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Low-sidelobe Pattern Synthesis for Sparse Conformal Arrays Based on PSO-SOCP Optimization

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ABSTRACT This paper addresses the constrained multi-objective optimization problem of sparse conformal arrays designing. The objective of array synthesis is to find an optimal element arrangement on a conformal surface and its associated excitation strategy, which generate the main radiation beam along a pre-selected spatial direction with maximum gain and, simultaneously, suppress sidelobe levels elsewhere. A hybrid algorithm PSO-SOCP, comprising of particle swarm optimization (PSO) and second-order cone programming (SOCP), each for a dedicated purpose, is proposed to fulfill this task in this paper. More specifically, the PSO algorithm is introduced to optimize sparse conformal array element positions, whereas the SOCP is applied to seek optimal excitation coefficients for each array layout obtained. After extensive simulation with the examples of sparse circular array and sparse conical arrays, we can find that our proposed method can synthesize better radiation patterns with regard to peak sidelobe levels, compared with those obtained through other traditional algorithms.

INDEX TERMS Antenna arrays, Antenna radiation patterns, Low sidelobe, Particle swarm optimization, Second-order cone programming.

I. INTRODUCTION

Conformal antenna arrays have received considerable attention in radar, communication, navigation and remote wireless sensing systems [1], [2]. Compared with linear and planar arrays, they exhibit many advantages, including: i) flexibility in attaching to arbitrary surfaces of vehicles and aircrafts while not compromising their aerodynamic performance; and ii) capability of offering greater angular coverage. However, the synthesis of conformal arrays is still challenging because of their inherent properties of nonlinearity and non-convexity associated with the synthesis procedures and the inapplicability of the array pattern multiplication rule, which make most classical pattern synthesis methods for linear and planar arrays, such as the Chebyshev and the Taylor methods [3], unusable for conformal arrays.

The pattern synthesis problems for conformal arrays can be broadly sorted into two categories, one is the conformal arrays attached on regular geometrical structures, and the other is the non-regular conformal arrays. For regular conformal array arrangements, a variety of radiation pattern synthesis approaches have been developed within the past few years [4]-[14]. Iterative numerical methods are commonly used, such as the projection-based methods [4], [5], the iterative least square techniques [6], the adaptive methods [7], and the linear programming approaches [8]. Evolutionary-type optimization algorithms are also feasible,
such as genetic algorithm [9], particle swarm optimization algorithm [10], differential evolution algorithm [11], and their hybrid alterations [12]-[14]. However, we noted that the above mentioned heuristic methods suffer from heavy computational cost and slow convergence speed.

In contrast, the pattern synthesis problems for non-regular conformal arrays have not been addressed efficiently to date. Intuitively, when the arrangements of array elements on conformal surfaces are involved in the pattern optimization process, enhanced array performance can be expected. In general, the resulting non-uniformly spaced arrays are called aperiodic or sparse arrays, which, when compared with their uniform counterparts, could exhibit favorable properties like narrower beamwidth, lower cost, and reduced complexity [15], [16]. Some efforts on exploiting these advantages have been made in [17]-[19]. A geometric synthesis procedure called orthogonal method for a given array excitation was presented in [17]. However, the acceptable synthesis outcomes highly rely on a good choice of the initial array state, which cannot be guaranteed as it is essentially a perturbation method. In [18], a versatile multi-task Bayesian compressive sensing strategy was proposed to design sparse conformal arrays with radiation patterns matching an arbitrary reference pattern, but this type of thinned arrays, which are derived by selectively zeroing some elements of initial equally spaced arrays, inevitably reduce the degrees of freedom that exist in sparse arrays with randomly spaced elements. In [19], the excitation coefficients and positions of conformal array elements were optimized with the least square method in order to suppress side-lobe and cross-polarization levels. It transforms the multi-objective problem into a single-objective problem by introducing weight coefficients associated with multiple objectives. Whereas the selections of weight coefficients, which have significant impacts on obtained optimization results, cannot be readily identified.

Beside the sidelobe levels, another important performance parameter for an antenna array is its directivity. Nevertheless, most of the state-of-the-art works on non-regular conformal arrays designing only focus on the sidelobe level suppression in some specified planes, without considering arbitrary main beam directions and/or optimization of their directivity. In this paper, our objective is to find an optimal sparse conformal array solution, namely, the array element positions and the associated array excitation coefficients, which generate a radiation pattern with maximum gain in a given spatial direction and, simultaneously, reduce the sidelobe levels. To handle this multidimensional nonlinear problem with multiple constraints, the particle swarm optimization algorithm is selected. In order to avoid local optima traps that PSO could fall into, the second-order cone programming [20] is combined with the PSO, resulting in a new hybrid algorithm proposed in this paper.

