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Advantages of Nonlinear Energy Harvesting with Dielectric Elastomers

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Abstract

The paper considers electrostatic energy conversion from ambient vibrations using dielectric elastomers. A novel device for vibration energy harvesting is proposed, studied and compared with a vibroimpact device previously investigated by the authors for the same purpose. The nonlinear dynamic behaviour of the new device is explored numerically and the results presented indicate excellent energy harvesting capabilities with an energy density of $9.15 \, J/kg$. The proposed device particularly benefits from stiffness nonlinearity of its dielectric membranes as is demonstrated by comparison with their linear behaviour. The influence of nonlinearity is further investigated, with results compared against recent reports on dielectric elastomer energy harvesting and other forms of vibration energy harvesting. Two electrical schemes that can be used for the electrostatic energy conversion are also introduced and compared in terms of their suitability for the proposed device.

Keywords:
Dielectric elastomer generator, Nonlinear energy harvesting, Energy density, Electrostatic energy harvesting

1. Introduction

Vibrations generated by humans, machines or structures provide an excellent opportunity for developing and testing new ideas and concepts of energy harvesting (EH) devices. Efforts in finding efficient solutions for such EH devices have been underway for some time (e.g. [1, 2, 3]) with much focus given to: (i) mechanical design of generators, and (ii) ways of converting mechanical energy into electrical. Both the streams play an important role in achieving the overall energy conversion efficiency, with the first stream being responsible for increasing the percentage of extracted energy, while the second one - for increasing the efficiency of energy transduction. Within the first stream various mechanical concepts for energy transfer and power take-off have been studied including linear, parametric, nonlinear (among them bi- and multi-stable) and strongly nonlinear (e.g. vibroimpact) systems [4, 5, 6, 7, 8, 9]. Since the development of an efficient EH system is very similar to that of an efficient energy sink, some ideas proposed for targeted energy transfer can also be relevant in this context [10, 11]. Majority of efforts in the second stream have been focused on the study of piezoelectric [12, 13] and electromagnetic [14, 15] devices due to their relatively clear principles of operation and application. However, a number of deficiencies attributed to piezoelectric and electromagnetic devices [3, 16, 17] made researchers think about other ways of energy conversion, with electrostatic conversion being one of them.

Electrostatic (ES) vibration EH generators utilise the electrostatic principle to convert mechanical energy into electrical one so that such a generator behaves, electrically, like a variable capacitance capacitor (VCC). Using the relationship between the charge and voltage of the capacitor:

$$Q = CV, \quad C = \frac{\varepsilon \varepsilon_0 A}{z} \quad (1)$$

where $\varepsilon$ is the relative permittivity of the medium between the capacitor plates, $\varepsilon_0$ the permittivity of free space ($= 8.854 \times 10^{-12} \, Fm^{-1}$), $A$ and $z$ the effective area and distance between the plates, one can generate power by applying either a constant voltage or charge to the plates and changing the distance between them, thereby changing the capacitance.

Conventional ES EH with help of a VCC suffers from low energy density when compared with other techniques [2]. However, the application of electroactive polymers (EAPs) instead of air enables to greatly increase the energy density. Dielectric elastomers (DEs) are the type of EAPs that have been proposed to serve as a VCC due to a variety of properties beneficial for the ES EH, such as a comparably high relative permittivity (e.g. for VHB 4905/10 $3M \, \varepsilon_\text{DE} = 3.4$) and high deformability [5, 18, 19, 20]. Often DEs are made from highly flexible materials such as acrylic, silicone, polyurethenes or rubber with some embedded particles to improve their mechanical and electrical properties. When a DE membrane is used as a VCC the second formula in (1) can be rewritten, assuming that the membrane
However, the di…

As one can imagine, both membranes will be engaged under any nonzero cyclic load (or vibrations of the device).

The distance between the membranes, has led to the study presented in this paper, in which the device performance with

Moreover, at some high values of the excitation amplitude [43] the ball was sticking to one of the membranes after a

At certain excitation amplitudes and frequencies the ball had time to accelerate to a relatively high speed

The effort to understand the optimal device size, more exactly the optimal ratio between the ball diameter, \( r_b \), and the distance between the membranes, has led to the study presented in this paper, in which the device performance with positive and negative clearance is investigated. The notion of “negative” clearance implies the layout of the device, shown in Figure 1a when the distance between the two opposite membranes is less than the diameter of the ball, i.e. \( d \leq 2 r_b \). In this case both membranes are pre-stretched, enveloping the ball, and their action is similar to that of a preloaded spring. However, because of the symmetric design adopted, the equilibrium position of the ball is stable.

