Abstract — In this paper, the effect of antenna coupling in an active array on the integrated power amplifiers (PAs) is studied. It is found that for multiple-input multiple-output (MIMO) applications some antenna elements inputs can experience negative impedance. When isolators between antennas and PAs are missing, which is preferable in highly integrated millimeter-wave (mm-Wave) massive MIMO transmitters due to the fact that isolators are bulky and integration-incompatible, this negative impedance can cause PA oscillating/damaged, or at least non-functional. This non-linear problem is studied theoretically using an example of a coupled antenna array driven by Doherty PAs. In addition, the experimental results validate the existence of the negative impedance and reveal its relationship with the array excitation and PA non-linearity.

Keywords — Coupled antenna array, MIMO, non-linear power amplifier, negative impedance

I. INTRODUCTION

The massive active antenna arrays, especially for mm-wave band (like 28 GHz for 5G application) operation, call for high integration for small form factors and reduced costs. Consequently, the use of isolators (normally placed between power amplifiers (PAs) and antenna elements) becomes unfavorable due to their bulky size [1]. This in turn leaves the PA exposed to non-ideal antenna impedance matching as well as load-pull effects associated with mutual coupling among the radiating elements.

This type of active antenna arrays with the absence of isolators has been studied for beam-forming applications. For coupled antenna arrays, because of the non-linear behavior of the exposed PAs, new methods of obtaining active array element patterns have been developed, so that beam-forming patterns can be synthesized. For example, in [1], [2], the PA non-linear Poly Harmonic Distortion (PHD) model [3] was derived for this purpose, and its extension [4] was proposed. Large active arrays, e.g., 64-element in [5], have also been studied.

In order to investigate the dynamic (time-variant) behavior, the time-domain PA models were developed, such as the Volterra series-based dual-input model [6], [7] and the PHD time domain model [8], [9]. In addition, a digital pre-distortion method for active antenna arrays was presented in [10]. Recently, an experiment of a coupled antenna array driven by Doherty PAs was given in [11], which implicated that the efficiency of PAs can be largely maintained for beam-forming application.

However, as we pointed out that these existing works mainly focused on the beam-forming applications wherein the magnitudes and phases of excitation vectors across the entire array are set according to a pre-defined profile, usually uniform magnitudes and progressive phases. While under the multiple-input multiple-output (MIMO) application scenarios, the amplitudes and phases of the driving signals input into PAs are determined by the MIMO precoding algorithms and the channel fading conditions. They can vary greatly across the array and be updated from time to time.

In this paper, with the theoretical and experimental studies, we find out that without isolators some PAs in a coupled antenna array can experience negative impedance at their outputs when MIMO applications are considered. In practice, a well-designed (for self-protection) PA shuts down under such hostile matching condition, which was observed in our experiment. Furthermore, we show that the negative impedance condition is non-linearly related to the power levels of PA input signals when isolators are removed.

II. IMPEDANCE OF COUPLED ANTENNA ARRAYS

In this section, the impedance at input ports of coupled antenna elements for two different architectures of PA and antenna element inter-connections, i.e. with and without isolators, are analyzed and compared. The negative impedance can occur under MIMO applications where coupled antenna array elements are excited with unbalanced power.

A. With isolators

In this traditional microwave-regime transmitter architecture where isolators are inserted between PAs and antenna elements, the calculation of the input impedance of each antenna element is a linear problem, with its real part \( R_q \) of the \( q^{th} \) antenna in an \( M \)-element coupled array being expressed as

\[
R_q = \operatorname{Re} \left\{ \frac{V_q}{I_q} \right\}
\]  

where \( V_q \) and \( I_q \) are, respectively, the voltage and current at the input port of the \( q^{th} \) antenna, and they can be written in the forms of

\[
V_q = V_q^+ + V_q^- = V_q^+ \left( 1 + \sum_{i=1}^{M} S_{qi} \frac{V_i^+}{V_q^+} \right)
\]

\[
I_q = \frac{1}{Z_0} (V_q^+ - V_q^-) = \frac{V_q^+}{Z_0} \left( 1 - \sum_{i=1}^{M} S_{qi} \frac{V_i^+}{V_q^+} \right)
\]  

(2)
Here \( V_q^+ \) and \( V_q^- (q = 1, 2, ..., M) \) are the incident and reflected voltage waves at the \( q^{th} \) antenna port. \( S_{ii} \) refers to the S-parameter between the \( q^{th} \) and the \( i^{th} \) elements, describing the coupling effects (when \( q \neq i \)) and input impedance mismatching (when \( q = i \)). \( Z_0 \) is the characteristic impedance, generally choosing 50 \( \Omega \).

