Effects of following and opposing vertical current shear on nonlinear wave interactions

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\textbf{A R T I C L E   I N F O}

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\textbf{A B S T R A C T}

A Navier-Stokes solver in OpenFOAM\textsuperscript{®} is combined with the Volume of Fluid (VOF) surface capturing method to investigate the wave interaction with depth-varying currents in intermediate and shallow waters. A special attention is paid to the separate effect of vertical current shear on near resonant triad wave interactions. It was found that in the presence of following vertical current shear, the wave exhibits a sharper crest and flatter trough, and the opposite is true in the presence of opposing vertical current shear. Our model results indicate that the wave steepness at which the current shear starts to affect the crest elevation is greater in deeper water than in shallower water. We found that adding vertical current shear to the uniform current further enhances the relative harmonic wave energy and the extent of triad interaction in the following current while weakens them in the opposing current. As a result, following and opposing current shear may cause wave to break at a lower and higher sea state respectively. Due to the increased wave nonlinearity in the presence of a following current shear, a linear superposition of the individual wave and current velocities is no longer adequate to represent the total horizontal velocity close to the free surface.

1. Introduction

Waves and currents coexist in the majority of marine environments, especially in the nearshore, estuarine and coastal regions. Wave-current interactions are known to contribute to the formation of extreme waves on inhomogeneous current with disastrous effect [1]. Extreme waves may be triggered and break when a stable wave packet encounters an opposing current [2]. Weakly nonlinear theory indicates that the linear vertical current shear may mediate resonant triad interactions, therefore contributing to directional spreading of the short wind-waves [3]. Wave interaction with current plays a dominant role in the hydrodynamic and morphological processes of river flow encountering the ocean. Better understanding these processes is of theoretical and practical significance for a variety of applications, such as turbulence and mixing, transport of nutrient, pollutant and sediment, morphodynamics, water quality, coastal ecosystem of beach, inlet and estuary, and interaction of wave and current with coastal and offshore structures.

The coupling between wave and current takes place through wave radiation stress (e.g. [4,5]), bottom stress (e.g [6,7]), surface stress [8,9] or vortex force [10,11]. The 2D depth-averaged wave radiation stress was proposed initially by Longuet-Higgins and Stewart [4]. Recently depth-dependent wave radiation stress formula was established for both deep and shallow water (e.g [5,12]). Wave and current interactions have been studied theoretically and experimentally for decades. Previous studies of this problem have been summarized by a number of review articles [13–15]. As a wave encounters a current that is uniform across the water column, the fifth-order Stokes wave theory is able to capture much of the current modulations of wave dispersion and kinematics [16]. When the current profile is not uniform, however, the current effect on wave motion becomes more complicated [17–19]. The studies by Kirby and Chen [20], Thomas [21], Swan and James [22], Swan et al. [23], Liu et al. [24] and Ellingsen and Li [25] focused on the effect of vertical current shear or mean flow vorticity on wave dispersion and wave kinematics. The individual effect of surface current and current shear on nonlinear wave profiles, however, is not well understood. As a rare exception, Nwogu [26] investigated the modulational instability of deep-water waves in an exponentially sheared current, and found that for a given surface current, the mean flow vorticity due to current shear increases the modulational instability in following currents. Francis and Khairf [27] investigated two-dimensional linear stability of finite amplitude steady waves on a linear shear current. It was found that increasing current shear may restabilize the modulational instability and that new instability bands were identified.

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Dong et al.’s [28] experimental study indicates that the opposing and following current affect the amplitude of the bound long wave over a mild slope in a different fashion. Lin et al. [29] found that wave reflection by a submerged flat plate increases in the following current and decreases in the opposing current.

The vertical shear of a current profile may be induced by the friction at the sea bottom or wind stress at the free surface. Much of the earlier experiments, e.g., [30–32], were designed to investigate how the wave–current interaction affects the bottom turbulent boundary layers and therefore the bottom shear stress, which in turn dictates sediment transport and coastal erosion. The wave-current interaction may manifest as either a current modulation of wave motion or a wave-induced change in the current or both. The laboratory experiments [31–33] show that the near-surface velocity of an otherwise uniform current is reduced by a following wave, but is enhanced by an opposing wave. Through a boundary-layer analysis, Huang and Mei [34] showed analytically that this phenomenon was largely due to the distortion of eddy viscosity and the non-zero mean shear stress in the opposite direction to the wave at the free surface. On the other hand, the mild-slope equation results for waves over a linearly sheared current by Touboul et al. [35] suggest that neglecting the wave effect on current profiles has little effect on the wave dynamics.

Wind-generated water waves are often accompanied by wind-driven currents with a vertical shear that decays rapidly with depth. Previous studies found that wind-driven currents play an important role in the evolution of extreme waves [36–39]. Zhang [40] investigated the short wind wave propagation on surface wind drift current with a vertical shear. Recently, Zou and Chen [41] numerically investigated the following and opposing wind and current effect on the evolution of a plunging breaking wave. Their model results compare well with the observed wave height evolution with and without wind by Tian and Choi [39]. It was found that the following and opposing winds shift the wave focus point downstream and upstream mainly due to the action of wind-driven current instead of direct wind forcing. Zou and Chen [41] also found that the vertical current shear plays an important role in wave propagation and transformation.

It has been a common practice to include a uniform current in numerical simulations in order to capture the observed downstream shift of the focus point of a wave train, where the extreme wave height occurs. However, the wind-driven currents are by no means uniform across the water depth. The presence of current shear is expected to play a key role in modulating the water surface elevation. Banner and Tian [42] and Banner and Song [43] used a fully nonlinear boundary integral method to examine the onset of wave breaking under the action of a linear vertical shear current. Using similar method, Moreira and Chacaltana [19] found that the presence of combined horizontal and vertical current shear may lead to more prominent wave breaking at blocking points. The individual effects of current shear strength and surface current velocity, however, have not been examined systematically. Quinn et al. [44] showed that neglecting the vertical current shear in spectral wave modelling might lead to significant errors in wave amplitude predictions.