SOCP is one form of convex optimization methods [20]. An important advantage of convex programing is its ability to guarantee the optimality of the solution to a convex problem if it exists. Therefore it has attracted much interest in utilizing convex optimization methods to solve pattern synthesis problems [21]–[23]. Convex models for different linear and planar arrays were developed for the synthesis of arbitrary beam patterns in [21], [22], and full-wave analysis was also introduced in [22] to take mutual coupling effects into account. However, the establishment of the convex models is only possible when the array excitations are conjugate symmetric, thus, lacking universality. In [23], an iterative SOCP method was proposed to synthesize beam patterns, via a sequence of linear approximations in order to form a convex sub-problem. However, this method, again, needs a good initial guess for the synthesis converging to acceptable solutions.

It should be pointed out that, convex programing can only be used for seeking unknown excitation coefficients because the far-field radiation patterns are linear functions of the array element excitations but not of array element positions. Therefore, the basic idea of our hybrid approach is to exploit the PSO as a global searcher to obtain the possible array layouts. And for each layout, the convex optimization SOCP as a local solver is used to find the optimal excitation coefficients by relaxing some constraints so that a standard second-order cone model is applicable. PSO algorithm is used to optimize the element positions of Y-type and Reuleaux triangle-type correlator antenna arrays in [24], and the coverage in the spatial frequency is increased and sidelobe level in the angular domain is reduced. Multi-objective Particle Swarm Optimization (MOPSO) is applied to optimize the uniform and non-uniform ultrawideband Vivaldi antenna array composed of 10 elements respectively in [25] and obtains optimum tradeoffs between sidelobe level and beamwidth in time domain. PSO-SOCP, which uses particle swarm optimization as global algorithm and second-order cone optimization algorithm as local algorithm, is an improvement on PSO algorithm. It is feasible to optimize the practical antenna array. This paper is an extended study of our previous work in [26], in which PSO-SOCP is applied to design a sparse circular array. Whereas in this paper, a more general application scenario, i.e., sparse conformal arrays, associated with different synthesis considerations is discussed.

Our paper is organized as follows. Section II presents the basic optimization model for sparse conformal arrays, taking the radiation pattern and layout constraints into account. In section III, the second-order cone model for the excitation synthesis of conformal arrays on fixed array arrangements is given and the details of the proposed hybrid optimization algorithm, i.e., PSO-SOCP, are considered.
elaborated. In Section IV, simulation results for some typical array design examples are provided to validate the effectiveness of the proposed algorithm. Finally, conclusions are drawn in Section V.

II. Sparse Conformal Antenna Array Model

The radiation field in the wave propagation direction \( \mathbf{u} \) from a sparse conformal array with \( N \) elements positioned arbitrarily on a given curved surface, as shown in Fig. 1, can be expressed as

\[
E = \sum_{n=1}^{N} \alpha_n E_n \exp(\textbf{j}k' \cdot \textbf{u})
\]

where \( K = 2\pi / \lambda \) is the propagation wave number with the \( \lambda \) being the operation wavelength, and \( \mathbf{u} = [\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta]^T \) is the unit vector in the spherical coordinate system. \( \theta \) and \( \phi \) are labelled in Fig. 1, and \( [\cdot]^T \) stands for transpose operation. For the \( n \)th element, \( r_n = [x_n, y_n, z_n]^T \) is the displacement vector in Cartesian coordinate system, \( \alpha_n \) denotes the complex excitation coefficient, and \( E_n(u) \) represents the electric field intensity along the wave propagation direction \( \mathbf{u} \). Operator \( [\cdot]^* \) conjugates enclosed complex numbers.

