Optimization of irrigation scheduling for spring wheat based on simulation-optimization model under uncertainty

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Abstract

Water scarcity is the major constraint to social-economic development in arid and semiarid regions, where irrigation needs to be scheduled properly for the main crops. In this study, a simulation-optimization model for crop optimal irrigation scheduling under uncertainty was developed to maximize the net benefit. The model integrated a water-driven crop model (AquaCrop) with the optimization model, and incorporated the generation technique for the interval values of hydrological parameters (i.e., precipitation and evapotranspiration) and crop market prices to deal with uncertainties in these variables. The water price was assumed constant. The model was calibrated based on field experimental data obtained in 2014 and validated using 2015 data. The field experiments involved spring wheat (Yongliang No. 4) at Shiyang River Basin Experiment Station in Wuwei City, Gansu Province of Northwest China. The model was then used to generate the optimal irrigation schedules under various irrigation amounts, irrigation events, initial soil water storage and crop market price under uncertainty. Results indicated that the model is applicable for reflecting the complexities of simulation-optimization under uncertainties for spring wheat irrigation water scheduling. The optimization results indicated that the optimal irrigation amount was $[185, 322]$ mm with the corresponding optimal net benefit of $[1.05, 2.77] \times 10^4$ Yuan/hm$^2$ and the corresponding yield of $[7.4, 7.6]$ kg/hm$^2$ for extremes in the basin (defined as the 5% precipitation combined with 95%
evapotranspiration) wet condition. For extreme dry conditions, the optimal irrigation amount was [442, 507] mm with the optimal net benefit of [0.85, 2.64] \times 10^4 \text{ Yuan/hm}^2 and the corresponding yield of [6.6, 7.4] \text{ kg/hm}^2. Results also showed that four irrigation events under higher initial soil water storage were more likely to get the higher net benefit and the optimal net benefit would increase with the increasing of the crop market price. This work can be used to guide irrigation management for local farmers.

**Key words:** irrigation optimization, AquaCrop, interval numbers, bootstrap, genetic algorithm, spring wheat
China, a big agricultural country, faces a great challenge of severe water scarcity (Wang et al., 2015). In China, more than 60% of water is used for agricultural purposes, so agricultural water consumption plays an important role in the overall water balance of the country (Wang et al., 2010; Deng et al., 2015). In the northern part of China, water shortage is very serious, because this region has half of the total area of China but less than 20% of total national available water resources (Deng et al., 2006). Especially in northwestern regions, natural rainfall cannot match crop water requirements and supplementary irrigation is needed to sustain and possibly increase crop yields (Zhou, 1996; Zhou, 2001; Deng et al., 2006). However, the water available for irrigation has been decreasing, partly as a consequence of climate change but also due to the increasing competition for water demand from other factors of the economy (Singh, 2012; Wang et al., 2017). Therefore, it is important that scarce water resources used in irrigation are optimally allocated in order to guarantee food security, improve farmers’ income and improve general social economic development in the region.

The fundamental work for irrigation water allocation in regional scales is to guarantee the crop yield with the limited irrigation water in point scale. Under this situation, irrigation should be timed and quantified, i.e., irrigation scheduling program in a way, that minimizes non-productive soil water evapotranspiration or drainage losses (Arora and Gajri, 1998). Thus, optimization of irrigation scheduling is basically for
optimization of irrigation water allocation. Moreover, programming optimal irrigation schedules is also essential to balance water saving and high net benefit for the local farmers in those regions.

To achieve the optimization of irrigation water scheduling, it requires knowledge about the response of crop growth/yield to soil water situation, and a model of the economic returns of crop production. The former used one of the numerous crop simulation models and the latter is an economic model depicting the net benefits for the project.

Crop models were developed in the last few decades for simulating the indices of dynamic crop growth under different irrigation schedules (Bouman et al., 1996). Water-driven models, one type of crop growth models, are based on crop growth controlled by phonological development processes, and they normally assume that crop growth rate is linearly proportional to transpiration through a constant of proportionality (Steduto and Albrizio, 2005). Water-driven models are the least complex and most parsimonious as compared to other crop growth models (Steduto et al., 2007; Steduto et al., 2009). It is particularly suitable for semi-arid and arid regions where water is the key limiting factor for crop production. One of the most popular water-driven crop models is AquaCrop (Steduto et al., 2009), which was developed by the Food and Agricultural Organization (FAO) of the United Nations. In recent years, AquaCrop has been widely used to simulate the crop water consumption and crop yield under different irrigation schedules (Salemi et al., 2011; Kiptum et al., 2013;
Lorite et al., 2013; Nazari et al., 2013; Vanuytrecht et al., 2014; Kim and Kaluarachchi, 2015; Paredes et al., 2015; Voloudakis et al., 2015; Li et al., 2016).

Although simulation models for crop growth are good at describing the effects of various irrigation schedules on the crop growth, they could only be used to get the answers to “what if” questions (Singh, 2014b). It means that the irrigation schedules are based on scenario analysis of several user-defined alternatives. In this case, a number of pre-specified irrigation schedules will be evaluated by comparing the results of crop yield and/or water use efficiency simulated by crop growth models. Then, the irrigation schedule with higher crop yield or net benefit will be recommended. However, whilst the recommended irrigation scheduling may be the best one among the chosen options, it is unlikely to be exactly the global optimal irrigation schedule (Shang and Mao, 2006). Under this consideration, optimization methods can be combined with simulation models to derive optimal irrigation scheduling (Singh and Panda, 2013; Singh, 2014a).