When a wave encounters a linear vertical current shear, the wave motion remains irrotational. Linear wave theory for this problem was proposed by Thompson [45]. Weakly nonlinear analytical solutions based on irrotational wave theory [46,47] and fully nonlinear numerical solutions [48,49] show that the linear current shear alters the wave surface profile. Jonsson et al. [50] argued that the depth-averaged current and constant vorticity associated with the linear shear play an important role in the wave amplitude evolution and set-down. When a wave encounters a current with an arbitrary vertical profile, however, the wave field is no longer irrotational so that the irrotational theory is no longer valid. Numerical models have been proposed by Dalrymple [51], Chaplin [52], Thomas [21], and Swan and James [22] for this problem instead. For example, Dalrymple [51] divided the water column into two layers. The current for each layer is approximated by a linear shear profile. A Fourier series based wave theory was used to predict the wave profile, which can be accurate to a given order through a numerical perturbation procedure.

Much of previous experimental studies of wave-current interaction considered the uniform current only [30–33,53,54]. Swan et al. [23] examined a 2D wave propagating over a depth-varying current with a non-uniform vorticity distribution. For a following vertically sheared current, such as that induced by the wind at the free surface, it was observed that the water particle velocity beneath a wave crest was substantially larger than that predicted by the irrotational wave theory such as that of Fenton [16]. However, Swan et al. [23] found a good agreement between their experimental results and the inviscid numerical model adapted from Dalrymple [51].

Chen et al. [55] applied an enhanced Boussinesq wave model to study the nonlinear interaction of a shallow water wave with a uniform current. Their model results indicate that the energy ratio between the higher-harmonic and primary wave is weakened by an opposing current but enhanced by a following current. In the presence of strong currents and large bathymetry variation, the set-down of mean water elevation is increased due to the Bernoulli effect. Zou et al. [56] incorporated this set-down into their formulation of Boussinesq-type equations including the dispersion relation. Son and Lynett [57] derived a set of Boussinesq equations for wave interaction with a weakly vertically sheared current. The turbulence induced by the wave-current interaction was parameterized by a Reynolds turbulent stress term. A dimensionless parameter was introduced to specify the vertical distribution of the Reynolds stress shear based on the experimental data of [31,32].

Recently, Navier-Stokes solvers have been applied to examine the complex free surface displacement associated with the wave-current interaction ([58–68]). Mayer et al. [66] used a height function to track the free surface, and solved the mass and momentum equations on a time-varying curvilinear grid. Olabarrieta et al. [67] incorporated the displacement of free surface by a vertical coordinate transformation and solved the governing equations on the transformed coordinate that follows the free surface. Teles et al. [68] used the Arbitrary Lagrangian Eulerian (ALE) method with the mesh following the moving free surface.

The Volume of Fluid (VOF) and Level Set method have been recently developed to capture the free surface and solves the Reynolds-Averaged Navier-Stokes (RANS) equations [69–73]. The RANS-VOF model has gained popularity and widespread applications in free surface flow problems including wave-structure interactions and wave transformation over beaches [74–77] and evolution of wave asymmetry over a low-crested structure [78]. Previous wave-current interaction studies using RANS-VOF models, however, have mainly focused on uniform currents [64,65]. It is difficult to generate a wave and a current with desired vertical profile at the same time in a physical wave flume. It is therefore more feasible to use numerical models to study the wave interaction with currents of arbitrary vertical profile and magnitude. Markus et al. [62,63] developed a procedure to implement RANS-VOF model to simulate nonlinear waves interacting with a depth-varying current. They found that the wave-current interaction significantly increased the local particle accelerations in the lower water column. Chen et al. [73] applied a novel Lagrangian model to iteratively reconstruct surface elevation and kinematics of a focusing wave group on sheared currents, which are then fed into a truncated numerical flume to study the loading of the combined wave-current flow on a surface piercing vertical cylinder.

The main objective of this paper is to investigate the effect of vertical current shear on nonlinear wave profiles and wave kinematics, with a special attention to the near-resonant triad wave-wave interaction and opposing current shear. The effects of the current shear and surface current will be examined individually. Following the introduction in Section 1, the methodology is described in Section 2. In Section 3, the numerical model is validated against the analytical solutions and the measurements for a linearly sheared current. In Section
4, the effect of current and current shear on harmonic generation and the nonlinear energy exchange among wave components is evaluated by examining the spatial evolution of harmonics strength, wave skewness and asymmetry. The predicted wave profile and crest kinematics by the present RANS-VOF model are also compared with our calculated results based on the multi-layered solutions by Dalrymple [51], in which the wave form is assumed to remain constant with time and thus time evolution of higher harmonics is not captured. Conclusions are drawn in Section 5.

2. Methodology

2.1. RANS-VOF model

The RANS-VOF model solves the Reynolds-Averaged Navier-Stokes (RANS) equation for the mean flow field, while the fluctuating turbulent components are represented by a turbulent closure model. As an open source CFD toolbox, OpenFOAM® (Open Field Operation and Manipulation) has a standard two-phase flow solver, “interFoam”, which solves the RANS equations for two incompressible phases using a finite volume discretization and the Volume of Fluid (VOF) surface capturing method [79–81].

The fluid motion is assumed to be governed by the Navier-Stokes equations for an incompressible fluid. The mass continuity and momentum equations are given by

\[ \nabla \cdot \mathbf{U} = 0 \]

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{U} \mathbf{U}) - \nabla p^* - \mathbf{g} \nabla \mathbf{z} + \nabla \cdot \left( \mu \nabla \mathbf{U} \right) = 0 \]

where \( \mathbf{U} \) is velocity vector, \( \rho \) density, \( p^* \) pseudo-dynamic pressure, \( \mathbf{g} \) acceleration due to gravity, \( \mathbf{z} \) position vector, and \( \mu \) effective dynamic viscosity, which takes into account of the molecular dynamic viscosity \( \mu \) and the turbulent eddy viscosity \( \omega \).

