Self Direction-Dependent Calibration For Wideband Radio-Interferometric Imaging

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Abstract—Radio-interferometric (RI) imaging can be formulated as an inverse problem, where the objective is to estimate a sky intensity image from degraded undersampled Fourier measurements. Future radio telescopes aim at improving imaging resolution and sensitivity by orders of magnitude. At this precision level, the reconstruction quality will be limited by the unknown calibration kernels. Accurate calibration of the array during imaging is thus needed, leading to a blind deconvolution problem. Recently, we have proposed a non-convex optimisation method to perform joint calibration and imaging in the context of monochromatic imaging, assuming that the calibration kernels are smooth both in space and time. In the current work, we extend this approach to wideband imaging by further assuming spectral smoothness of the calibration kernels. The resulting problem is solved using an alternating forward-backward scheme, benefiting from convergence guarantees.

I. OBSERVATION MODEL

Radio interferometry is a technique probing the radio sky at high sensitivity and angular resolution and large bandwidth via a collection of $n_a$ antennas. In particular, RI wideband imaging aims to jointly recover the spatial and the spectral information of the radio emission. Considering $L$ channels, for each channel $l \in \{1, \ldots, L\}$, the objective is to find an estimate of the unknown image $x_l \in \mathbb{R}^N$, from degraded Fourier measurements $y_l \in \mathbb{C}^{M_l}$. Formally, each measurement $y_{l, t, \alpha, \beta} \in \mathbb{C}$, acquired by the antenna pair $(\alpha, \beta) \in \{1, \ldots, n_a\}^2$ at instant $t \in \{1, \ldots, T\}$ and spatial frequency $k_{l, t, \alpha, \beta}$, can be written as

$$y_{l, t, \alpha, \beta} = \sum_{n=-N/2}^{N/2-1} d_{l, t, \alpha, \beta}(n) d_{l, t, \beta}(n)^* x_l(n) e^{-2i\pi k_{l, t, \alpha, \beta} \frac{n}{N}},$$

where $d_{l, t, \alpha, \beta} = (d_{l, t, \alpha}(n))_{-N/2 \leq n \leq N/2-1} \in \mathbb{C}^N$ represents the unknown direction-dependent effect (DDE) related to antenna $\alpha$, at instant $t$ and channel $l$. When spatially constant, $d_{l, t, \alpha}$ is dubbed the direction-independent effect (DIE). This work aims to estimate jointly the wideband images and the DDEs.

II. PROPOSED APPROACH

The proposed joint wideband imaging and self calibration approach combines the wideband imaging model developed in [1] with the DDE calibration approach for monochromatic imaging proposed in [2], [3]. On the one hand, for the imaging part, it is shown in [1] that estimating the wideband model cube simultaneously exploiting the spectral correlations leads to better estimates than imaging each channel separately. On the other hand, for the monochromatic calibration part [3], the DDEs are assumed to be smooth functions of both sky and time. Building on this work, we further assume possible spectral smoothness of the DDEs. For antenna $\alpha$, the associated DDEs $(d_{l, t, \alpha})_{1 \leq l \leq L, 1 \leq t \leq T}$ are modeled as 4D compact-support kernels $u_{\alpha} \in \mathbb{C}^{Q \times P \times S}$, where $S \ll N$ is the 2D square kernel size in the spatial frequency domain, $P \leq T$ and $Q \leq L$ are the respective sizes in the temporal and spectral domains. Following this DDE’s model, the number of degrees of freedom in the calibration and imaging problem is significantly reduced. Formally, we propose to

$$\min_{U, X} \frac{1}{2} \| \Phi(U, X) - Y \|^2 + r_1(X) + r_2(U),$$

where $U = (u_{\alpha})_{1 \leq \alpha \leq n_a}$, $X = (x_l)_{1 \leq l \leq L}$, $Y = (y_l)_{1 \leq l \leq L}$, and $\Phi$ is the measurement operator mapping the convolution between the DDEs and the images in the Fourier domain. Functions $r_1$ and $r_2$ introduce prior information on the images and the DDEs, respectively. More specifically, $r_1$ encompasses the positivity constraint, the nuclear norm to promote low rankness, and the $\ell_2,1$ norm to promote joint sparsity; and $r_2$ introduces bounds on the amplitude of the DDE kernels. Following the method developed in [2], the measurement model is linearised by introducing auxiliary variables $U_1$ and $U_2$ such that $U_1 = U_2 = U$. Therefore, the minimisation in (2) is done over $(U_1, U_2)$ rather than $U$, and the equality is ensured by minimising $\| U_1 - U_2 \|$. The resulting minimisation problem is solved using a block-coordinate forward-backward algorithm [4], which alternates between the estimates of $U_1$, $U_2$ and $X$, and benefits from convergence guarantees.

III. ILLUSTRATION ON REAL DATA

We present reconstruction results performed on wideband observations of the Cygnus A galaxy via the Very Large Array (courtesy R.A. Perley, NRAO, USA), spanning a bandwidth of 20MHz, centered at 8.368GHz with a frequency step of 2MHz. The data have been pre- calibrated via standard DIE calibration techniques in the array’s pipeline. We consider imaging a data cube consisting of $L = 11$ channels each having $M = 1.8 \times 10^7$ measurements. The recovered wideband images are of dimension $N = 192 \times 384$. In our approach, we estimate DDE kernels with a compact support such that $(S, P, Q) = (25, 432, 1)$. In the figure, estimated images obtained via wideband imaging [1] (left) and by our joint wideband imaging and calibration approach (right) are displayed in log$_{10}$ scale as embedded animations. One can see the efficiency of our approach in recovering images with higher dynamic range.

REFERENCES