Genetic algorithm (GA) introduced in the 1970s (Holland, 1975), one of the traditional algorithms for optimization model, is based on the analogy of the mechanics of biological genetics and imitate the phenomenon of selection of the fittest individuals (Baron, 1998). The solution set in GA is represented by a population of strings, which comprises a number of blocks. Each block represents the individual decision variables of the optimization problem. Strings are processed and combined according to their fitness in order to generate new strings that have the best
features of two parent strings. Selection, crossover, and mutation are the three fundamental operations involved in GA to manipulate strings and move to a new generation. Compared with other traditional methods (linear method, nonlinear method and dynamic programming), GA is more likely to be used in solving the simulation-optimization model and it has been widely used in irrigation scheduling optimization or irrigation water allocation (Wu et al., 2007; Moghaddasi et al., 2010; Wen et al., 2017).

In previous simulation-optimization models, the simulation model part was usually integrated by crop water production functions (Jensen, 1968) and water balance equation. For example, Shang and Mao (2006) developed a simulation-optimization model based on crop water production functions and produced the optimal irrigation date series for winter wheat in North China. Yu and Shang (2016) determined the optimal irrigation scheduling on a crop rotation system with a multi-objective simulation-optimization model by integrating water balance model, crop water production functions and optimization model. Wen et al. (2017) analyzed the optimal irrigation schedules for spring wheat under plastic mulching using a simulation-optimization model by coupling water balance model, crop water production functions and optimization model. However, crop water production functions were traditionally obtained from long-term field experiment, which are site-specific, expensive and time-consuming. To our best knowledge, there are few irrigation scheduling simulation-optimization modelling schemes, that have coupled
crop growth simulation model and optimization model. This is mainly because most
of the crop growth simulation models are complex and not convenient to be readily
coupled with the other models. Our current work is therefore an effort at closing this
knowledge gap.

Irrigation scheduling optimizations in real field conditions are more challenging
because many uncertainty factors are involved, such as climate parameters and
economic parameters (Li and Guo, 2014; Li et al., 2016). These climate parameters
usually change temporally and are complicated by various uncertainties. Such
uncertainties will compound the complexity of irrigation scheduling optimization by
simulation-optimization models or other traditional methods (Li et al., 2016). Most of
the previous simulation-optimization models used the average values for the
uncertainty parameters, which would neglect the randomness and complexity in both
simulating and optimizing. Accordingly, introducing uncertainty theory into
traditional simulation-optimization method can help to tackle various uncertain
factors of parameters and to reflect the complexity and reality of irrigation system.

Among the widely used uncertainty methods, the interval mathematical programming
approach is popular because of its computational efficiency (Li et al., 2018). It
considers the uncertainty by approximating the lower and upper boundaries of the
variables concerned. In addition, as the major driving factors, hydrological elements,
such as precipitation and evapotranspiration usually exhibit various degrees of
stochasticity in their behavior that must be accommodated. Therefore it is more
thorough for the simulation-optimization based irrigation scheduling to consider the
stochasticity occurring in these inputs by fully specifying their complete probability
distribution function from the uncertainty characterization of the optimization
decision variables and objective function evaluation.

Wheat, one the most important food crops, is the staple food for about 34% to 40% of
the world’s population and 50% of Chinese population (Jia, 2013). China is the largest
wheat-producing country with the highest wheat production of the world, and in
China the perennial wheat planting area accounts for 25% of the total food crops
planting area (Yang, 2010). In the arid regions of northwest China, spring wheat is
also a widely cultivated and irrigated crop (Tong et al., 2007; Jiang et al., 2012), and
it has a high seasonal water requirement for maximum yields. Border irrigation,
sprinkler irrigation and drip irrigation are the main types of irrigation systems.

Although drip irrigation and sprinkler irrigation are more efficient than border
irrigation (Deng et al., 2006), farmers in arid regions of China prefer to adopt border
irrigation because of its low cost of irrigation equipment (He et al., 2013). Thus,
spring wheat and border irrigation were selected as the target crop type and the
irrigation technology because of their popularity, respectively, for the purpose of
investigating irrigation scheduling optimization in this study.

Taking into account the considerations above, the aim of this study is to develop a
simulation-optimization model for crop irrigation scheduling on typical crop type and
irrigation technology to obtain the maximum net benefit under uncertainties. The
model will integrate a simulation model for crop growth (AquaCrop) and the optimization model formulated to maximize the net economic benefit from the project. Uncertainty in both hydrological and economic inputs was handled using the interval parameter approach because of its relative simplicity compared to other more formal and sophisticated stochastic optimization approaches.

This study thus entailed several elements as listed below:

(i) The performance of AquaCrop was evaluated for predicting soil water storage, canopy cover, above-ground biomass and crop yield based on the field experiment data from 2014 to 2015.

(ii) Interval numbers of hydrological elements for different frequencies and crop market prices were generated.