The two immiscible fluids of air and water are considered as one effective fluid and solved simultaneously throughout the domain, where the volume fraction of water in a cell, \( \alpha \), the VOF function, serves as an indicator function to mark the location of the air-water interface. The VOF function \( \alpha = 1 \) if the cell is full of water, \( \alpha = 0 \) if the cell is full of air, and \( 0 < \alpha < 1 \) if the cell is a mixture of the two fluids. The local density \( \rho \) and the local viscosity \( \mu \) of the fluid in each cell are given by

\[ \rho = \rho_0 (1 - \alpha) \rho_0 \]

\[ \mu = \mu_0 (1 - \alpha) \mu_0 \]

where the subscripts 1 and 2 denote water and air, respectively.

The VOF function is tracked by the advection equation [81]

\[ \frac{\partial \alpha}{\partial t} + \nabla \cdot (\mathbf{U} \alpha) - \nabla \cdot (\alpha \mathbf{U} (1 - \alpha)) = 0 \]

where an extra compression term, \( \alpha \), is added to the classic VOF transport equation to limit the smearing of the interface [69]. This artificial convective term is active only in the thin interface region because the multiplication term \( \alpha (1 - \alpha) \) vanishes when \( \alpha = 1 \) (water side) and \( \alpha = 0 \) (air side). \( \mathbf{U} \) is a velocity field introduced to compress the interface, acting only in the direction perpendicular to the interface. More details about the VOF method can be found in Rusche [80] and Berberović et al. [82].

The present study considers only non-breaking regular waves and neglects the effect of turbulence. Without turbulence closure modeling, Eqs. (1)–(5) are solved using the Pressure Implicit with Splitting of Operators (PISO) algorithm. Details of the numerical algorithm can be found in Jasak [79]. An extended version from the OpenFOAM’s standard “interFoam” solver, the waves2Foam package [83], is employed to investigate the wave-current interaction in this study. This package includes wave generation and absorption using the relaxation zone technique. The numerical schemes and solver settings in general follow those of the “Breaking of a dam” tutorial in OpenFOAM’s user manual.

2.2. Model setup

A numerical wave-current flume was developed to investigate the transformation of nonlinear regular waves over a steady current field. Fig. 1 illustrates the computational domain and set-up of the numerical flume. A steady current field of an arbitrary profile \( \mathbf{U}_s(z) \) is introduced into the flume before the wave is generated. A current velocity is assigned to each cell occupied by the water in the whole computational domain according to the vertical coordinate \( z \) to initialize the simulation. The wavemaker is then activated by specifying the additional velocity due to wave \( \mathbf{U}_w(z, t) \) at the inlet boundary. The total velocity specified at the inlet is \( \mathbf{U} = \mathbf{U}_w(z, t) + \mathbf{U}_s(z) \), while the velocity specified at the outlet is \( \mathbf{U} = \mathbf{U}_s(z) \). At the beginning of the wave generation at the inlet, a ramping function \( \sin(\pi t / T_{\text{ramp}}) \), where \( t \) is the current time and \( T_{\text{ramp}} \) is the ramping time interval, was applied to the wave velocities and the surface elevations so that a series of waves are slowly built up and propagate into the existing steady current field.

A uniform grid size is adopted for the model runs in this paper. Our modeling results indicate that to achieve accurate wave predictions, an adequate grid resolution requires 100–300 grids per wavelength in the horizontal direction and 10–20 grids per wave height in the vertical direction. The time step is dynamically adjusted at runtime to maintain a maximum Courant number of 0.3.

2.2.1. Waves at the inlet boundary

Analytical solution exists for a linear wave interacting with a uniform current with speed \( U_s \) [84], for which the dispersion relation is

\[ \omega - k U_s = g k \tanh kd \]

where \( \omega \) is the apparent wave circular frequency, \( d \) the water depth, and \( k \) the wave number. For a linear shear current, \( U_s(z) = U_s - \Omega_s z \), where \( U_s \) is surface velocity and \( \Omega_s = U_s / d \) is the constant vorticity across the water depth, the dispersion relation becomes [26]

\[ \omega - k U_s = (g - \Omega_s \omega - k U_s) \tanh kd \]

At fifth-order, the dispersion relation in the presence of a uniform current \( U_s \) is given by

\[ \omega - k U_s = g [C_0 + (0.5 H^2) C_2 + (0.5 H^2)^2 C_4] \]

where \( H \) is the wave height, \( g \) is the gravitational acceleration, \( C_0 = \sqrt{\tanh kd} \), \( C_2 = C_0 (2 + 75\sqrt{4(1 - S)^2}) \), and \( C_4 = C_0 (4 + 325 - 116S^2 - 400S^3 - 71S^4 + 146S^5)/[32(1 - S)^3] \)

with \( S = \sec h(2kd) \). In the deep water limit \( kd \to \infty \), \( C_0 = 1/2 \) and \( C_4 = 1/8 \) (see Tables 1 and 2 in [16]).

The analytical solutions, however, are not readily available for depth-dependent currents. Therefore, in this study, a superposition of the current and wave velocities without current effect is used to specify the water particle velocity at the inlet, while the full wave-current interaction comes into effect and reaches an (quasi-) equilibrium state further downstream. Unless noted otherwise, the wave induced water
particle velocities were calculated by the fifth-order Stokes theory [16].