After substituting (2) into (1), it can be noticed that when antenna coupling is strong, i.e. \( |S_{ii}| \) is large, and array excitations are unbalanced, i.e. \( |V_i^+| \gg |V_q^+| \) which happens with high probability for MIMO applications, the real part \( R_q \) of the impedance can be negative. However, the resulting negative impedance has no effect on PA operation and performance as PAs are shielded by the isolators.

**B. Without isolators**

The absence of the isolators leads to non-linear PAs interact with linearly coupled arrays, making the impedance calculation more complicated. In this sub-section, using the same array architecture discussed in [11], i.e. coupled antenna arrays driven by Doherty PAs, as an illustrative example, the approach of impedance calculation is theoretically presented.

Fig. 1 shows an \( M \)-element active array configuration with parameters and variables labelled. In each Doherty module, a carrier amplifier and a peaking amplifier are modelled as current source \( i_{c} \) and \( i_{p} (i = 1, 2, ..., M) \), respectively, \( x \) and \( y \) are the scaling factor related to the input power level. For class-B biased amplifiers (used here for calculation), in order to normalize the drain bias voltage \( V_{DD} \) and optimal load \( R_{opt} \) to unity, the \( x_{max} (x \in [0, x_{max}]) \) has been set as \( 2V_{DD}/x_{max} = 2 \). The relationship between \( x \) and \( y \) determines the input back-off (IBO) power point where Doherty PA efficiency is optimized. Here 6 dB is assumed in the calculation, which indicates

\[
y = \begin{cases} 
0, & x \in [0, 1] \\
2(x - 1), & x \in (1, 2] 
\end{cases}
\]

In addition, \( \alpha \, e^{j\beta t} \) is the complex excitation signal that for MIMO applications can be arbitrary for different path \( i \) because of the MIMO precoding and random wireless channel fading.

The normalized PA drain current \( i_{p} \) (we drop the subscript \( 'c' \) or \( 'p' \) for simplicity as the formulations for both carrier and peaking PAs take the same form. The same rule applies to other notations hereafter), is related to the corresponding drain voltage \( v_{i} \) of the associated PAs as [12]

\[
i_{p} = A(\theta)k(v_{i}) \\
v_{i} = 1 - W_{c}\cos(\theta + \phi_{i})
\]

(4)

Here it is assumed that all harmonics are short terminated. \( k(v_{i}) = 1 - (1 - v_{i})^N \) is the knee voltage function with even integer \( N \) as polynomial order. \( \theta = 2\pi f_{0} t \) with \( f_{0} \) of operation frequency and \( t \) of time. \( W_{c} e^{j \phi_{i}} \) is the corresponding drain voltage of the fundamental frequency component \( V_{i} \). \( A(\theta) \) is the baseline function determined by the transconductance angle (here \( \pi \) for class B operation), which can be Fourier expanded as

\[
A(\theta) = A_{0} + \sum_{n=1}^{\infty} [A_{n}\cos(n\theta) + B_{n}\sin(n\theta)]
\]

(5)

where \( A_{n} = T_{n}\cos(\beta_{n}), B_{n} = -T_{n}\sin(\beta_{n}), \) \( \beta_{n} \) is the phase of the excitation signal at the path \( i \), see Fig. 1, and \( T_{n} \) are the expansion coefficients for ideal class B operation when \( \beta_{i} = 0 \).

\[
\lambda(wavelength) \quad \mathbf{Z}^{\prime} \text{[impedance matrix]} \quad \text{Antenna array}
\]

Combining (4) and (5), the fundamental frequency component \( I_{i} (= I_{R,i} - jI_{Q,i}) \) of \( i_{p} \), can then be obtained as

\[
I_{R,i} = A_{1}k_{0} + \frac{1}{2} \sum_{n=1}^{\infty} ([A_{2n-1} + A_{2n+1}]k_{2n,R} + (B_{2n-1} + B_{2n+1})k_{2n,Q})
\]

\[
I_{Q,i} = B_{1}k_{0} + \frac{1}{2} \sum_{n=1}^{\infty} ([A_{2n-1} - A_{2n+1}]k_{2n,Q} + (B_{2n+1} - B_{2n-1})k_{2n,R}],
\]

(6)

where \( k_{0}, k_{2n,R} \) and \( k_{2n,Q} \) are the Fourier expansion coefficients of \( k(v_{i}) \) [12].