(iii) The simulation-optimization model was developed for irrigation scheduling based on the generation of interval parameters.

(iv) The model was applied to the optimal irrigation scheduling for spring wheat in Northwest China.

2 Simulation-optimization model for irrigation scheduling under uncertainty

2.1 AquaCrop model description and evaluation

2.1.1 Model description

The AquaCrop crop growth simulation model (version 5.0) was used to assess the
response of spring wheat to different irrigation treatments. AquaCrop simulates daily water balance in the root zone and crop development with a small number of inputs, e.g., meteorological conditions, initial values of the model parameters, soil characteristics and management practices. A full description of the theory and functions of AquaCrop can be found in previous research (Hsiao et al., 2009; Raes et al., 2009; Steduto et al., 2009), consequently only the key components of AquaCrop for simulating crop yield are provided here.

The biomass produced over the growth period $B$ (kg/m$^2$) is represented as:

$$B = WP^* \sum_{l} \frac{T_{rl}}{ET_{0l}}$$

(1)

where $T_{rl}$ is the actual crop transpiration in $l$th day (mm/day) and is given by:

$$T_{rl} = K_s \cdot CC^* \cdot K_{c_{tr,x}} \cdot ET_{0l}$$

(2)

the resulting yield $Y$ (kg/m$^2$) is,

$$Y = B \cdot HI$$

(3)

where $WP^*$ is the normalized water productivity (kg/m$^2$), $ET_{0l}$ is the reference crop evapotranspiration in the $l$th day (mm/day), $K_s$ is the water stress coefficient, which is a function of water content in the root zone and expressed as a fractional depletion of the total available water (non-dimensional), $CC^*$ is the adjusted canopy cover (%), $K_{c_{tr,x}}$ is the coefficient for maximum crop transpiration (non-dimensional), and $HI$ is harvest index, respectively.
2.1.2 Model evaluation

In this study, the normalized root mean square error \( (NRMSE) \) and the determination coefficient \( (R^2) \) were used to evaluate the AquaCrop model as the evaluation indicators of goodness of fit. The equations are as follows,

\[
NRMSE = \frac{100}{M_{ave}} \sqrt{\frac{1}{K} \sum_{k=1}^{K} (M_k - S_k)^2} \tag{4}
\]

\[
R^2 = \frac{\left[ \sum_{k=1}^{K} (M_k - M_{ave})(S_k - S_{ave}) \right]^2}{\sum_{k=1}^{K} (M_k - M_{ave})^2 \sum_{k=1}^{K} (S_k - S_{ave})^2} \tag{5}
\]

where \( K \) is the number of the evaluated points, \( S_k \) is the simulated value, \( M_k \) is the measured value, \( M_{ave} \) and \( S_{ave} \) are the average of the measured values and the simulated values, respectively. The simulation results are considered excellent when \( NRMSE<10\% \), good if \( NRMSE \) is in the range of 10%-20%, acceptable if \( NRMSE \) ranges 20%-30%, and poor if \( NRMSE>30\% \) (Ran et al., 2017; Ran et al., 2018).

Regarding the value of \( R^2 \), higher values indicate less error variance, and normally values greater than 0.5 are considered acceptable (Legates and McCabe, 1999; Ran et al., 2017; Ran et al., 2018).

The model was calibrated and validated by the field observations including the soil water storage in 1 m depth, the canopy cover, the above ground biomass and the crop yield. The measured canopy cover was converted from the observed LAI according to the empirical equation (Iqbal et al., 2010). The measured data in 2014 were used to calibrate the model. For details, the parameters of the model (including initial canopy...
size, canopy growth coefficient, and maximum canopy cover etc. Please see Table 3) were verified to simulate the crop growth in 2014 through an iterative process using a trial and error method until the evaluation indicators were good. After that, the calibrated parameters were tested in simulating the crop growth with the climate and irrigation data in 2015. Then, the simulated and observed values of soil water storage, canopy cover, above ground biomass and crop yield were compared to validate the model (Moriasi et al., 2007).

2.2 Optimal model for irrigation scheduling

With consideration of crop market price, irrigation water price and the other costs, the target for the objective function is to maximize the net benefit for farmers:

\[
\text{max } NB = Y \cdot P_{\text{crop}} - I \cdot P_{\text{water}} - P_{\text{other}}
\]

Subject to \[
\begin{cases}
I_{\text{min}} \leq I \leq I_{\text{max}} \\
I > 0
\end{cases}
\]

where \(NB\) is the net benefit for the farmers (Yuan/hm\(^2\), and Yuan is the monetary unit in China), \(Y\) is the crop yield (kg/hm\(^2\)), \(P_{\text{crop}}\) is the crop market price (Yuan/kg).

\(I = \sum_{j=1}^{n} i_j\) is the optimal irrigation amount per hectare (m\(^3\)/hm\(^2\)), \(i_j\) is the irrigation volume for the \(j\)th irrigation event per hectare (m\(^3\)/hm\(^2\)) and \(n\) is the total irrigation times. \(P_{\text{water}}\) is the water price (Yuan/m\(^3\)), which includes two parts, i.e., fundamental water fee (30 Yuan/hm\(^2\)) and quantitative water price (0.157 Yuan/m\(^3\)) (Su, 2014).