2.2.2. Current profiles at the inlet boundary

With time variation in water depth due to surface displacement in the presence of waves, the current profile may be vertically stretched or compressed. In the spirit of the Wheeler stretching method, which is used for predicting wave kinematics above \( z = 0 \) for linear waves, the stretching technique is applied to the current profile in still water [86]. The basic idea is to introduce a stretched vertical coordinate, \( z_s \), such that the current velocity \( U_s(z) \) at depth \( z \) in the still water is specified at the stretched coordinate \( z_s \) depending on the instantaneous water depth. The linear coordinate stretching technique by Det Norske Veritas [87] is adopted in this study. The stretched vertical coordinate is defined as

\[
z_s = \frac{d(z + d)}{d + \zeta} - d = \frac{z - \zeta}{d + \zeta}
\]

where \(-d < z_s < \eta \) and \(-d < z_s < 0\). The current velocity at the free surface remains constant with time as desired using this technique, so is the current velocity at the other part of the water column (see Fig. 2).

2.2.3. Wave absorption

To avoid wave reflection at the inlet and outlet, a relaxation zone was implemented at both the inlet and outlet boundary (see Fig. 1). The velocity and VOF function inside these zones are relaxed towards the target field according to

\[
U = \gamma U_{\text{computed}} + (1 - \gamma)U_{\text{target}}
\]

\[
\alpha = \gamma \alpha_{\text{computed}} + (1 - \gamma)\alpha_{\text{target}}
\]

where the subscript “computed” indicates the variables computed by the CFD model, the subscript “target” indicates the target values specified for the relaxation zone, and \( \gamma \) denotes a relaxation parameter which is a function of local coordinate \( \sigma \) in the relaxation zone and varies from \( \gamma(\sigma = 0) = 1 \) to \( \gamma(\sigma = 1) = 0 \) across the relaxation zone [83]. The relaxation weight \( \gamma(\sigma) = 1 - [\exp(\sigma^p) - 1]/[\exp(1) - 1] \) with a default exponent of \( p = 3.5 \) is used in this study.

The target field inside the inlet relaxation zone is the superimposed wave and current field. The relaxation zone provides a transition from the wave field without current to that with current. Increasing the length of this zone would make the wave-current interaction occur further downstream in the flume. The target field for the relaxation zone at the outlet is the current field only. Inside this zone, the total velocities are relaxed so that the total water particle velocity approaches the target current velocity while the wave velocities are attenuated to zero at the end of the zone. The length of relaxation zone at the inlet is minimal (0.1 m), while that of the relaxation zone at the outlet is about twice the wavelength in order to minimize wave reflections. With this model setup, it is demonstrated in the Appendix that the target current profile is well maintained throughout the numerical flume.

2.3. Solution for multi-layer piecewise-linear current

Focusing on the equilibrium conditions arising from the interaction of waves with a depth-varying current, Dalrymple [51] proposed a two-layer bi-linear solution to represent the steady wave formed in the presence of a piecewise-linear current. The model’s accuracy can be carried to any order (default to 10 in this study) through a numerical perturbation technique. Swan et al. [23] extended this solution to a five-layer solution in order to adequately represent a strongly sheared current near the free surface. Readers are referred to these papers for detailed model descriptions. In Section 4.3, we will compare the predictions by the RANS-VOF CFD model described in Section 2.1 and the multi-layered solution to illustrate the current shear effect on the nonlinear wave interactions.

2.4. Wave shape parameters

As suggested by Chen et al. [55], the presence of ambient current contributes to the higher harmonics generation and the energy exchange between wave components. To measure the contributions of uniform and sheared current, the ratio of energy in higher harmonics to that in the primary wave is defined as

\[
\eta = \sum_{n=2} \eta_n^2 / \eta_1^2
\]

where \( \eta_n \) is the instantaneous surface elevation, \( \eta_1 \) is the mean surface elevation, and \( \eta_2, \eta_3, \eta_4 \) are, respectively, the wave amplitude, circular frequency, and phase angle of the \( n \)th wave component, with \( n = 1 \) being the primary wave and \( n \geq 2 \) the higher harmonics.

The wave skewness, \( S_s \), is the lack of symmetry of the wave profile relative to the horizontal axis, while the wave asymmetry, \( A_s \), is the lack of symmetry relative to the vertical axis. Both parameters are indicators of the wave nonlinearity, more specifically, how much and in what direction energy is moving up or down the frequency spectrum. They are defined as

\[
S_s = \frac{\langle (\eta - \bar{\eta})^2 \rangle}{\langle (\eta - \bar{\eta})^2 \rangle}^{1/2}
\]

\[
A_s = \frac{\langle (\eta - \bar{\eta})^2 \rangle}{\langle (\eta - \bar{\eta})^2 \rangle}^{1/2}
\]

where \(< >\) denotes the time-average over multiple wave periods, and \( \hat{H} \) denotes the Hilbert transform. Increased crest-trough asymmetry in the surface elevation signifies larger wave skewness and more contributions from higher harmonics on a nature beach with and without low-crested structures [78,88,89]. The bispectral analysis indicates that the sum-frequency wave-wave interactions increases skewness and decreases asymmetry while difference-frequency wave-wave interactions have opposite effects [78].

3. Model validation

The numerical model was first verified with analytical solutions for a uniform current by Fenton [16] (the model results are omitted here for brevity). It was found that although the current-free wave field was specified at the inlet, the wave kinematics reaches the equilibrium with the ambient current within about one wavelength. This result is consistent with previous laboratory studies by Thomas [21] and Swan et al. [23]. In this section the numerical results are compared with the analytical solutions and the experimental measurements for a following and opposing linear shear current.
Fig. 3. Wave profiles in the presence of a linearly sheared current. (a) Following current, \( U_c = 0.35 + 1.0 z \); (b) opposing current, \( U_c = -0.35 - 1.0 z \). Wave parameters \( T = 1.412 \) s, \( H = 0.0702 \) m, \( d = 0.35 \) m. Dimensionless surface current \( |U_s|/\omega = 0.21 \) and dimensionless vorticity \( |\Omega_s|/\omega = 0.22 \) where \( U_s \) is surface velocity and \( \Omega_s \) is the constant vorticity across the water depth. Dotted lines: third-order analytical solution [46]; dashed lines: multi-layered fully nonlinear numerical model based on Dalrymple’s [51] theory.