In this paper, we aim to find the optimal conformal array element arrangement on an given curved surface and its associated excitation coefficients so that the generated far-field radiation patterns meet the following requirements: i) the maximum directivity along the selected spatial direction; ii) the minimum average power projected in the given sidelobe region; and iii) the spacing between adjacent elements is greater than a given value in order to minimize antenna mutual coupling. The above multi-objective model can be mathematically expressed as

\[
\begin{align*}
\min \quad & P_{av}, P_{\Omega} \\
\text{s.t.} \quad & E(u_n) = 1 \\
& \Delta r_{mn} \geq d_{\text{min}}, 1 \leq m, n \leq N, m \neq n
\end{align*}
\]

where \( E(u_n) \) is the electric field intensity along the desired radiation direction \( u_n = (\theta_n, \phi_n) \), which is normalized to be unity; \( P_{av} \) and \( P_{\Omega} \) represent the average power in the solid angles of \( 4\pi \) and the sidelobe region \( \Omega \), respectively. In order to reduce the coupling effects, another constraint on the minimum element spacing \( d_{\text{min}} \) is introduced in the optimization process. \( \Delta r_{mn} \) is the distance between \( m \)th element and \( n \)th element, then we have

\[
\Delta r_{mn} = \sqrt{(x_m - x_n)^2 + (y_m - y_n)^2 + (z_m - z_n)^2}.
\]

In the matrix forms,

\[
\begin{align*}
P_{av} &= \frac{1}{4\pi} \int_{-\pi}^{\pi} \int_{0}^{2\pi} |E(u)|^2 \sin \theta \phi d\theta d\phi = w^H \Omega w \\
P_{\Omega} &= \frac{1}{\Omega} \int_{\Omega} |E(u)|^2 \sin \theta \phi d\theta d\phi = w^H \Omega w
\end{align*}
\]

where the excitation coefficient vector \( \mathbf{w} \), and array steering vectors \( \mathbf{b}(u) \) are given by

\[
\mathbf{w} = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_N \end{bmatrix}, \quad \mathbf{b}(u) = \begin{bmatrix} E_1 \exp(\textbf{j}kr_1 \cdot \mathbf{u}) \\ E_2 \exp(\textbf{j}kr_2 \cdot \mathbf{u}) \\ \vdots \\ E_N \exp(\textbf{j}kr_N \cdot \mathbf{u}) \end{bmatrix}
\]

and

\[
\begin{align*}
\Omega &= \frac{1}{4\pi} \int_{-\pi}^{\pi} \int_{0}^{2\pi} |\mathbf{b}(u)|^2 \sin \theta \phi d\theta d\phi \\
\Omega &\Delta = \frac{1}{\Omega} \int_{\Omega} |\mathbf{b}(u)|^2 \sin \theta \phi d\theta d\phi
\end{align*}
\]

Therefore, the multi-objective optimization problem in (2) becomes

\[
\begin{align*}
\min \quad & w^H \Omega w, w^H \Delta \Omega w \\
\text{s.t.} \quad & w^H \mathbf{b}(u_n) = 1 \\
& \Delta r_{mn} \geq d_{\text{min}}, 1 \leq m, n \leq N, m \neq n
\end{align*}
\]

III. Proposed PSO-SOCP Algorithm

In this section, a new algorithm combining of PSO and SOCP is described to address the problem stated in (6). Every particle generated in the process of the PSO represents one possible array layout satisfying \( \Delta r_{mn} \geq d_{\text{min}} \). While the SOCP algorithm is applied to seek the optimal excitation coefficients for every particle. With iterations,
the optimal or near-optimal sparse conformal array solutions can be obtained. In Section III-A, the second-order cone model for conformal arrays in any given array layouts is developed first, followed by the combination of the PSO and the SOCP in Section III-B.

A. SOCP in Given Array Layout
Since the terms $P_{av}, P_{\Omega}$ are exponential functions, the problem described in (6) is non-convex. Thus, the SOCP algorithm cannot be applied directly. It can be readily proven that the matrices $Q$ and $Q_b$ are Hermitian. Thus, using the Cholesky decomposition we have:

$$
\begin{align*}
    w'Qw &= w'U^H U w = \|Uw\|^2 \\
    w'Q_b w &= w'U^H \alpha U w = \|U\alpha w\|^2
\end{align*}
$$

(7)

Which, under the condition that the array layout is fixed, becomes a linear function of the complex element excitations $w$.