\(P_{\text{other}}\) is the other costs for irrigation and planting, which included the cost of seed, pesticide, fertilizer and labor (about 3750 Yuan/hm\(^2\) for spring wheat according to the
field experiment and in situ investigation. $I_{\text{min}}$ and $I_{\text{max}}$ are the minimum and maximum irrigation volume for irrigation.

### 2.3 Interval parameter programming

In this study, there are some uncertain parameters (e.g., precipitation, reference evapotranspiration ($ET_0$) and crop market price) in both simulation model and optimization model. The interval numbers for them were considered and the optimization model with interval parameters was solved by best-worst method (Huang et al., 1995).

The data series of precipitation and $ET_0$ obtained from China Meteorological Data Sharing Service System (http://data.cma.cn/site/index.html), are usually more than 30 years. The bootstrap method (Hu et al., 2015) was used to generate the interval numbers for them. The steps for generating the interval numbers for precipitation and $ET_0$ are as shown below.

Firstly, calculate the empirical distribution of the data (precipitation or $ET_0$), and determine the certain theoretical frequency curve by comparing the fit with the empirical frequency curve. Secondly, use Monte Carlo method to resample from the original sample, repeat the sampling for 1000 times, estimate the parameters for each new sample, and obtain the probability distribution of a certain frequency. Finally, obtain the distribution of each frequency and generate the corresponding interval numbers. In this study five scenarios were set according to the commonly used classification standard of wet and dry conditions of China. For details, scenario 1
(extreme wet condition) corresponds to the combination of precipitation with frequency 5% and evapotranspiration with frequency 95%; scenario 2 (wet condition) corresponds to the combination of precipitation with frequency 25% and evapotranspiration with frequency 75%; scenario 3 (normal condition) corresponds to the combination of precipitation with frequency 50% and evapotranspiration with frequency 50%; scenario 4 (dry condition) corresponds to the combination of precipitation with frequency 75% and evapotranspiration with frequency 25% and scenario 5 (extreme dry condition) corresponds to the combination of precipitation with frequency 95% and evapotranspiration with frequency 5%.

The data series of crop market price collected from Agricultural Product Price Net (http://www.3w3n.com/index/goIndex) were from 2012 to 2017. The frequency of the price data were analyzed to obtain the probability density function. 95% confidence interval was chosen to get the interval numbers for market price.

2.4 Framework for simulation-optimization model under uncertainty

The framework for simulation-optimization model under uncertainty contains mainly three parts (Fig. 1). The first part focused on the generation of interval parameters, the second part was the application of AquaCrop model, and the third part was the solution for the optimization model. In the first part, the uncertainties for hydrometeorological parameters and socioeconomic parameters were considered. For hydrometeorological data, the interval numbers of parameters were generated by bootstrap method, i.e., precipitation and reference evapotranspiration. As to
socioeconomic parameters, e.g., the market price for crop, frequency distribution was
analyzed and 95% confidence interval were chosen to obtain the interval numbers. In
the second part, AquaCrop model was calibrated and validated with the experimental
data, and then applied to simulate the corresponding crop yield under various
irrigation schedules. Based on the first two parts, the optimal irrigation scheduling for
maximum net benefit was solved by the genetic algorithm (GA) (Holland, 1975).

The framework was realized on the platform of MATLAB (R2016a, MathWorks Inc.,
MA, USA). First, the interval numbers were generated by the functions of MATLAB.
Then, the initial inputs of AquaCrop model were prepared, and AquaCropplug-in.exe
was called by the MATLAB command “dos” to simulate the corresponding yield.
After that, the objective function of optimal model was calculated and the optimal
irrigation scheduling was solved by genetic algorithm toolbox through the functions
on MATLAB.

3 Field experiment

Field experiment was carried out at Shiyang River Basin Experiment Station in
Wuwei City, Gansu Province of Northwest China (37°52′N, 102°50′E, and 1581 m
above sea level) in 2014 and 2015. The experiment station lies in a typical arid region
with 164 mm mean annual precipitation and 2000 mm pan evaporation (E601) (Jiang
et al., 2016). The soil at the experiment site is loam with an average bulk density of 1.44 g/cm³ and a filed capacity of 270 mm in 0-100 cm soil layer. The groundwater depth is more than 30 m in recent years.

Spring wheat (Yongliang No. 4) was selected as the target crop, which was sowed on March 26 and harvested on July 24 in 2014, and sowed on March 21 and harvested on July 19 in 2015. The experimental design was a randomized block and each plot had an area of 5.5×7.5 m². The treatments included mulched and non-mulched cases, although here we only concentrated on the non-mulched ones. The non-mulched cases include one sufficient irrigation treatment and four deficient ones with different water stress in growing stages (Table 1), each treatment with three replicates in 2014 and two in 2015. Spring wheat was irrigated through border irrigation with the water pumped from the aquifer, and irrigation volume was measured by the flow meter. In addition, pre-sowing irrigation was applied to promote seed emergence and ensure seedling growth.