Fig. 4. Measured vertically sheared current profiles in Swan [85]. (a) Case 1 and 3, following current; (b) Case 2, opposing current.

Table 1

<table>
<thead>
<tr>
<th>Case</th>
<th>Wave height ( H ) (m)</th>
<th>Water depth ( d ) (m)</th>
<th>Wave period ( T ) (s)</th>
<th>Wave steepness ( 0.5Hk )</th>
<th>( kd )</th>
<th>Current profile</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Eq. (6)</td>
<td>Fenton [16]</td>
<td>Eq. (6)</td>
<td>Fenton [16]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 [85]</td>
<td>0.063</td>
<td>0.35</td>
<td>1.418</td>
<td>0.076</td>
<td>0.075</td>
<td>Linear shear, following current, Fig. 4a.</td>
</tr>
<tr>
<td>2 [85]</td>
<td>0.123</td>
<td>0.35</td>
<td>1.420</td>
<td>0.196</td>
<td>0.185</td>
<td>Linear shear, opposing current, Fig. 4b.</td>
</tr>
<tr>
<td>3</td>
<td>0.123</td>
<td>0.35</td>
<td>1.420</td>
<td>0.146</td>
<td>0.141</td>
<td>Linear shear, following current, Fig. 4a.</td>
</tr>
</tbody>
</table>

Swan [85] conducted laboratory experiments of waves over a strong linearly vertically sheared current. The strong linear shear was generated by placing honeycomb directly in front of the inflow and outflow circulating pipes. The measured current profiles are shown in Fig. 4. The corresponding wave and current conditions are listed in Table 1. In the present study, we focus on steep waves interacting with strong currents. The wave steepness, \( 0.5Hk \), and relative water depth, \( kd \), were calculated using the linear wave dispersion relation, Eq. (6), and the fifth-order Stokes wave dispersion, Eq. (8), by Fenton [16] including uniform current. The depth-averaged current velocity was used in the calculations. As shown in Table 1, the wave steepness calculated from the former is larger than that from the latter, since nonlinear wave theory tends to predict a longer wavelength than linear wave theory. This observation has been confirmed by the wave profile predictions in Fig. 3. These waves are within the intermediate-depth regime, \( kd = O(1) \).

The predicted surface elevation and horizontal wave velocities are compared with the measurements by Swan [85] in Fig. 5 for a linearly sheared opposing current. In the presence of an opposing current shear, the wave trough becomes deeper. Both the present predictions and the experimental measurements show larger negative horizontal wave velocities at the trough for the linearly sheared opposing current than the wave only case. Fig. 6 shows that good agreement is achieved between the present model results and the experimental measurements of maximum wave velocities for cases 1 and 2, respectively. The Root Mean Square (RMS) error averaged over the water column is 0.008 m/s for case 1 and 0.019 m/s for case 2. For the Boussinesq-type model results by Son and Lynett [57], the RMS error rises to 0.015 m/s and 0.039 m/s for case 1 and case 2 respectively, despite that the effects of turbulence and radiation stress were included in their model. Same as uniform current, the effect of sheared current on wave kinematics is
first and foremost a manifestation of the change in the dispersion relation. The presence of current shear with current magnitude decaying with depth not only acts to limit the current induced change in the wave number by decreasing the effective depth-weighted current velocity used in the wave dispersion relationship [20,25], but may also introduce additional wave components associated with the current vorticity distribution. For following sheared currents, these current vorticity induced wave components may be large enough to compensate the expected reduction in near-surface velocities due to the reduced wave number and wave steepness [23]. This phenomenon will be further explored in section 3 where the current shear effect on the crest kinematics is investigated.

**4. Results and discussions**

The vertical current shear effect on the harmonics generation is examined by specifying only first-order current-free wave boundary condition at the inlet of the numerical flume, which triggers the near resonant triad interaction for shallow-water waves. The effect of current shear on wave nonlinearity is thus evaluated by the comparison of the harmonics strength, wave skewness and asymmetry in the presence of uniform and sheared current. The predicted wave profile and crest kinematics are also compared with the predictions by the multi-layered model extended from Dalrymple [51], which assumes steady wave form and, therefore, neglects higher harmonics evolution.

**4.1. Effects of vertical shear on near resonant triad interaction**

The near resonant triad interactions occur in shallow and intermediate water depths due to weak wave dispersion, which leads to energy exchanges between three interacting wave components. It can cause substantial cross-spectral energy transfer in relatively short distances. One of the simplest examples of triad interactions can be observed in a physical flume, or a numerical flume setup by the present RANS-VOF model, when the first-order wave solutions are applied in shallow water [55,90]. This will unintentionally generate parasitic free higher harmonics in addition to bound higher harmonics, which are phase-locked to the first-order waves. The mismatch between the bound and the free wave numbers governs the so-called beat-length and hence the strength of the triad interaction, which causes the energy to cycle between the primary wave and higher harmonics and therefore a spatial variation of wave amplitude downstream from the wavemaker [90].

Chen et al. [55] found that the triad interaction is enhanced by a uniform following current and is suppressed by a uniform opposing current. In this section, we will examine the additional effect of vertical current shear on the near resonant triad interaction.

The numerical wave-current flume is 35 m long and 0.6 m high, with the still water depth being 0.4 m. A uniform grid size of 0.015 m in horizontal and 0.008 m in vertical is adopted. The first-order current-free wave with period $T = 2.5 \text{s}$, height $H = 0.084 \text{m}$, and Ursell number 29.5 is specified at the inlet. Fig. 7 shows the predicted spatial evolution of the primary wave, the second and the third harmonic amplitude in comparison with the experimental measurements by Chapalain et al. [101]. Apart from the slight decay in the experimental data due to bottom friction, the observed amplitude evolution is well predicted by the present model within 15 m from the wavemaker.