The first DE generator was proposed by Pelrine et al in 2001 [21]. Basic theory describing the electromechanical behaviour of DEs was set out by Suo et al [22]. The material properties have been extensively investigated [23, 24] and review papers compiled in the field [25, 26], with specific research having been done on the commonly used research materials, acrylates VHB 4905 and VHB 4910 [27, 28]. Huang et al presented a DE generator which used a new electrical circuit and produced 550 J/kg [29]. Their electrical circuit was based on the constant voltage scheme with other schemes such as constant charge and constant field being investigated [30, 31, 32]. Another idea of utilising this principle for wave energy converter has been proposed and studied [33, 34, 35]. In recent years, a number of review papers on the use of DEs for EH have been published (e.g. [36, 37, 38, 39, 40]).

Recently, the authors proposed a vibroimpact device that used DE generators to convert mechanical energy of impacts into electrical energy [41, 42, 43]. This device consisted of two membranes, coated with compliant electrodes and located at the ends of a hollow tube at distance \( d \) from each other. A free moving ball was placed between the membranes; friction was neglected in the device analysis. When the device was subject to ambient vibrations, the ball moved inside the tube colliding with the membranes, which worked as DEs generators converting the mechanical energy from the ball’s impacts into electrical one. The distance \( d \) between the membranes clearly affected the efficiency of this device. At certain excitation amplitudes and frequencies the ball had time to accelerate to a relatively high speed before impacting one of the membranes, thus causing its large deformation and producing higher electrical output. At the same time, there were some threshold values of the excitation amplitude and frequency, which depended on the distance \( d \), below which the ball was not impacting either membrane when the device was in a horizontal state. Moreover, at some high values of the excitation amplitude [43] the ball was sticking to one of the membranes after a number of insignificant chatter-type impacts, resulting in poor energy efficiency. A similar effect had been observed earlier by other researchers (e.g. [4, 44, 45]). In addition, at any nonzero angle of the device inclination, the ball dropped down on the bottom membrane and when the device acceleration was below a certain minimum value the ball did not reach/impact the top membrane. These factors led to poor performance of the device both in horizontal and inclined positions, despite some “help” of gravity force in the latter case.

The volume stays unchanged, as:

\[
C = \frac{\epsilon_{DE} \epsilon_0 \text{Vol}}{\epsilon^2}
\]  

where \( \text{Vol} \) - volume of the membrane used and \( \epsilon \) the membrane thickness. Thus, mechanically loading the membrane and changing its thickness will induce conversion of mechanical energy to electrical one.

The section results of numerical studies are presented and comparison of two electrical schemes is demonstrated. The third section shows numerical results for some specific cases that validate the developed numerical code. The fourth section describes the basic equations of motion used to simulate the device dynamics and an electrical scheme to collect the harvested power. The third section shows numerical results for some specific cases that validate the developed numerical code. In the fourth section results of numerical studies are presented and comparison of two electrical schemes is demonstrated. The paper ends with brief conclusions.

![Figure 1: Potential generator layouts: (a) is cases where \( d < 2 r_b \), (b) is a case where \( d = 2 r_b \), and (c) is cases where \( d > 2 r_b \)](image-url)
2. Theoretical development

The DE generator used in this study comprises a cylindrical device (capsule) with a hollow interior (tube) containing a ball \( m \), that can freely move in it (the friction is neglected). Two DE membranes (shown in orange) enclosing the tube at each end, as shown in Figure 1. Each membrane is encased between a housing and the tube, so that the distance between the two membranes is \( d \). The generator may be inclined at an angle \( 0^\circ \leq \beta \leq 90^\circ \), with the excitation force, \( F(t) \), acting along the inclined axis \( z \). The ball of radius \( r_b \) is free to move along the same angle as the generator and has the positive displacement, \( z_m \), pointing downwards. Similarly the generator as a whole has a mass \( M \) and can move along the same axis under the excitation force, so that the generator’s positive displacement \( z_M \) to the left. The ball moves within the hollow internal section of the tube with radius of \( R_0 \), which is marginally larger than \( r_b \) to let the ball move freely.

Depending on the values of \( z_m, z_M \) and the length of the generator, \( d \), the ball can either stay in touch with just one or both membranes. It is assumed that the effect of the ball on the motion of the generator is negligible and is disregarded. The ball however will experience a change in the direction and magnitude of the velocity due to a soft impact model.

2.1. Dynamic Analysis

To fully understand the nonlinear behaviour of the ball and energy harvesting capabilities the governing equation of motion should be written. For that two fundamentally different cases of \( d < 2r_b \) and \( d \geq 2r_b \) should be considered separately from the mechanical point of view. Then an electrical equations responsible for energy harvesting should complement the set of equations.