Place Table 1 here

Time domain reflectometry (IMKO Micromodultechnik GmbH, Germany) was used to measure volumetric soil water content periodically (every 6-9 days) at the plot center along the soil profile (every 20 cm depth to 100 cm). A canopy analysis system (SunScan, Delta-T Devices Ltd, Cambridge, UK) was used to record leaf area index.
with 3 replicates in each plot. The above ground biomass was measured by oven-drying method. The crop yield was determined from two uniform areas of 1×1 m² each, with the ears air-dried naturally and weighed by scale. Soil samples were taken in five soil layers with three replications along the soil profile to measure soil properties (Table 2) in laboratory after harvest.

Place Table 2 here

4 Results and discussion

4.1 Calibration and validation for AquaCrop

Results of model calibration and validation are shown in Fig. 2 and the calibrated parameters are presented in Table 3. Results showed that the simulated values were in good agreement with the measured values in both model calibration and validation. All the evaluation indicators were within acceptable ranges. For details, the determination coefficient ($R^2$) was all above 0.65 and most of them were above 0.90 for model calibration. In model validation, the values of $R^2$ were a little lower than calibration, but all of them were above 0.57. In terms of NRMSE, they ranged from 2.44% to 15.1% for calibration and ranged from 5.41% to 13.7% for validation. Results showed a good performance of AquaCrop and indicated it was capable to be used for predicting the soil water storage, canopy cover, above ground biomass and crop yield for spring wheat at
the field site.

Place Figure 2 and Table 3 here

Fig. 3 shows the measured and simulated values of soil water storage in 1 m soil layer, canopy cover and above ground biomass for two irrigation treatments (irrigation treatment I and V) in 2014. Each irrigation depth in irrigation treatment V (170 mm for total) was half of the depth in irrigation treatment I (340 mm for total). Results of soil water storage in 1 m soil layer (Figs. 3a and 3b) showed that the simulated values were in accordance with the observations, with the sharp increase in soil water storage responding to water input through irrigation/precipitation, followed by a gradual decrease due to the continuous evapotranspiration. Soil water storage after the first irrigation in treatment V was significantly lower than that in treatment I. It indicated some of the soil water was used for crop evapotranspiration because of the insufficient water input under treatment V (Feng et al., 2014). Results of canopy cover (Figs. 3d and 3e) showed that the simulated canopy cover was in good agreement with the measured values. The maximum canopy cover reached 99% in irrigation treatment I and 97% in irrigation treatment V, which indicated that deficit irrigation could decrease the canopy cover for spring wheat. Figs 3e and 3f showed that the simulation results of above ground biomass fitted well with the measured values, both increasing almost linearly during the growth period. In the end of the growth stage,
above ground biomass in irrigation treatment was 16.34 t/hm², and it reduced to 13.34 t/hm² when the irrigation amount was cut down to 50%. For the crop yield, the values ranged from 5.11 t/hm² to 7.48 t/hm² under various irrigation treatments, which were consistent with previous study in the same study area (He et al., 2013; Yang et al., 2017; Yang et al., 2018). Results also confirmed those of Lamm et al. (1995), Pandey et al. (2000) and Igbadun et al. (2008), who stated that deficit irrigation would reduce the crop yield. Therefore, it is very necessary to balance the precious irrigation water and the crop yield/net benefit. In other words, irrigation scheduling optimization is very essential to the local farmers in the arid regions.

4.2 Interval numbers for parameters

4.2.1 Precipitation and reference evapotranspiration

Time series for precipitation and reference evapotranspiration are 55 year (from 1951 to 2016), and they were collected from Wuwei hydrological station (37°55′N, 102°40′E, and 1532 m above sea level) through the China Meteorological Data Sharing Service System. In this study, ten-days precipitation and reference evapotranspiration were analyzed and the interval numbers of them were obtained using the bootstrap method. Through hydrological curve fitting, the probability
distribution of ten-days precipitation or reference evapotranspiration was determined and parameters were estimated. The Pearson type-III distribution was fitted to the values of ten-days precipitation or reference evapotranspiration, both the distribution parameters and the parameters of these hydrological elements were estimated using the least square method. The eleventh ten-days in spring wheat growing period (110 to 120 day after sowing) precipitation and reference evapotranspiration were used as examples to demonstrate the generation of interval numbers by the bootstrap method and the probability distributions are shown in Fig. 4.

Place Figure 4 here

The eleventh ten-days reference evapotranspiration ($ET_0$) in spring wheat growing period was taken as an example. Fig. 5 presents the frequency histogram and the normal probability plot of ten-days $ET_0$ values under the frequencies of 5%, 25%, 50%, 75% and 95%. The figure shows that the normal distribution function fitted the frequency histogram well under each frequency. The scatters were evenly distributed around the 45° line, showing the distribution values of ten-days $ET_0$ under each frequency was approximately a normal distribution. Therefore, using the normal distribution, the interval number of each frequency was obtained for the 95% confidence interval. Similarly, the interval numbers of the other ten-days $ET_0$ and all ten-days precipitation were obtained and listed in Table 4.
4.2.2 Market price for spring wheat

The market price for spring wheat in Gansu Province (Fig. 6) was collected from Agricultural Product Price Net (http://www.3w3n.com/index/goIndex). The frequency distribution of market price (Fig. 7) was fitted according to the series values by Kernel Density Estimation (Rosenblatt, 1956; Parzen, 1962) and 95% confidence interval was chosen to get the interval numbers for spring wheat market price, i.e., [2.03, 4.21] Yuan/kg.