**Table 2** lists the calculated wave numbers, phase mismatch, and beat-length in the presence of a uniform/linear shear current, by theory and the present RANS-VOF model. This table indicates that the uniform following current reduces the phase mismatch considerably, therefore enhancing the triad interaction. The following linear shear current also reduces the phase mismatch considerably, although not as much as the uniform current, therefore, also enhancing the triad interaction in comparison with the current-free case. So the extent of triad interaction for the linear shear current is expected to be somewhere in between the no current case and the uniform current case. The opposite is expected to be true for the opposing linear shear current.
Fig. 8 shows the spatial evolution of relative harmonic energy and the wave shape parameters (see definitions in Section 2.4) along the flume in the presence of both uniform and linear shear current. As expected based on our discussion above, the uniform following current enhances the extent of triad interaction significantly, since the phase mismatch is reduced by half and beat-length doubled by the following current (see Table 2). In contrast, the uniform opposing current suppresses the triad interaction with its beat-length reduced to about one third of that of no current case. Contrary to our expectation, however, the vertical current shear in the following current enhances the energy exchange between the primary wave and higher harmonics further, while the vertical current shear in the opposing current suppresses it, albeit to a lesser extent. The beat-length in the presence of linear shear current, on the other hand, falls between the no current case and the uniform current case, consistent with our calculated wave numbers in Table 2.

It should be noted that using the uniform current velocity equal to the depth-averaged velocity for the linear shear current would decrease/increase the beat-length for following/opposing current, but it would not enhance/suppress the energy exchange between the primary wave and higher harmonics as much as their linear shear counterparts. Namely, the current shear effect cannot be fully captured by using the depth-averaged current, therefore, needs to be considered separately.

4.2. Effects of current surface velocity and shear on wave shape evolution

A simple superposition of the individual current and current-free wave velocities is specified at the inlet, the wave-current interaction hence comes into effect further downstream from the wavemaker along the flume. When strongly sheared current is present, however, the discrepancy between the equilibrium condition and the specified inlet

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### Table 2

Theoretical and RANS-VOF model predictions of wave numbers and beat-lengths with and without following and opposing uniform and linear shear current for wave parameters $T = 2.5\,\text{s}$, $H = 0.084\,\text{m}$, $d = 0.4\,\text{m}$.

<table>
<thead>
<tr>
<th>Wave number $k$ ($\text{m}^{-1}$) / Beat length (m)</th>
<th>No current</th>
<th>Uniform current$^1$</th>
<th>Linear shear current$^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bound primary wave $k^{(1)}$</td>
<td></td>
<td>Following</td>
<td>Opposing</td>
</tr>
<tr>
<td>Free second harmonic $k^{(2)}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Phase mismatch $2\pi/(k^{(2)} - 2k^{(1)})$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Beat-length by Theory $2\pi/(k^{(2)} - 2k^{(1)})$</td>
<td>15.3</td>
<td>30.6</td>
<td></td>
</tr>
<tr>
<td>Beat-length RANS-VOF model $2\pi/(k^{(2)} - 2k^{(1)})$</td>
<td>15.3</td>
<td>26.7</td>
<td></td>
</tr>
</tbody>
</table>

1. Following: $U_c = 0.3\,\text{m/s}$; opposing: $U_c = -0.3\,\text{m/s}$.
2. Linear shear current profile is of the form, $U_c(z) = U_s - \Omega_s z$, where $U_s$ is surface velocity and $\Omega_s$ is the constant vorticity. Following: $U_c = 0.3 + 0.75z$; opposing: $U_c = -0.3 - 0.75z$.
3. Calculated from dispersion relations, Eqs. (6) and (7), for uniform and linear shear current, respectively.
4. Calculated from $(\nu_0 - k^{(0)}\ell) = gk^{(0)}\tanh k^{(0)}d$ for uniform current and $(\nu_0 - k^{(0)}\ell) = [gk^{(0)} - \Omega_s (\nu_0 - k^{(0)}\ell)]\tanh k^{(0)}d$ for linear shear current where $n = 2$.
5. Approximately taken as the distance at which the relative harmonic wave energy defined in Eq. (12) becomes zero.

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**Fig. 8.** Effect of (solid lines) uniform current, $U_c = 0.3\,\text{m/s}$, and (dash-dotted lines) linearly sheared current, $U_c = 0.3 + 0.75z$, on the spatial evolution of (Upper) relative harmonic energy, (Middle) normalized wave skewness and (Lower) asymmetry in comparison with (dashed lines) wave-only case for (Left) following and (Right) opposing current. Wave parameters $T = 2.5\,\text{s}$, $H = 0.084\,\text{m}$, $d = 0.4\,\text{m}$.
boundary would result in a modulation of the wave form along the flume, the extent of which depends on wave nonlinearity and the near-resonant triad interactions occurring in shallow and intermediate water depths [55,90].

It is seen in Fig. 9 that the wave-only case (dotted lines) exhibits only a slight oscillation of wave skewness and asymmetry downwave from the wavemaker. This demonstrates that without current, using higher-order wave theory at the inlet may weaken the generation of higher harmonics. However, this is not the case for the uniform current (dashed lines) and the shear current (solid lines) with the same velocity at the water surface. A much higher peak of wave skewness and asymmetry appears for the linear shear current than for the uniform current. Larger wave skewness indicates increasing crest-trough asymmetry and thus more contributions from higher harmonics. As discussed in Zou et al. [91], velocity skewness and asymmetry are related to the phase coupling between primary and harmonic waves (bound/forced waves), which is quantitatively described by the bispectrum. For unimodal, narrow banded waves, skewness and asymmetry are mainly due to the self-self interaction [88,92–94]. Significant evolution of wave skewness and asymmetry has been observed on nature beaches by Elgar et al. [88] and over low-crested structures [78,89] and due to bottom slope and friction [91]. This is consistent with the model results of current shear effect on harmonic wave generation studied in Section 4.1. It is expected that a uniform current with a smaller velocity, which equals the depth-averaged velocity of the linear shear current, would lead to less deviation from the wave-only case but more deviation from the wave plus linear shear current case.