Assume that the generator is subject to a harmonic excitation, the capsule’s motion is governed by the following equation (3)

\[
\ddot{z}_M = \frac{A}{M} \cos(2\pi f_0 t)
\]

(3)

where \( A \) and \( f_0 \) are the amplitude and frequency of the excitation. From this the velocity and displacement of the generator can be found:

\[
\begin{align*}
\dot{z}_M(t) &= \frac{A}{M} f_0 \sin(2\pi f_0 t) + C \\
\ddot{z}_M(t) &= -\frac{A}{M} f_0^2 \cos(2\pi f_0 t) + Ct + D
\end{align*}
\]

(4)

where the constants \( C \) and \( D \) can be determined from the system initial conditions.

The balls behaviour is more complex though and is acted on by forces created in an interaction with the membranes as well as being under the influence of gravity, \( +mg \sin \beta \) (\( g = 9.8\text{m/s}^2 \) and it is positive due to selected positive direction of \( z_m \)). Hence the acceleration of the ball is for the case of \( d \geq 2r_b \) is:

\[
\ddot{z}_m = g \sin \beta
\]

(5)

To study the effect of interaction of the membranes and the ball let us introduce a relative displacement of the ball with respect to the generator:

\[
\Delta z = z_m - z_M
\]

(6)

The deflection of the membranes is very important indicator of the energy generation since the larger the displacement the greater the change in capacitance resulting in more energy produced. In this paper the membrane deflection is calculated as the absolute value of displacement of the membrane centre point along the ball direction of motion from its rest position. Whenever the ball is in contact with a membrane that membrane is considered to be deflected, regardless of relative velocity. Then the equations (7) and (8) are derived, which represent the deflections of the upper/right and lower/left membranes respectively.

\[
\delta_u = \begin{cases} 
-\Delta z + r_b - d/2, & (-\Delta z + r_b - d/2 > 0) \\
0, & (-\Delta z + r_b - d/2 < 0)
\end{cases}
\]

(7)

\[
\delta_l = \begin{cases} 
\Delta z + r_b - d/2, & (\Delta z + r_b - d/2 > 0) \\
0, & (\Delta z + r_b - d/2 < 0)
\end{cases}
\]

(8)

For the purpose of dynamic analysis the membranes are treated as nonlinear springs, thus the ball will experience the following force when impacting upper and lower membranes correspondingly:

\[
f_{us} = K \delta_u^n, \, f_{ls} = -K \delta_l^n
\]

(9)
Figure 2: Net spring force from the DEs dependant on generator length, \(d\).

where the values \(K\) and \(n\) are the material constants validated experimentally earlier in [41] and have the values \(K = 4.0847 \times 10^5\) and \(n = 2.6\). Figure 2 demonstrates the net restoring force acting on the ball from both membranes for different values of \(d\). The preloading case presented differs from classical preloaded systems [46, 47, 48] as both DE membranes are free to move without blockers thereby the resultant force between the two should be considered as they cancel each other out when \(d < 2r_b\). It can be seen that with the increase of \(d\) ”flat” part of the curve widens out and when \(d > 2r_b\) the ball can freely move \(d - 2r_b\) distance inside the capsule before impacting one of the membranes. The difference in the curves can be explained by looking at the net force experienced by the ball from the membranes when it is displaced a little from its equilibrium position. Whereas in the case of \(d > 2r_b\) the ball will experience the net force from one of the membranes it is much smaller than that from both membranes when \(d = 0\) for instance, taking into account the stiffness nonlinearity.

Most of the DE materials exhibit viscoelastic properties resulting in energy losses during their deformation. A viscous damper is used to capture this effect, however because the ball is not rigidly attached to either membrane the energy losses happen only when the ball deforms the membrane slowing down, but not when the ball is rebounded from the membrane accelerating. Assuming the upper and lower membranes’ deflection velocity (the centre point) are \(\dot{\delta}_u\) and \(\dot{\delta}_l\) one can write the following conditions for the energy losses at the impacts by each membrane:

\[
\delta'_u = \begin{cases} 
\delta_u, & (\Delta z + r_b - d/2 \geq 0) \text{ and } (\Delta \dot{z} < 0) \\
0, & \text{Otherwise}
\end{cases}
\]

\[
\delta'_l = \begin{cases} 
\dot{\delta}_l, & (\Delta z + r_b - d/2 \geq 0) \text{ and } (\Delta \dot{z} > 0) \\
0, & \text{Otherwise}
\end{cases}
\]

Then, the ball will experience the following forces:

\[
\begin{align*}
\text{f}_{ud} &= \alpha \delta'_u, \\
\text{f}_{ld} &= -\alpha \delta'_l
\end{align*}
\]

where \(\alpha\) represents the equivalent viscous damping coefficient. Combining equations (3), (5), (9) and (12) the static equation for the ball can be derived as:

\[
m\Delta \ddot{z} = mg \sin \beta + K \delta'_u + \alpha \delta'_u - K \delta'_l - \alpha \delta'_l - mF(t)/M
\]