After the matrix decomposition, the problem in (6) is still non-convex due to its multiple objectives. In practical scenario, we want to suppress the sidelobe to a certain level according to different applications. Thus, a power threshold $\varepsilon$ is assigned to control the sidelobe power. Consequently, the problem in (6) can be formulated as

$$
\begin{align*}
    \min_{w} & \|Uw\|_2 \\
    \text{s.t.} & \quad w' b(u_0) = 1 \\
    & \quad \|U\alpha w\|_2 \leq \varepsilon
\end{align*}
$$

(8)

which is a convex problem [20]. Making $y = (\alpha, w^x, w^y)^T$, where $w_x$ and $w_y$ are the real and imaginary parts of the excitation coefficient vector $w$, the standard second-order cone form of (8) can, then, be expressed as

$$
\begin{align*}
    \min_{h, y} & \quad h' y \\
    \text{s.t.} & \quad \begin{bmatrix} 0 & m & -n \\ 0 & n & m \end{bmatrix} y \leq \sqrt{\varepsilon} \\
    & \quad \begin{bmatrix} 0 & m_x & -n_x \\ 0 & n_x & m_x \end{bmatrix} y + \begin{bmatrix} -1 \\ 0 \end{bmatrix} + \begin{bmatrix} m_y & n_y \end{bmatrix} y = 0
\end{align*}
$$

(9)

where $m, n$ are the real and imaginary parts of $U$, $m_x, n_x$ are the real and imaginary parts of $U_x$, $m_y, n_y$ are the real and imaginary parts of $b'(u_0)$, and $h = (1, 0, 0)^T$.

B. Combining PSO and SOCP
The above second-order cone model is developed based on a fixed array layout and a given threshold $\varepsilon$ for sidelobe power. From (4) and (5) we know that the vector $b(u_0)$ and the matrices $U, U_\alpha$ are the functions of element positions. It is a non-convex problem when element positions need to be optimized. Therefore, the PSO algorithm proposed by Coello et. al [27] is applied in our approach to search optimal array arrangement, resulting in a hybrid algorithm of PSO and SOCP.

In the particle swarm initialization updating stage, all the particles are flying in a search space $L$ which is defined according to the array aperture. In each iteration, their position updates have to satisfy the minimum element spacing constraint in (6), otherwise adjustments of their trajectories are repeated.

For every particle position, when $\varepsilon$ is given, the SOCP in the prior subsection is utilized to get the optimal element excitation coefficients, and then calculate the multi-objective optimization values of $P_{av}$ and $P_{\Omega}$. Based on the concept of Pareto dominance, the PSO algorithm updates the non-dominated vectors through comparison between the new solution vectors in each iteration.

The main procedure of the proposed method is summarized in Fig. 2, comprising of:

1) Set the sparse conformal antenna array configuration and the objectives for the pattern

![FIGURE 2. Procedures of the proposed PSO-SOCP algorithm](image)
synthesis, and initialize the particle swarm in the
given layout constraints;

2) Apply SOCP to get the optimal $w$ with (7)-(9) for
the initialized particle swarm, and then calculate the
fitness values, i.e. $P_{rv}$ and $P_{0v}$. Initialize the
external repository of particles. The external
repository adopts the adaptive grid technique, and is
used to keep a historical record of the non-dominated vectors found along the search
process. Details of the external repository and the
adaptive grid technique can be found in [27];

3) Update the speed and positions of the particles with
the following expressions:

$$v_{id}(t + 1) = av_{id}(t) + c_1 r_1 [p_{id}(t) - x_{id}(t)]$$
$$+ c_2 r_2 [p_{pd} - x_{id}(t)]$$

$$x_{id}(t + 1) = x_{id}(t) + v_{id}(t + 1)$$

where $x_{id}(t + 1)$ and $v_{id}(t + 1)$ are the position and
velocity of $i$th particle on the $d$th dimension in the
$(t + 1)$th iteration, $a$ is the inertia weight, $c_1$ and $c_2$
are acceleration constants, setting as 2 in the
simulation. $p_{id}$ is the best position that $i$th particle
has had and $p_{pd}$ is the best position of all particles.
$r_1$ and $r_2$ are random numbers within the range 0 to 1;

4) Determine whether the new particle positions satisfy
the minimum array spacing constraint. If not, the
positions need to be re-generated;

5) Calculate the fitness value of every particle, and
update the external repository;

6) Repeat from the Step 2) until the maximum number
of iterations is reached;

7) Output the optimal sparse conformal array layout
and its associated excitation coefficients.

IV. Numerical Results

In this section, demonstrative examples are presented in
order to validate the effectiveness of the proposed method.
First, an example of pattern synthesis using a
three-dimensional sparse circular array is provided. The
effects of sparse ratio on array pattern are investigated.
Sparse ratio [28] is a parameter to describe the sparse degree
of sparse arrays,

$$\gamma = \frac{N}{M} \times 100\%$$

where $N$ is the number of the antenna elements in the sparse
array, $M$ is the number of the required antenna element if
they are half-wavelength spaced. Later, conical array pattern
synthesis is presented.

