumatic damping coefficient is
dependent on the wave period. For a longer wave, the optimal pneumatic damping coefficient is
larger and hydrodynamic efficiency is not sensitive to the pneumatic damping coefficient at the
vicinity of optimal value, and the opposite is true for the shorter waves. For weaker wave
nonlinearity, the hydrodynamic efficiency increases with the wave amplitude. Conversely, the
hydrodynamic efficiency decreases with with the wave amplitude for the stronger wave
nonlinearity from a critical value, approximately $kA_i =0.10$ according to the present study.

If a proper pneumatic damping coefficient for a specified OWC device is chosen properly,
without being influenced by the wave conditions, the developed potential numerical wave flume is
expected to be powerful enough that it could be an ideal tool for the optimal design, site selection,
and safety analysis of land-based OWC devices with many numerical calculations. The developed
numerical model can be extended straightforwardly to irregular wave cases in a future study.

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