4.3 Optimal irrigation scheduling of spring wheat

4.3.1 Influence of irrigation amount on optimal net benefit

The simulation-optimization model was used to solve the optimal net benefit for spring wheat under various irrigation amounts ($I_{\text{min}}$ and $I_{\text{max}}$ in Eq. 6b as detailed in Table 5) and the initial soil water storage was set at field capacity (0.28 m$^3$m$^{-3}$) considering pre-sowing irrigation. Results are shown in Fig. 8.
As shown in Fig. 8, the optimal net benefit increased almost linearly with the increase of irrigation amount at lower level and declined slightly at higher level under different scenarios. The corresponding yield of spring wheat with the optimal net benefit was also closely related with the irrigation amount, which increased with the increasing of irrigation amount at lower level and became stable at higher level. It is because irrigation is crucial to the crop yield when the crop water demand was not satisfied. When it had been satisfied, over-irrigation would help little on crop yield. Under this condition the extra irrigation water would not produce more crop yield but waste more money on water fee, and finally contributed to the decrease of net benefit. It can be seen that the upper optimal net benefits were around $2.70 \times 10^4$ Yuan/hm$^2$ and the lower optimal net benefits were around $9.97 \times 10^3$ Yuan/hm$^2$ (Fig. 9). The optimal irrigation amount increased with the increasing of precipitation frequency, while the optimal net benefit decreased slightly with increasing of precipitation frequency. Under the extreme wet condition (5% precipitation frequency), the optimal net benefit was the highest ($[1.05, 2.77] \times 10^4$ Yuan/hm$^2$) with the irrigation amount ($[185, 322]$ mm). Under the extreme dry condition (95% precipitation frequency), the optimal net benefit decreased to $[0.85, 2.64] \times 10^4$ Yuan/hm$^2$ for the irrigation amount ($[442, 507]$ mm). When the optimal net benefit was obtained, the corresponding yields under different frequencies were around 6.6 t/hm$^2$ to 7.6 t/hm$^2$. The upper and lower
corresponding yields would be approximately equal when the irrigation amount was large enough.

In previous study on optimal irrigation scheduling of spring wheat in the same study area, Feng et al. (2014) used a crop growth simulation model to simulate the crop yields under different irrigation schedules and selected the scheduling with the highest crop yield as the optimal irrigation schedule. They finally obtained the optimal irrigation amount of 322 mm, 328 mm and 400 mm for wet condition, normal condition and dry condition, respectively. The results were similar to our study, i.e., [300, 400] mm for wet condition, [350, 433] mm for normal condition and [383, 473] mm for dry condition. The reasons for this discrepancy were that the optimal result by simulation method was the best one among the defined alternatives, it may be not the global optimal irrigation scheduling. In this research, we used both simulation method and optimization method. In addition, uncertainties on both hydrometeorological data and socioeconomic data were considered in searching for the optimal irrigation schedules.

4.3.2 Influence of irrigation times on optimal net benefit

The optimal irrigation amount in section 4.3.1 (Fig. 9) were used to investigate the influence of irrigation times on optimal net benefit and its corresponding yield. The
initial soil water storage was also set at the field capacity. Results are shown in Fig. 10.

Fig. 10 shows that under almost all the scenarios the optimal net benefit of four irrigation events was the highest, then was the three irrigation events, and the last one was the two irrigations. Under scenario 1 (extreme wet condition), the upper boundary of optimal net benefit under three irrigation events was a little higher than the others. In other words, irrigation times had little influence on the optimal net benefit under the extreme condition. It can also be seen from the figure that under the four irrigation events the optimal net benefit decreased slightly when the condition become drier, with the average intervals \([1.0, 2.7] \times 10^4\) Yuan/hm². Under four irrigation events, the optimal net benefits would decrease by 22% for the lower boundary and 6% for the upper boundary, when the precipitation frequency increased to 95%. While under two irrigation events, the optimal net benefit decreased sharply with the increasing of the precipitation frequency, with the average intervals \([0.6, 2.3] \times 10^4\) Yuan/hm². The optimal net benefits would decrease by 55% for the lower boundary and 35% for the upper boundary, when the precipitation frequency increased to 95% under two irrigation events. As the figure present, the intervals of optimal net benefit under higher irrigation frequency would be smaller when the precipitation frequency
become larger. Which is to say, fewer irrigation events would cause larger uncertainties because of the weather variations as He et al. (2013) reported. It indicated that four irrigation events were preferred to get higher net benefit under the higher precipitation frequency (i.e., dry conditions) and the acceptable optimal net benefit could be obtained only if the reasonable irrigation date was programed by the model despite the difference of climate conditions (e.g., wet condition, normal condition, dry condition and extreme dry condition). As to its corresponding yield, results were similar with the optimal net benefit. The yield of four irrigation events was the highest under all scenarios. It confirmed the results by He et al. (2014) that four irrigation events were more likely to be the best choice for spring wheat in Shiyang River basin. Therefore, four irrigation events can be set as the optimal irrigation frequency for spring wheat in the study area.