A. Example 1: three-dimensional sparse circular array

Considering a 30-element circular array with a radius of
$R = 5\lambda$, that is placed on the $x$-$y$ plane, as shown in the Fig. 3.

Its far field radiation pattern in $x$-$y$ plane, i.e., only $\theta = 90^\circ$
is considered in the pattern synthesis, is a function of $\phi$, as is
shown in (13),

$$E(\phi) = \sum_{n=1}^{N} \alpha_n E_n \exp[jKR\cos(\phi - \phi_n)] = w^H b$$

(13)

where $\phi_n$ is the azimuth angle of the $n$th element, and the
steering vector is

$$b(n) = [\exp(jKR\cos(\phi - \phi_1)) \cdots \exp(jKR\cos(\phi - \phi_n))]^T$$

(14)

It is assumed that the desired beam pointing direction is
$\phi_0 = 200^\circ$. The minimum spacing of adjacent elements is
$d_{min} = \lambda / 2$, equivalent to the minimum angular separation
$\phi_{min} = 5.73^\circ$.

It needs to be pointed out that our method is inspired by the
algorithm proposed in [31], which used convex optimization
to obtain excitation coefficients. This approach in [31], as
well as other two multi-objective algorithms also used for
reference: PSO in [27] and NSGA-II in [29], are chosen as
benchmarks to prove the superiority of the proposed method
in this paper.

Each algorithm was run for 10 independent trials, with a
population of 300 individuals and a maximum of 200
iterations in each trial. The parameters used in the
TABLE I

PERFORMANCE STATISTICS OF DIFFERENT ALGORITHMS IN TERMS OF PSLL AND TIME TO CONVERGENCE PER TRIAL

<table>
<thead>
<tr>
<th>Method</th>
<th>Lowest PSLL (dB)</th>
<th>Highest PSLL (dB)</th>
<th>Average PSLL (dB)</th>
<th>PSLL SD</th>
<th>Average t (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PSO-SOCP</td>
<td>-21.20</td>
<td>-20.08</td>
<td>-20.68</td>
<td>0.10</td>
<td>77.67</td>
</tr>
<tr>
<td>Method in [31]</td>
<td>-16.75</td>
<td>-12.995</td>
<td>-15.10</td>
<td>1.89</td>
<td>40.49</td>
</tr>
<tr>
<td>PSO</td>
<td>-10.06</td>
<td>-6.14</td>
<td>-7.68</td>
<td>1.41</td>
<td>40.39</td>
</tr>
<tr>
<td>NSGAII</td>
<td>-12.13</td>
<td>-9.04</td>
<td>-10.12</td>
<td>0.91</td>
<td>63.86</td>
</tr>
</tbody>
</table>

FIGURE 4. PSLL with iterations for the proposed algorithm.

simulations are set as follows: in the PSO a repository size of 100 particles is used, \( c_1 = c_2 = 2 \) with an inertia weight \( a = 0.73 \), and 20 divisions for the adaptive grid; the NSGA-II was programmed with a crossover rate of 0.8 (uniform crossover), tournament selection, and a mutation rate of 0.2.

The statistical results of the performance of these algorithms are presented in Table I. We can observe that, our proposed method outperforms the others, in terms of the lowest, highest, average, and the standard deviation (SD) of the peak sidelobe levels (PSLLs). It is because the evolutionary algorithms like PSO or NSGA-II can be easily trapped in a local optimum when dealing with this type of high-dimensional optimization problem. And the method in [31] achieves some improvements because it simply optimizes the result of PSO once and has a limited suppression of PSLL, but still underperforms the proposed hybrid method. The convergence time for each algorithms is also listed in Table I. We can see that the proposed method requires the longest calculation time for each iteration. However, within a given time period, for example around 40 minutes, the PSO-SOCP method is able to synthesis a sparse array with far better radiation properties.

FIGURE 5. Comparison of best radiation patterns in a 30-elements sparse circular array obtained by different algorithms.