It should be noted that most formulations for nonlinear waves propagating in the presence of linear shear current assume that the linear shear current profile has a zero velocity at the bottom or at the mean free surface (as in [27]). But this is not the case for the current profiles generated in the experimental flume by Swan [85], which was used in this study to validate the CFD model. Implementing fully nonlinear wave solutions in the presence of the depth-varying current profiles at the inlet would help to suppress the wave beat observed numerically here. In the future we would implement a more advanced nonlinear combined wave/current inlet boundary condition and assess the stability of the waves propagating in the presence of the corresponding current.

Fig. 10 shows the vertical profiles of skewness and asymmetry for wave horizontal and vertical velocities at three locations along the wave flume for a following linearly sheared current. It is seen that consistent with the surface elevation skewness and asymmetry in Fig. 9, velocity skewness and asymmetry vary horizontally with x and vertically with z as well. The former is largely in response to the horizontal evolution of wave profile and its skewness and asymmetry shown in Fig. 9 since the wave velocity is linearly correlated with the local surface elevation not only at the peak frequencies but also at their harmonics over a flat and sloping bottom [91]. The latter is due to the frequency-dependent vertical variation of wave velocity shown in Fig. 6. Zou et al. [91] and Zou & Hay [95] demonstrated that the effect of bottom slope and bottom friction on vertical variation of wave velocity is dependent on frequency, which alters the vertical distribution of wave velocity skewness and asymmetry and possibly the vertical variation of current. Since the horizontal velocity is in phase with the surface elevation, peak skewness for horizontal velocity appears at

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**Fig. 9.** Spatial evolution of (a) wave skewness and (b) asymmetry and (c) time history of surface elevations at varying locations for a following linearly sheared current indicated in Fig. 4a (case 3 in Table 1), no current, and uniform current. Surface elevations in (c) corresponds to the locations, $x = 4.7$ m for linear shear current and no current, and $x = 4.6$ m for uniform current, where the wave skewness is at the peak and the asymmetry is roughly zero as shown in Fig. 9(a) and (b). The uniform current and linear shear current profiles have the same current velocity at the water surface.

**Fig. 10.** Vertical profiles of skewness (solid lines) and asymmetry (dashed lines) for wave horizontal velocity (Left) and vertical velocity (Right) at $x = 2.0$, 3.5 and 4.7 m from the wavemaker ($x = 0$) for a following linearly sheared current in Fig. 4a (case 3 in Table 1). See Fig. 9 for the spatial evolution of surface elevation skewness and asymmetry at these locations.
x = 4.7 m where the peak wave skewness is, corresponding to a roughly zero skewness for vertical velocity. Thus the presence of current shear modifies the skewness and asymmetry for both surface elevation and wave velocity. The latter is closely related to net sediment transport.

Next we will assess the individual contribution by the surface current velocity, vertical shear strength, and the water depth to the current shear effect on the wave. Fig. 11a,b shows the comparison of wave skewness and asymmetry for 3 linearly sheared current profiles with different surface current velocity and shear strength. Fig. 11c shows the time history of surface elevations corresponding to the locations where the wave skewness is at the peak and the asymmetry is roughly zero as shown in Fig. 11(a) and (b).

Due to the strong current shear and the near-resonant triad interaction, the wave form experiences changes along the flume. When the wave skewness attains the largest value and the asymmetry roughly zero (e.g., x = 4.7 m for case 3 in Figs. 9 and 11), wave nonlinearity is at its peak. Fig. 12a shows the comparison between the predicted wave shape by the present model and the fully nonlinear multi-layered solution extended from Dalrymple [51], in which the wave form is assumed to constant in space.

For the same wave height, Fig. 12 indicates that the triad interaction affected wave has sharper crest and flatter trough, thus is more nonlinear, than that predicted by the fully nonlinear wave theory. This is due to the fact that the fully nonlinear theory neglects the spatial evolution of higher harmonics evolution. Fig. 12b shows the corresponding wave amplitude spectrum. Both model results show a multitude of higher-order wave components. The present model predicts a lower primary wave amplitude and higher harmonics amplitudes because of the sharper/narrower wave crest in Fig. 12a.

Fig. 12c shows the predicted total horizontal velocities under wave crest at the same location. Both predictions take into account the effect of wave-current interaction. The maximum total horizontal velocity is the sum of the steady current velocity and the maximum wave orbital velocities affected by the presence of a current. This parameter is of particular interest for calculating drag forces on offshore and coastal structures. In engineering practice, however, it has been a common practice to use the sum of the current and wave velocities to represent the total water particle velocity and neglect the nonlinear wave-current interaction. As demonstrated in Fig. 12c, linear superposition of wave and current leads to an underestimate (by 16%) of the total horizontal velocity close to the water surface. The increased velocity under the crest is due to the additional wave components (irrotational and/or rotational) introduced by the presence of the current shear [23].

The effects of the current shear strength and surface current velocity are also examined individually using this multi-layered wave theory solution by Dalrymple [51]. In each case the wave height (H = 0.153 m) and wave period (T = 1.42 s) are the same and assumed to remain unchanged for the combined wave-current flow. The uniform current has the same velocity at the free surface as that of the linearly sheared current profile. A third current profile with exponential shear (Fig. 1) is considered here, which has a larger magnitude of vorticity at the water surface than the linearly sheared current.