4.3.3 Influence of initial soil water storage on optimal net benefit

Pre-sowing irrigation was popular to improve the initial soil water storage, but in some places pre-sowing irrigation was not implemented and the initial soil water storage would not reach the field capacity. Therefore, it is essential to program the optimal irrigation schedules under different initial soil water storage. The optimal irrigation amount in section 4.3.1 (Fig. 9) and four irrigation events during the crop growing period were used to investigate the influence of initial soil water storage on the optimal net benefit and its corresponding yield. The initial soil water storage was set as 20%, 40%, 60% and 80% of the field capacity. Results are shown in Fig. 11.
Fig. 11 shows that the optimal net benefit increased with the increase of the initial soil water storage under all scenarios, with the average intervals of \([0.4, 1.2] \times 10^4\) Yuan/hm\(^2\) under 20% field capacity, \([0.6, 1.8] \times 10^4\) Yuan/hm\(^2\) under 40% field capacity, \([0.8, 2.4] \times 10^4\) Yuan/hm\(^2\) under 60% field capacity, and \([0.8, 2.5] \times 10^4\) Yuan/hm\(^2\) under 80% field capacity. They were all smaller than the result under the initial storage of field capacity \(([1.0, 2.7] \times 10^4\) Yuan/hm\(^2\)), which means increasing the initial soil water storage would help to increase the net benefit for spring wheat.

As to its corresponding yields, they also increased with the increase of initial soil water storage but differed distinctly under different scenarios, from 2.65 t/hm\(^2\) to 7.46 t/hm\(^2\), indicating pre-sowing irrigation was essential to promote crop yield and net benefit. The results were consistent with previous study (Wen et al., 2017) in the same study area that the higher initial soil water storage would produce higher crop yield.

**4.3.4 Sensitivity analysis of market price on optimal net benefit**

The simulation-optimization model was used to solve the optimal net benefit for spring wheat under various crop market price to analyze the influence of market price on the results. In this section, the initial soil water storage was set at field capacity \((0.28 \text{ m}^3\text{m}^{-3})\) considering pre-sowing irrigation. Results are shown in Fig. 12.
As shown in Fig. 12, the optimal net benefit increased almost linearly with the increase of crop market price at both upper and lower boundary. The upper optimal net benefit ranged from $1.01 \times 10^4$ Yuan/hm$^2$ to $3.37 \times 10^4$ Yuan/hm$^2$ and the lower optimal net benefit were from $0.83 \times 10^4$ Yuan/hm$^2$ to $3.29 \times 10^4$ Yuan/hm$^2$. Under the lowest market price (2 Yuan/kg), the optimal net benefit was $[1.05, 1.10] \times 10^4$ Yuan/hm$^2$ in Scenario 1, $[1.03, 1.07] \times 10^4$ Yuan/hm$^2$ in Scenario 2, $[1.01, 1.04] \times 10^4$ Yuan/hm$^2$ in Scenario 3, $[0.97, 1.03] \times 10^4$ Yuan/hm$^2$ in Scenario 4 and $[0.83, 1.01] \times 10^4$ Yuan/hm$^2$ in Scenario 5, respectively. When the crop market price reached to 5 Yuan/kg, the optimal net benefit would increase to $[3.29, 3.37] \times 10^4$ Yuan/hm$^2$ in Scenario 1, $[3.26, 3.32] \times 10^4$ Yuan/hm$^2$ in Scenario 2, $[3.23, 3.27] \times 10^4$ Yuan/hm$^2$ in Scenario 3, $[3.12, 3.26] \times 10^4$ Yuan/hm$^2$ in Scenario 4 and $[2.80, 3.22] \times 10^4$ Yuan/hm$^2$ in Scenario 5, respectively. It can be seen from the picture that the corresponding yield would not change with the crop market yield, and it would reach the highest value when the optimal net benefit reached to the max value. Under scenarios of 1, 2, 3 and 4, the corresponding yield spring wheat were all around $[7.4, 7.5]$ t/hm$^2$. On the extreme dry condition (scenario 5), the corresponding crop yield was $[6.5, 7.4]$ t/hm$^2$. As to the optimal irrigation amount, it neither differed with the crop market prices. As a conclusion, the crop market price was the crucial factor to the optimal net benefit, and it would not influence the corresponding crop yield and optimal irrigation.
Conclusions

To program the irrigation scheduling of spring wheat in northwest China and obtain the optimal net benefit, we proposed a simulation-optimization model considering the uncertainty of both hydrological parameters and crop market price. This model integrated AquaCrop model with optimization model, and incorporated the bootstrap method. This study constitutes a framework which was capable of: (1) simulating the response of different irrigation schedules on crop yields based on crop growth model, (2) searching out the global optimal irrigation scheduling by optimization model solved by genetic algorithm, and (3) considering the uncertainties on hydrological elements and economic parameters by generating their interval numbers.

The developed model was firstly calibrated and validated based on experiment data in 2014 and 2015. Then, interval numbers of crop market price and hydrological elements, such as precipitation and reference evapotranspiration, were generated. Lastly, the optimal irrigation scheduling for spring wheat under various irrigation amount, irrigation times, initial soil water storage and crop market price were solved. Results show that the model is applicable for reflecting the complexities of simulation-optimization under uncertainties for spring wheat irrigation scheduling.