Fig. 4 shows the convergence properties of our proposed algorithm for the mean, the highest and the lowest PSLLs as a function of the iteration number. The small gap between the best and the worst convergence curves, which is just over 2dB, indicates the stability of the proposed algorithm, consistent to the small SD shown in Table I. At the beginning of the iterations, the PSLL for the best member is the same as the worst member, while the mean of the population is about 1dB higher than that for them. After 105
TABLE II

POSITIONS AND THE ASSOCIATED EXCITATION COEFFICIENTS OF CIRCULAR ARRAY ELEMENTS

<table>
<thead>
<tr>
<th>Index</th>
<th>Angel (°)</th>
<th>Excitation</th>
<th>Amplitude</th>
<th>Phase (°)</th>
<th>Index</th>
<th>Angel (°)</th>
<th>Excitation</th>
<th>Amplitude</th>
<th>Phase (°)</th>
<th>Index</th>
<th>Angel (°)</th>
<th>Excitation</th>
<th>Amplitude</th>
<th>Phase (°)</th>
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<td>0.028</td>
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<td>156.05</td>
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<td>17</td>
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<td>136.26</td>
<td>0.391</td>
<td>22.57</td>
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<td>0.223</td>
<td>60.86</td>
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<td>0.987</td>
<td>72.97</td>
<td>30</td>
<td>354.00</td>
<td>0.003</td>
<td>118.61</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

Fig. 6. Circular array elements’ distribution obtained by the proposed algorithm at $\lambda = 0.06 \text{m}$.

iterations, the best member attains a PSLL of about -21.18dB, the corresponding PSLLs for the worst and the average are -20dB, -20.5dB, respectively. This is also the exact number of iterations when the proposed algorithm can achieve the basic stability. It can be inferred that it took a while for the worst member to escape from the local optimum, thus their end-ups exist a gap even though the start points of the best and the worst member are the same.

Fig. 5 illustrates the far-field radiation patterns of the synthesized arrays obtained with different approaches. The synthesized circular array with our proposed method, including its element positions and the associated excitation coefficients, is provided in Table II. Note that in order to improve searching efficiency, we fix positions of the first and last elements thus giving a limited aperture in our simulation, a common trick which can be seen in [30]−[32]. In theory, it is preferred that these two fixed element radiators should be terminated as the radiation direction is 200°, which is also evidenced with the synthesized results in Table II, namely small amplitudes for antenna 1 and 30. An intuitionistic view of the rest 28 elements’ distribution is given in the Fig. 6.

Then the effects of sparse ratio on synthesized pattern are investigated. The same array configuration is used while the array number is varied from 10 to 62. The equivalent number of $M$ in this circular array is 62, and the sparse ratio is 48.4% in the above discussion when $N$ is selected to be 30.

Table III lists achieved PSLL of the circular array for different synthesis methods described, with another clear illustration given in the Fig. 7. From Fig. 7, the performance superiority of our method is well established no matter what sparse ratio is set. As the sparse ratio increases, the PSLL through our method reduces rapidly until N reaches 30. The trend for other algorithms are similar, expect with higher
Fig. 7 shows the radiation pattern at different sparse ratios synthesized using the proposed method. First, let’s take a look at the 3dB bandwidth. It decreases slightly as the sparse ratio goes up until $N$ reaches around 30, when $\lambda$ goes over about 50%, the mainlobe beamwidth of the radiation patterns seems to reach a limit. This is because that the mainlobe

PSLL floors. Also, it is worth pointing out that when sparse ratio is 100%, there is no need for optimizing element layout, and our method and the method in [31] would turn into a simple SOCP, hence resulting in the same result -21.53dB, much better than the results that the evolutionary optimization algorithms can achieve.

**TABLE III**

<table>
<thead>
<tr>
<th>N</th>
<th>$\gamma$</th>
<th>MOPSO</th>
<th>Method in [31]</th>
<th>MOPSO</th>
<th>NSGAII</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>16.13%</td>
<td>-12.40</td>
<td>-7.60</td>
<td>-6.18</td>
<td>-6.18</td>
</tr>
<tr>
<td>20</td>
<td>32.26%</td>
<td>-18.29</td>
<td>-13.69</td>
<td>-8.94</td>
<td>-9.38</td>
</tr>
<tr>
<td>40</td>
<td>64.52%</td>
<td>-21.65</td>
<td>-18.95</td>
<td>-10.20</td>
<td>-12.17</td>
</tr>
<tr>
<td>50</td>
<td>80.64%</td>
<td>-22.12</td>
<td>-17.65</td>
<td>-9.25</td>
<td>-12.01</td>
</tr>
<tr>
<td>62</td>
<td>100.00%</td>
<td>-21.53</td>
<td>-21.53</td>
<td>-12.73</td>
<td>-11.00</td>
</tr>
</tbody>
</table>

Fig. 8 shows the radiation pattern at different sparse ratios synthesized using the proposed method.
achievable bandwidth greatly depends on the physical aperture of array, which in this case is fixed to the circular diameter.