This equation is valid when the ball is in contact with both membranes, however for cases where \(d \geq 2r_b\) and in some specific cases for \(d < 2r_b\) when the excitation amplitude, \(A^*\) and frequency make the ball depart from one of a membrane the next interaction will result in an impact. Impacts will occur between the ball and the membranes when the relative displacement is larger than the distance to a membrane, which can be written as following:

\[
\begin{align*}
\Delta z > d/2 - r_b & \quad \text{impact occurs on the bottom/left membrane} \\
-\Delta z > d/2 - r_b & \quad \text{impact occurs on the top/right membrane}
\end{align*}
\]

However, the soft impact model used here does not require an instantaneous change in the ball velocity, but rather engage the nonlinear stiffness and viscous damping terms discussed above and presented in (14). The proposed highly nonlinear problem will be solved numerically where the relative displacements between the ball and each membrane is monitored at every time step.
2.2. Electrical Analysis

Recently a new electrical scheme has been designed and was first set out by Shian et al [49]. This scheme, also called the ‘triangular’ scheme seeks to maximise the working area within the failure limits of the generator set out by [19] as shown in Figure 3, where also the dielectric breakdown (DB) and electromechanical instability (EMI) failure limits in red are shown. This makes it clear how the triangular scheme aims to optimises the harvested energy, when compared to the constant voltage and constant charge schemes, shown with green and blue shading respectively. The triangular scheme claimed to have the highest energy density to date of 780 J/kg.

![Figure 3: Triangular Scheme Cycle](image)

DEs convert electrical energy to mechanical energy by cyclically stretching and relaxing. This change in dimensional properties at various points in the cycle is what allows for this conversion to take place. Hence in order to fully understand the conversion, the dimensional properties of the membrane need to be understood at each time step. Since the volume of the membrane is considered to be constant, by knowing the area of the membrane the new thickness can be found using:

\[ Vol = A(t_k)h(t_k) \]  

(15)

where \( Vol \), \( A \) and \( h \) are the volume (kept constant), area and thickness of the DE with the subscript \( k \) referring to the individual time instances. The initial volume of the membrane is known, however, changes in the area due to the ball impacting the membrane has to be analytically derived based on the geometry of the ball and membrane. When the ball deforms the membrane its surface area can be split into two parts - a conical fulcrum and a spherical cap, assuming that the ball has almost the same diameter as the membrane:

\[ A = A_1 + A_2 \]  

(16)

where \( A_1 \) and \( A_2 \) represent the spherical cap and conical fulcrum respectively for each membrane and their dependence on time \( t_k \) has been skipped for brevity. These in turn can be derived as:

\[
\begin{align*}
A_1 &= 2\pi r_b(r_b - r_b \cos \psi) = 2\pi r_b^2(1 - \cos \psi) \\
A_2 &= \frac{\pi R_0^2 - \pi(r \sin \psi)^2}{\cos \psi}
\end{align*}
\]  

(17)

where all the values are known constants apart from \( \alpha \) - the angle the membrane has been deflected from its normal state and is directly related to the deflection \( \delta \) of the membranes. Hence one can geometrically find \( \alpha \) using the deflection and the initial undeformed radius of the DE, \( R_0 \) [41]:

\[
\cos \psi = \frac{-2r_b(\delta - r_b) + 2R_0\sqrt{R_0^2 + \delta^2 - 2\delta r_b}}{2\left[R_0^2 + (\delta - r_b)^2\right]} 
\]  

(18)
From equations (16), (17) and (18) the area of the membrane for each point can be determined. The area is proportional to the deflection, $\delta$, this means that for excitation conditions that lead to larger deformations will induce larger changes in the capacitance. The difference in voltage and charge between the time instances of maximum and minimum capacitance is directly responsible for energy harvesting mechanism, thus the deflection of membranes and their capacitances should be monitored as every time step.

To find the capacitance the thickness of the membrane must first be calculated using (15) and used in (19):

$$C = \frac{\epsilon_0 \epsilon_{DE} Vol}{h^2}$$  \hspace{1cm} (19)

where each $DE$ is to be treated equally and separately. The electrical system used in this paper was first introduced by Shian et al [49]. The system works by initially charging the DE at it’s maximum stretch point, before disconnecting the DE from the supply, which is labelled as line 1 in Figure 3. The charge over the DE at this point, point A, can be found using the input voltage, donated as $V_{in}$ and equation (20) and is:

$$Q_A = V_{in} C_A$$  \hspace{1cm} (20)

where the subscript $C_A$ indicate the maximum stretch state, $\lambda_{max}$ for a cycle respectively. The DE is then allowed to relax back towards its unstretched state. As the DE is relaxing some of the present charges will be transferred over the transfer capacitor at a rate determined by the ratio of the capacitances of the DE and the transfer capacitor $C_T$.