Table 3 lists two scenarios for significant surface elevation changes due to the current shear. The first scenario corresponds to case 3 of the present study, where the maximum crest elevation increases about 15% when compared with the case of the same wave height in the absence of the shear current. The second scenario corresponds to the case of Fig. 13c in Swan et al. [23], where the maximum crest elevation increases by 12%. The normalized surface vorticity |Ωs|/ω, by the wave frequency ω, is the same for the two cases, while there are large differences in the wave steepness 0.5Hk and relative water depth kd. The wave steepness for the first scenario is much smaller than that in the second. This suggests that the wave steepness at which the same current shear starts to affect the crest elevation increases with increasing relative water depth. In other words, larger wave nonlinearity is required for the same current shear effect to be observed at increasing relative water depth. Beji [96] introduced a modified form of Ursell number, 0.5Hk/[tan(hk)]2, which becomes identical to the classical Ursell number for small kd and tends to 0.5Hk for large kd. It is seen in the last column of Table 3 that the modified Ursell number for the two cases are close to each other, indicating comparable nonlinearity and thus similar crest elevation changes.

4.3. Current shear strength
linear, and exponential shear current. For uniform current, the relative harmonic wave energy is enhanced by the following current and reduced by the opposing current. The same conclusion can be drawn for a linearly sheared current with the same velocity at the free surface. This is consistent with previous findings considering only the uniform current effect on the extent of triad interactions [29,55], and the vertical current shear effect on higher harmonics explored in sections 4.1 and 4.2 of this study. In comparison with the linear shear, the exponential shear enhances the skewness and harmonics energy further for the following current, while it has less effect for the opposing current.

5. Conclusions

A Navier-Stokes solver is coupled with a Volume of Fluid (VOF) surface capturing method to investigate wave interactions with a depth-varying current in the intermediate and shallow waters, with a special attention to the separate effect of vertical current shear on nonlinear wave-wave interactions. The model was validated against the analytical solutions and the experimental measurements for a linearly sheared current. The effect of vertical current shear on wave nonlinearity and near resonant triad interaction is then explored by comparing the harmonics strength and wave skewness and asymmetry with and without adding the current shear to the uniform current.

Changes in wave velocity by the presence of uniform current are mainly through the current modulation of wave length and wave number. Our model results indicate that the wave velocity is further modified by the presence of current shear through its influence on the wave profile. The waves have sharper crests and flatter troughs, therefore, larger wave skewness in the following current and become more sinusoidal in the opposing current. Similar phenomenon is observed for waves in deep water [19]. In the presence of current shear, we found that the wave steepness at which the surface current shear starts to affect the crest elevation increases with relative water depth. A modified Ursell number was found to be a good representation of wave nonlinearity influenced by current shear in both shallow and deep water.

The separate effect of current shear on harmonic generation and energy exchange among wave harmonics due to near resonant triad
interactions was examined for the first time in this study. Spectral analysis of the surface elevations show that, while the presence of a following uniform current increases the relative harmonic wave energy and the extent of triad interaction, the presence of a vertical current shear further enhances them. The opposite is true for an opposing current. Both the RANS-VOF model and our numerical result based on the multi-layer solution by Dalrymple [51] predict the same trend of wave profile changes in the presence of vertical current shear. However, the triad interaction is neglected in Dalrymple’s [51] solutions. Because of the stronger shear near the surface, the exponential shear tends to modify the wave profiles more than the linear shear. For constant shear current, the present results are consistent with the fully nonlinear solutions by Simmen & Saffman [48] and da Silva & Peregrine [49], as well as the experimental measurement of waves propagating in currents with strong surface shear [23]. All these studies show that the vertical current shear or vorticity has a profound effect upon the wave profiles. The present study demonstrates further that the vertical current shear plays an important role in near-resonant triad interactions.

The higher and sharper wave crests due to enhanced nonlinearity by the following current shear produce larger horizontal wave velocities near the free surface. As a result, a simple linear superposition of the individual wave and current velocities without wave-current interaction would lead to an underestimate of the total horizontal velocity close to the water surface. The increased crest elevation also gives rise to greater water particle acceleration, therefore, larger wave loading and larger freeboard required for the design of coastal and offshore structures. The increased nonlinearity in the presence of following current shear may contribute to premature wave breaking at a lower sea state than that without current or with uniform current. The presence of current shear also has a significant effect on wave velocity skewness and asymmetry, which has important implications on sediment transport.

On the other hand, the present study shows that the opposing current and current shear reduce the wave skewness so that the wave profile becomes more sinusoidal. This may lead to wave breaking at larger wave steepness than that without current or with uniform current and change the breaking type and intensity. Similar phenomena were found for in-cipient wave breaking of unsteady wave groups [97,99]. When the horizontally varying opposing current is large enough to block and break the incoming waves, however, Chen and Zou [98] found that the steepness-limited, current-induced wave breaker may be observed at a much lower wave steepness than the current-free deep water waves. Eventually, the interaction between wave and current and their transformation across the coastal region has significant impact on coastal flooding during severe storms through their influences on water level [100].

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Appendix A. Current profile evolution in the numerical flume

Fig. A1 shows the evolution of wave-averaged current profiles from the wavemaker to the middle of the numerical flume. A superposition of linear shear current and current-free wave (case 1 and 2 in Table 2) is specified at the inlet. It is evident that there is little change in the current profiles across the numerical flume due to the presence of wave motion. The relaxation zone at the outlet (see Fig. 1 for illustration), within which the target current profile is specified, is able to maintain the same current profile across the numerical flume.

Fig. A1. Current profiles in a numerical flume specified at the wavemaker (x = 0 m) and predicted by the present RANS-VOF model downwave from the wavemaker at (Left) x = 1.3, 2.6 and 5.3 m for a following current and (Right) at x = 2, 3 and 5 m for an opposing current.
References