B. Example 2: Sparse conical conformal array

In this example, our method is applied to a sparse regular conical array, see Fig. 9, consisting of \( J \) stacked arc subarrays. The radiating elements are arranged on the conical surface, where \( k = 0 \) represents the arc subarray at the bottom, whereas \( k = J - 1 \) represents the apex.

Assuming the desired sparse conical array is composed of 4 arc subarrays, i.e. \( J = 4 \), each of which has 10, 9, 8 and 7 elements, respectively. The radius of the cone is \( r = 0.5m \), and the half conical angle is \( \delta = 45^\circ \). The spacing between adjacent arcs along the conical surface is \( \lambda \), where \( \lambda \) is the free space wavelength at 20 GHz. Then the radius of the \( k \)th stacked arc subarray and its height in z-direction are given as

\[
R_k = r - \lambda \cdot \sin(\delta) \times k
\]
\[
h_k = \lambda \cdot \cos(\delta) \times k
\]

In the case of \( \lambda / 2 \) uniform spacing, the equivalent antenna number on this conical surface is 213, which means the sparse ratio of the synthesized example is \( (10+9+8+7)/213=15.96\% \). Fig. 10 shows distribution of the sparse conical array with uniform spacing.

In this example, the active element patterns of all antenna radiators are assumed to be identical, following

\[
f(\phi, \theta) = \cos[(\pi / 2) \sin \theta]
\]

the same as used in [31]. It is
TABLE IV

<table>
<thead>
<tr>
<th>Index</th>
<th>Position</th>
<th>Excitation</th>
<th>Index</th>
<th>Position</th>
<th>Excitation</th>
<th>Index</th>
<th>Position</th>
<th>Excitation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\alpha_n$ ($^\circ$)</td>
<td>$h(\lambda)$</td>
<td>Amplitude</td>
<td>Phase ($^\circ$)</td>
<td></td>
<td></td>
<td>$\alpha_n$ ($^\circ$)</td>
<td>$h(\lambda)$</td>
</tr>
<tr>
<td>1</td>
<td>0.00</td>
<td>0</td>
<td>0.088</td>
<td>31.43</td>
<td>13</td>
<td>89.17</td>
<td>0.71</td>
<td>0.730</td>
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<tr>
<td>2</td>
<td>74.29</td>
<td>0</td>
<td>0.570</td>
<td>-137.56</td>
<td>14</td>
<td>100.08</td>
<td>0.71</td>
<td>0.960</td>
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<tr>
<td>3</td>
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<td>0</td>
<td>0.861</td>
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<td>15</td>
<td>120.13</td>
<td>0.71</td>
<td>0.547</td>
</tr>
<tr>
<td>4</td>
<td>100.35</td>
<td>0</td>
<td>1</td>
<td>123.68</td>
<td>16</td>
<td>131.83</td>
<td>0.71</td>
<td>0.412</td>
</tr>
<tr>
<td>5</td>
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<td>0</td>
<td>0.031</td>
<td>58.15</td>
<td>17</td>
<td>185.39</td>
<td>0.71</td>
<td>0.142</td>
</tr>
<tr>
<td>6</td>
<td>120.17</td>
<td>0</td>
<td>0.655</td>
<td>87.44</td>
<td>18</td>
<td>294.69</td>
<td>0.71</td>
<td>0.004</td>
</tr>
<tr>
<td>7</td>
<td>131.82</td>
<td>0</td>
<td>0.353</td>
<td>7.05</td>
<td>19</td>
<td>349.09</td>
<td>0.71</td>
<td>0.053</td>
</tr>
<tr>
<td>8</td>
<td>241.94</td>
<td>0</td>
<td>0.002</td>
<td>123.44</td>
<td>20</td>
<td>0.00</td>
<td>1.41</td>
<td>0.079</td>
</tr>
<tr>
<td>9</td>
<td>281.52</td>
<td>0</td>
<td>0.001</td>
<td>-132.00</td>
<td>21</td>
<td>64.89</td>
<td>1.41</td>
<td>0.521</td>
</tr>
<tr>
<td>10</td>
<td>351.22</td>
<td>0</td>
<td>0.064</td>
<td>-122.33</td>
<td>22</td>
<td>90.64</td>
<td>1.41</td>
<td>0.691</td>
</tr>
<tr>
<td>11</td>
<td>0.00</td>
<td>0.71</td>
<td>0.085</td>
<td>-118.90</td>
<td>23</td>
<td>105.64</td>
<td>1.41</td>
<td>0.963</td>
</tr>
<tr>
<td>12</td>
<td>68.91</td>
<td>0.71</td>
<td>0.545</td>
<td>-149.09</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