As the charge over the DE decreases and the capacitance decreases there will be an increase in the voltage over the remaining charges corresponding to equation (21). The charge at this point, point C, can be derived by considering the total electrical energy input into the system and the relative capacitances, as is shown in equation (21). This process is graphically depicted in Figure 3 and marked by line 2.

$$Q_B = V_B C_B$$  \hspace{1cm} (21)

where $Q_B$ and $V_B$ represents the charge and voltage over the DE at the minimum stretch of the DE, $\lambda_{min}$ which correlates the the minimum capacitance, $C_B$. It should be noted that the gradient of line “2” is directly related to the transfer capacitor $1/C_T$ [49] and therefore can be used to optimise the DE performance covering the largest avoiding the failure domains. Then the amount of charge after a cycle can be expressed as:

$$Q_B = \frac{C_B C_A V_{in}}{C_T + C_A}$$  \hspace{1cm} (22)

At the end of each cycle when the DE is at it’s minimum capacitance the second switch in the electrical circuit is closed allowing both the DE and transfer capacitor to both fully discharge into the load or storage circuit, as shown in
process 3 in Figure 3. After this the DE will be stretched back out to its maximum capacitance before the electrical cycle repeats. This means that the enclosed area shown in Figure 3 represents the total energy gain per cycle and can be calculated by considering the 2 triangles of the shaded green section, and the triangle above in Figure 3, this gives the result as:

$$E_{\text{net}} = \frac{1}{2} V_B (C_B - C_A)$$

(23)

where $V_B$ is found computationally. Whilst the net energy can be useful indicator of the device performance in practical applications the generated power is used more frequently. To calculate electrical power the energy can be multiplied by the frequency of the cycle:

$$P_{\text{net}} = E_{\text{net}} f_0$$

(24)

Although it seems obvious from Figure 3 that the triangular scheme is superior compare to two other options, it requires a sophisticated electrical circuit with switches. On the other hand the constant voltage scheme is much easier to implement and thus it is important quantitively compare them keeping in mind the fact that the vibroimpact interaction depends highly on the excitation’s amplitude and frequency and therefore various significantly. The constant voltage scheme has been studied by the authors earlier [41] and presumes a constant voltage is kept on the membrane all the time. The power harvested in a given cycle is shown is:

$$P_{\text{net}} = \frac{1}{2} V_{\text{in}} V_T$$

(25)

where $V_T$ is the averaged voltage gain removed from the DE. The nonlinear dynamics behaviour of the device and its EH potential is analysed in next two sections.

3. Free Vibrations

To validate the code describing the system performance first the response of the system to free vibrations for various values of parameters is studied. It should be noted that during the process of unstretching, corresponding to line ”2”, it is essential to calculate the area of the large triangle in the triangular scheme. This is done by calculating the area of the bottom part (shaded in green and corresponding to the constant voltage scheme) and top parts separately and then adding them together.

Let’s first consider the case of $d = 3r_b$, $\beta = 90^\circ$ and $\alpha = 0$ with the results shown in Figure 4. In this case the initial position and velocity are both equal to zero, therefore the ball is placed at the centre of the vertically $\beta = 90^\circ$
positioned capsule. Due to the gravity effect the ball will repeatedly fall and impact off of the bottom membrane before returning to its original position since the energy is conserved. This can be seen from the top left plot that demonstrates the ball’s relative displacement, whereas the relative velocity is shown in the top right plot. Two red lines in the left plot indicate the distance to the upper and lower membranes, therefore crossing the red line implies the deformation of the corresponding membrane. It can also be seen that the bottom red line corresponding to the upper membrane is not crossed indicating that it is not engaged thus should not be deformed and harvest energy.

The next three rows in Figure 4 express the deflection, force and capacitance of the bottom (left column) and top (right column) membranes. The red-dash line at the bottom row indicates the lowest value of the corresponding capacitance, which is related to the membranes’ undeformed state. As expected, the ball deforming the low membrane will experience force from it and this deformation will result in the periodic changes in the capacitance associated with the low membrane. The time instances when the low membrane’s deflection (the force and capacitance) are staying zero correspond to the free flying motion of the ball.