The optimization results indicated that the optimal net benefit was around $[9.97, 27.0] \times 10^3$ Yuan/hm$^2$ and the optimal irrigation amount increased with the increase of drought degree, from $([185, 322] \text{ mm})$ for the extreme wet condition to $([442, 507] \text{ mm})$.
for the extreme dry condition). The net benefit with four irrigation events during the crop growing period were higher than the cases with three or two irrigation events, and the net benefit was the highest with the largest initial soil water storage through pre-sowing irrigations for spring wheat in the study area. Crop market price was the crucial factor to the net benefit and the optimal net benefit increased almost linearly with the increase of market price.

Note that the above conclusions were drawn under two conditions. Firstly, this study was for the point scale in the farmland, and only the typical crop type (spring wheat) and irrigation method (border irrigation) were considered. More crop types and irrigation methods should be considered to get the optimal water allocation in the future study. Secondly, the market price was a random variable and it did not change with time or crop production. The analysis of relationship between market price and time or crop production depends on much data available. Therefore, further market research about the price and its related data is required in order to analyze the influence of prices on the irrigation scheduling optimization.

**Acknowledgements**

This work was supported by the National Key Research and Development Program (2016YFC040106-3) and the Chinese National Natural Science Fund (51790535, 51679234). The valuable comments from the editor and anonymous reviewers are greatly appreciated.
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Yang, J., Mao, X., Wang, K., Yang, W., 2018. The coupled impact of plastic film


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<td>91</td>
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<td>Saturated hydraulic conductivity (mm/day)</td>
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### Table 3 Calibrated parameters of AquaCrop

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<th>Symbol</th>
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<td>CC₀</td>
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<td>CGC</td>
<td>Canopy growth coefficient (%/growing degree day)</td>
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<td>CCₓ</td>
<td>Maximum canopy cover (%)</td>
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<td></td>
<td>Time from sowing to start senescence (growing degree day)</td>
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<td>CDC</td>
<td>Canopy decline coefficient (%/growing degree day)</td>
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<td>Time from sowing to maturity (growing degree day)</td>
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<td>Time from sowing to flowering (growing degree day)</td>
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<td>Length of the flowering stage (growing degree day)</td>
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<td>Kₓfr,x</td>
<td>Crop coefficient when canopy is complete but prior to senescence</td>
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<tr>
<td>WP⁺</td>
<td>Water productivity normalized for $ET₀$ and CO₂ (g/m²)</td>
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</tr>
<tr>
<td>HI₀</td>
<td>Reference harvest index (%)</td>
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<td></td>
<td>Soil water depletion threshold for canopy senescence</td>
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<td>Minimum growing degrees required for full biomass production</td>
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Table 4: Interval numbers for ten-days precipitation and reference evapotranspiration

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<td>[1.3, 2.6]</td>
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<td>[1.8, 3.8]</td>
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<td>[0.8, 1.3]</td>
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<td>Third</td>
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<td>[3.4, 6.6]</td>
<td>[2.1, 3.5]</td>
<td>[0.9, 2.1]</td>
<td>[0.7, 1.4]</td>
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<td>Fourth</td>
<td>[5.6, 15.5]</td>
<td>[3.4, 6.1]</td>
<td>[1.9, 3.1]</td>
<td>[0.5, 1.7]</td>
<td>[0.2, 1.1]</td>
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<td>Fifth</td>
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<td>[13.2, 21.6]</td>
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<td>[1.7, 4.8]</td>
<td>[1.3, 3.2]</td>
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<td><strong>Ten-days reference evapotranspiration (mm)</strong></td>
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<td>[40.3, 51.1]</td>
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Table 5 Irrigation amount applied

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Parameter estimation

Interval estimating

Resampling from $F_n$

Obtaining distribution of parameter

Interval numbers

Precipitation

Reference evapotranspiration

Interval numbers

Market price for crop

Model calibration

Model validation (Field experiment data)

Model application

Crop yield

Irrigation amount

Initial irrigation scheduling

Selection

Crossover

Mutation

New irrigation scheduling

Constraints

Objective function

Optimal irrigation schedule—Maximum net benefit

Yes

Termination?

No

Uncertainty

Hydrometeorological parameters

Socioeconomics parameters

Interval parameters generation

Bootstrap method

Frequency analysis

Probability density function

95% confidence interval

Interval numbers

Socioeconomics parameters

Hydrometeorological parameters

Fig. 1 Framework for simulation-optimization model under uncertainty
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Fig. 6 Market price for spring wheat in Gansu Province during 2012 to 2017
Fig. 7 Frequency distribution of market price for spring wheat and the interval number of 95% confidence interval
Fig. 8 Relationship between optimal net benefit/corresponding yield with irrigation amount under different scenarios
Fig. 9 Optimal net benefit and irrigation amount under different scenarios
Fig. 10 Optimal net benefit (a) and its corresponding yield (b) of various irrigation times under different scenarios.
Fig. 11 Optimal net benefit (a) and its corresponding yield (b) of initial soil water storage under different scenarios (FC means field capacity)
Figure 12 Optimal net benefit, its corresponding yield and optimal irrigation amount under different crop market price.