FIGURE 12. Top view of conical array optimized by MOPSO-SOCP.

assumed that the desired main beam pointing direction is $(\phi_b = 100^\circ, \theta_b = 45^\circ)$. The minimum spacing of adjacent elements in the same arc is $d_{\text{min}} = \lambda / 2$, which corresponds to the minimum angular separation $k_{\text{min}} \phi = d_{\text{min}} / R_k$, where $k$ refer to the $k$th arc subarray. Each array element has the same type and the same pattern function in the element coordinate system, which can be expressed as $f_\phi(\phi, \theta) = \cos[(\pi / 2) \sin \theta]$. Because of the different position and direction of each element in the array, the pattern of each element in the array coordinate system is different. Assuming that the pattern function of the $n$th array element in the array coordinate system is $f_n(\phi, \theta)$, where $\phi$ and $\theta$ are the azimuth and pitch angles respectively in the array coordinate system. The element direction of each element is perpendicular to the cone of the location of the element and is determined by the position angle $\alpha_n$ in Table IV and the half cone angle. Using the coordinate transformation relation in [33], we can get the pattern function of the $n$th element in the array coordinate system:

$$f_n(\phi, \theta) = \cos \left[ \frac{\pi}{2} \sqrt{-\sin \theta \sin \sin(\phi_\theta + \phi) - \cos \theta \cos \theta_\theta} \right]$$

where $(\phi_\theta, \theta_\theta)$ is the direction of the $n$th element.
In Fig. 11 the synthesis results for our proposed method and the three benchmark algorithms are compared. Again, we can see that the proposed method outperforms the other three algorithms, featuring the lowest PSLL of the co-polarization pattern, about -24.49dB. One may notice that the PSO results in unacceptable radiation pattern with PSLL of only -4.95dB. Even after introducing convex model to optimize elements’ excitation coefficients, as adopted in [31], the PSLL only improved by less than 6dB, suggesting the limitation of method in [31].

The three-dimensional spatial conical array can be seen as a stack of two-dimensional circular arrays with different radius in the Z-axis direction (vertical), so the simulation results of the circular array can be extended to the conical array. PSO-SOCP optimization algorithm for conical array optimization can also effectively suppress peak sidelobe level, obtain an ideal radiation pattern, and has good convergence properties.

Table IV holds the amplitudes and phases of the best obtained pattern found by PSO-SOCP. Fig. 12 depicts a more intuitive two-dimensional illustration of the array layout, which has a big difference compared with Fig. 10. Sparse array antennas are widely used, especially in satellites. In such circumstances, the beam is pointing to a certain direction. The asymmetrical properties shown in Fig. 11 does not affect the realization of specific functions of the related antenna array.

V. Conclusion

A novel hybrid algorithm PSO-SOCP is proposed for the purpose of sidelobe suppression in sparse conformal arrays. The basic idea of our approach is to take the PSO as a global searcher to optimize the array layout. For each obtained layout, convex optimization as a local solver is applied to generate the optimal excitation coefficients by relaxing some constraints so that a standard second-order cone model, which overcomes the possibility of being trapped into the local optimal solution, can be applied.

As examples, we have utilized PSO-SOCP to design sparse circular and conformal arrays. In addition, we investigated the effects of sparse ratio on synthesized patterns. Numerical results showed that the proposed method is an effective tool, which can synthesize sparse conformal arrays with better pattern characteristics compared with other normal evolutionary algorithms. Further efforts will be devoted to introducing polarization into our model. The suppression of cross-polarization level is another important aspect in the pattern synthesis of conformal arrays, since the radiating elements are directed in different directions in the conformal arrays, and the cross-polarization level caused by the surface curvature can be high even though all the elements are linearly polarized.

REFERENCES

[11] Guo, Jing Li, and J. Y. Li, “Pattern synthesis of conformal array antenna in the presence of platform...


[29] Kesong Chen, Zishu He, and Chunlin Han, “A modified real GA for the sparse linear array synthesis with


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