The numerical results for the similar set of parameters are shown in Figure 5 for $\beta = 0^\circ$. For the sake of consistency with the previous case the initial relative displacement is kept zero however the initial relative velocity is set to be the same as the impact velocity from the Figure 4, which is found to be $\Delta z(0) = \sqrt{gd}$. As can be seen from the relevant
time histories the ball has a symmetric periodic motion with the respect to its initial position impacting the right and left membranes in sequence with the absolute values of the ball’s velocity staying the same due to the elastic impact \( \alpha = 0 \) assumption. It can be seen that once a contact is made the ball’s velocity reduces until the DEs deformation reaches its maximum, as well as its capacitance, and then the pattern repeats.

Two phase portraits corresponding to the above cases are shown in Figure 6, where the asymmetry observed in the left plot is associated with the influence of the gravity.

Very similar to the previous case of \( \beta = 0^\circ \), \( \alpha = 0 \) results are presented in Figure 7 for \( d = 0 \) and \( \Delta z(0) = 0 \) and \( \Delta z(0) = 0.5687 \text{ms}^{-1} \). As can be seen the ball oscillates around the central point of the generator engaging both membranes all the time and changing their capacitances. Since the initial velocity is not sufficient enough for the ball to loose the contact with either of the membranes, the ball will stay in contact with both of them at all time, consequently the capacitance of both DEs will be above their minimum positions. It should be stressed that although the absolute change in the deformation in this case is smaller than that for the case of \( d \geq 2r_b \), because the membranes have a stiffening type nonlinearity, the change in the capacitance is greater than that in the previous case due to the nonlinearity.

4. Forced Vibrations

4.1. Implementing the Triangular Scheme

Having validated the code the study of the forced response of the system will be presented next. For all cases investigated below the excitation force is of the form \( F = A \cos 2\pi f_0 t \) and the zero initial conditions have been selected. The input voltage in all the cases was \( 2000V = 2kV \) and the triangular scheme will be used first to assess the performance of the device.

The next four sets of plots will demonstrate the power maps, namely the averaged amount of power harvested by the device of a specific design and orientation under different values of the excitation amplitude and frequency. First, a set of results for the case of \( \alpha = 7.6 \) and \( \beta = 0^\circ \) is shown in Figure 8.

It should be stressed that the maximum value of energy in the first two plots is \( 60mW \), whereas in the bottom four maps is \( 20mW \). It can be seen by comparing the maximum value of energy throughout all the plots that with the increase of \( d \) it drops. This is somewhat unexpected, because one would think that the larger \( d \) the higher the ball speed before the impact and consequently the higher the deformation of the membrane. This is not what is observed in this case. One can see that the maximum amount of energy harvested in this case occurs at \( d = 0 \) and within a certain frequency range, between \( 30Hz \) and \( 40Hz \). Of course more energy can be harvested by applying larger excitation amplitude. With the increase of \( d \) the peak of harvested energy shifts left and for \( d = r_b \) is found around \( 20Hz \), which corresponds to \( 35mW \). Further increase of \( d \) keeps shifting the peak to the left reducing its absolute value. The dark blue or zero level at the bottom right side corresponds to the combination of amplitudes and frequencies of the excitation where the ball does not impact the membranes at all. It should also be mentioned that the case of \( d = 0 \) is better for almost all the cases, for instance the maximum power for \( d = 4r_b \) is achieved at \( 7Hz \) and is equal to \( 11.7mW \), which is smaller than \( 13mW \) achieved at the same frequency and \( d = 0 \); for \( d = 2r_b \) the maximum power harvested at \( 12Hz \) and is \( 20.5mW \), which is smaller than \( 23mW \) achieved at the same frequency but \( d = 0 \). The only exceptions to this may be observed at some parts around the wake of the high energy domains in the cases of \( d \geq 2r_b \), for instance at \( \beta = 90^\circ \) and \( d = 5r_b \), shown in Figure 9, the peak power is \( 9.6mW \) that is generated at \( 5Hz \), which corresponds to a power of \( 9.1mW \) at the same frequency when the generator length is \( d = 0 \). However, such an almost negligible superiority at a particular frequency does not significantly changes the overall observation that the shorter device is superior.

In the case of \( \beta = 90^\circ \) shown in Figure 9, one can observe the similar trend in terms of maximum values of harvested power and the peak shift to the left with the increase of \( d \). However, a different pattern of level lines can be observed to the right from the peak values at the bottom four plots. This is associated with the action of the gravity, which engages the bottom membrane all the time. One can also see that the level lines on the bottom four plots are organised and focused around the top right corner staying unchanged for increasing values of \( d > 2r_b \). Moreover the curvature of the level lines allows to reach the same power level at two different frequencies.

The next set of maps in Figure 10 and Figure 11 demonstrates the results of numerical simulations for \( \alpha = 48.9, \beta = 0^\circ \) and \( \beta = 90^\circ \) correspondingly. The structure of level lines has significantly changed showing almost everywhere a flat pattern indicating the same level of harvested power over a wide frequency range. Apparently, the absolute value of gained power is much lower than that for \( \alpha = 7.6 \) due to high energy losses. The difference between these two sets can be observed in the bottom plots, where the zero level of harvested power is shown in the right low part of the plots for \( \beta = 0^\circ \) case. The gravity in Figure 11 makes the low membrane harvest energy continuously and therefore some non-zero values of harvested power are present. The fact that almost the same amount of energy can be harvested by the device over a relative large frequency range can be thought of as a somewhat positive outcome.

These results can be viewed in more detail by analysing selected state space plots, all of which are taken from cases when \( \beta = 0^\circ \), and are shown in Figure 12. The first figure, Figure 12a shows the case for \( d = 0 \) under 3 different
cases: the red line indicates the phase space for the peak conditions when $\alpha = 7.6$, which occur when $A = 25N$ and $f_0 = 35Hz$ as can be seen in Figure 8a; the green line represents a non peak condition from the same simulation for $A = 5N$ and $f_0 = 5Hz$, from the same simulation; the blue line represents the peak conditions when $\alpha = 48.9$ ($A = 25N$ and $f_0 = 35Hz$). In Figure 12b the case of $d = r_b$ is presented the peak conditions $A = 25N$, $f_0 = 20Hz$ for $\alpha = 7.6$. The state-space plots demonstrates the case when the ball moves from being in contact with two DE’s to just one DE. This occurs at the two ‘tips’, once this happens the ball immediately begins decelerating before the DE reaches its maximum deflection, corresponding to $\Delta z = 0$, before accelerating in the reverse direction and getting back in contact with the opposite DE.

The other cases presented have impacting interaction only, with Figures 12c and 12d giving detail to the case of $d = 2r_b$ at peak conditions of $A = 25N$ and $f_0 = 12Hz$ and $\alpha = 7.6$. The relative displacement (blue) and velocity (black) of the ball shown in Figure 12d correspond to the phase plot in Figure 12c, where some “skew-symmetric” portrait can be observed. This happens because when the ball is displaced from its equilibrium position it is in contact with one of the membranes only. Thus, when the ball moves towards its equilibrium position the membrane helps the ball to accelerate until it crosses the equilibrium position and hits another membrane, which will resists its motion forward. Thus one can see the discontinuity in the acceleration at $\Delta z = 0$ reflected in different curvature of lines approaching $\Delta z = 0$ line.

Figures 12e and 12f represent state space plot and time response of the ball for $d = 4r_b$, $A = 25N$, $f_0 = 7Hz$ and $\alpha = 7.6$. In the presented case the ball undergoes a double impact from each membrane apparently because the ball’s rebound speed is less than that of the capsule.

Looking at these power maps an important question may be asked: how the degree of nonlinearity influences the harvesting potential of the proposed device. To address this question a set of maps is presented in Figure 13 for $\beta = 90^\circ$, $\alpha = 7.6$, $d = 0$ and different values of $n$ (see (9)), and the maximum energy densities achieved for each case as well as for $n = 2.6$ (61.8$mW$ from Figure 8a), corresponding to VHB4910, are presented in Table 1. It can be seen that for a linear restoring force, Figure 13a, the maximum value of the gained power is the lowest one out of all presented cases. It can also be observed that with the increase of nonlinearity the absolute amount of gained power increases nonlinearly moving the peak frequency to the low frequency range. However, for large values of $n$ a larger external force is required to deform the membranes and therefore a large portion of Figure 13d shows low energy output. Besides, the electric field applied to the material in [51] was three times larger, the diameter of the dielectric elastomer was approximately four times larger and the thickness was only a third of the thickness used in this work. However, when the same dimensions were used in our simulation coupled with an equal deformation then a similar energy density was achieved. Alternatively, the results are compared well against 8.66$mJ$/kg obtained in [52] as the two generators are of a similar volume and electric fields. Nonlinear energy harvesting has been explored in other types of generators, for example in piezoelectric (PE) harvesters. Majority of PE EH devices reported in [53], for instance, demonstrate much higher energy density due to either larger size or much higher (orders of magnitude) excitation frequencies. However, when comparing with the device of similar size under an equivalent excitation frequency the presented results indicate greater energy density than reported for PE EH devices in [54] for the existing material ($n = 2.6$) and orders of magnitude higher when $n > 3$.

### 4.2. Triangular vs. Constant Voltage scheme

The results are set out to compare the two schemes using equations (24) and (25) for a range of input conditions. In Figure 14 the system is compared when $\beta = 0^\circ$ and $\alpha = 7.6$, with the top row and bottom rows of figures representing the cases of $d = 0$ and $d = 4r_b$ respectively. It can be seen that at all sets of initial conditions that the ‘Triangular’ scheme outperforms the constant voltage scheme by a factor of 49.7% for $d = 0$ and 33.2% for $d = 4r_b$. These estimates however only apply when the dynamics of the generator allow for the influence of the electrical generation.

<table>
<thead>
<tr>
<th>Nonlinearity, $n$</th>
<th>1</th>
<th>1.8</th>
<th>2.6</th>
<th>3</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak Frequency (Hz)</td>
<td>50</td>
<td>50</td>
<td>35</td>
<td>14</td>
<td>3</td>
</tr>
<tr>
<td>Maximum Energy Density (J/kg)</td>
<td>0.017</td>
<td>0.373</td>
<td>9.15</td>
<td>32.2</td>
<td>4359</td>
</tr>
</tbody>
</table>

Table 1: Results of the numerical modelling for various values of $n$
For example in Figure 14c after the system peaks at $f_0 = 13\, \text{Hz}$ it begins to decrease in energy harvesting capability, and from $18\, \text{Hz}$ the generator will produce very little power. In these conditions the difference between the two schemes is negligible. Similarly in Figure 14d, since the generator length is $d = 4r_s$ the system needs a large enough amplitude to begin impacting the DEs, below this amplitude the difference in electrical systems is negligible. This shows that as expected the from Figure 3, the triangular scheme is capable of optimising the electrical working area within the bounds of the dynamical system and the failure limits of the DE technology. In application, however, the implementation of a constant voltage system is simpler to achieve due to the lack of need for complex switching.

5. Conclusion

This paper has presented a comprehensive dynamic analysis of a novel nonlinear oscillator for energy harvesting from ambient vibrations. The device uses an electrostatic method of converting mechanical energy into electrical one incorporating dielectric elastomer in a form of a membrane mimicking the behaviour of a variable capacitance capacitor. The presented numerical results show the superiority of the proposed design, with an energy density of $9.15\, J/kg$, compared to the vibroimpact harvester, which produced $0.652\, J/kg$ [41, 42, 43]. Apparently, the reported finding has a positive influence on the EH technology indicating that this new generator design, smaller than the previous one, can harvest and produce significant amount of energy. The comparison of numerical results has also demonstrated the positive influence of stiffness nonlinearity on the device’s harvesting capabilities. Moreover, based on comparison of the results with other authors it has become obvious that the device harvesting potential depends not only on the material nonlinearity and applied electrical field, but also on the size of active material, i.e. the part of the generator’s membranes involved in energy harvesting. Finally, as expected, the implemented ‘triangular’ electrical scheme proved to be more efficient than the conventional constant voltage scheme.

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Figure 8: Average Power for $\beta = 0^\circ$, $\alpha = 7.6$ and different values of $d$ such that in: (a) $d = 0$, (b) $d = r_p$, (c) $d = 2r_p$, (d) $d = 3r_p$, (e) $d = 4r_p$, and (f) $d = 5r_p$. 
Figure 9: Average Power for $\beta = 90^\circ$, $\alpha = 7.6$ and different values of $d$ such that in; (a) $d = 0$, (b) $d = r_b$, (c) $d = 2r_b$, (d) $d = 3r_b$, (e) $d = 4r_b$, and (f) $d = 5r_b$. 

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(a) 

(b) 

(c) 

(d) 

(e) 

(f)
Figure 10: Average Power for $\beta = 0^\circ$, $\alpha = 48.9$ and different values of $d$ such that in; (a) $d = 0$, (b) $d = r_b$, (c) $d = 2r_b$, and (d) $d = 3r_b$. 
Figure 11: Average Power for $\beta = 90^\circ$, $\alpha = 48.9$ and different values of $d$ such that in; (a) $d = 0$, (b) $d = r_b$, (c) $d = 2r_b$, and (d) $d = 3r_b$.
Figure 12: Comparison of different systems and conditions: (a) $d = 0$, (b) $d = r_b$, (c) and (d) $d = 2r_b$, (e) and (f) $d = 4r_b$
Figure 13: Average Power for $\beta = 90^\circ$, $\alpha = 7.6$, $d = 0$ and different values of $n$: (a) $n = 1$, (b) $n = 1.8$, (c) $n = 3$, and (d) $n = 5$

Figure 14: Comparison of two schemes for $\alpha = 7.6$, $\beta = 0^\circ$. (a) $A = 10N$ and $d = 0$, (b) $f_0 = 15Hz$ and $d = 0$, (c) $A = 10N$ and $d = 4r_b$, and (d) $f_0 = 15Hz$ and $d = 4r_b$. 