From the network topology in Fig. 1, after some derivations we can obtain

\[
\mathbf{I}_{c} = \mathbf{Z}^{\prime} \mathbf{V}_{c} + jZ_{0}\mathbf{I}_{p}
\]

(7)

where \( \mathbf{I}_{c} = [I_{1,c}, I_{2,c}, ..., I_{M,c}]^{T}, \mathbf{V}_{c} = [V_{1,c}, V_{2,c}, ..., V_{M,c}]^{T}, \mathbf{I}_{p} = [I_{1,p}, I_{2,p}, ..., I_{M,p}]^{T} \) and \([\cdot]^{T}\) is matrix transposition. We add subscript ‘c’ or ‘p’ to refer to the carrier or peaking amplifiers. It needs to be pointed out that here \( I_{i,c} \) and \( I_{i,p} \)
are the fundamental frequency components associated with the carrier and peaking PAs, respectively, and they are both functions of \( V_e \) (see (6) wherein \( k_0 \) and \( k_{2n}(R,Q) \) are the functions of \( V_e \)). The vector \( V_p = [V_{1,p}, V_{2,p}, ..., V_{M,p}]^T \) does not contain independent variables as \( V_{i,p} = -jI_{i,c}Z_0 \), thus being eliminated in (7). \( Z^2 \) is the impedance-matrix of the antenna array including quarter-wavelength transmission lines, see illustration in Fig. 1. In (7), there are \( 2M \) unknowns (magnitude \( W_{i,c} \) and phase \( \phi_{i,c} \) of \( V_{i,c} \)) with \( 2M \) equations (real and imaginary parts). Thus, \( V_e \) can be computed. The loads of the carrier and peak current sources can then be calculated via

\[
Z_{i,c} = \frac{V_{i,c}}{I_{i,c}}, Z_{i,p} = -\frac{jI_{i,c}Z_0}{I_{i,p}},
\]

and the input impedance at ports of the antenna array can be obtained as

\[
Z_{in,i} = \frac{V_{i,c} + jI_{i,p}Z_0}{2I_{i,c}}.
\]

Thus, it can be observed that \( Z_{i,c} \) and \( Z_{i,p} \) are the fundamental frequency components associated with the carrier and peaking PAs, respectively, and they are both functions of \( V_e \) (see (6) wherein \( k_0 \) and \( k_{2n}(R,Q) \) are the functions of \( V_e \)). The vector \( V_p = [V_{1,p}, V_{2,p}, ..., V_{M,p}]^T \) does not contain independent variables as \( V_{i,p} = -jI_{i,c}Z_0 \), thus being eliminated in (7). \( Z^2 \) is the impedance-matrix of the antenna array including quarter-wavelength transmission lines, see illustration in Fig. 1. In (7), there are \( 2M \) unknowns (magnitude \( W_{i,c} \) and phase \( \phi_{i,c} \) of \( V_{i,c} \)) with \( 2M \) equations (real and imaginary parts). Thus, \( V_e \) can be computed. The loads of the carrier and peak current sources can then be calculated via

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\]

and the input impedance at ports of the antenna array can be obtained as

\[
Z_{in,i} = \frac{V_{i,c} + jI_{i,p}Z_0}{2I_{i,c}}.
\]

Thus, it can be observed that \( Z_{i,c} \) and \( Z_{i,p} \) are determined by the antenna coupling (non-diagonal items in \( Z^2 \) are not zero), PA non-linearity (knee voltage profile \( k(V_i) \)) as well as the excited signals \( \alpha_i e^{j\beta_i x} \).

Table 1. Measured S-parameters of 2 × 2 antenna array at 3.5 GHz (dB/degree).

<table>
<thead>
<tr>
<th>( S_{ij} )</th>
<th>( j = 1 )</th>
<th>( j = 2 )</th>
<th>( j = 3 )</th>
<th>( j = 4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( i = 1 )</td>
<td>-19.1/-124°</td>
<td>-18.4/-12°</td>
<td>-13.7/5°</td>
<td>-20.8/-120°</td>
</tr>
<tr>
<td>( i = 2 )</td>
<td>-18.4/-12°</td>
<td>-19.4/-101°</td>
<td>-20.6/-121°</td>
<td>-13.4/-2°</td>
</tr>
<tr>
<td>( i = 3 )</td>
<td>-13.3/-5°</td>
<td>-20.6/-121°</td>
<td>-19.2/121°</td>
<td>-17.9/-17°</td>
</tr>
<tr>
<td>( i = 4 )</td>
<td>-20.8/-120°</td>
<td>-13.3/-2°</td>
<td>-17.9/-17°</td>
<td>-21.3/-105°</td>
</tr>
</tbody>
</table>

Table 2. Array excitation examples used for theoretical calculation (amplitude/phase).

<table>
<thead>
<tr>
<th>( \alpha_i / \beta_i )</th>
<th>( i = 1 )</th>
<th>( i = 2 )</th>
<th>( i = 3 )</th>
<th>( i = 4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>0.3/-114°</td>
<td>0.5/94°</td>
<td>1/140°</td>
<td>1/-170°</td>
</tr>
<tr>
<td>Case 2</td>
<td>0.4/-48°</td>
<td>0.6/170°</td>
<td>1/-167°</td>
<td>0.8/139°</td>
</tr>
<tr>
<td>Case 3</td>
<td>0.5/-122°</td>
<td>0.8/116°</td>
<td>0.9/28°</td>
<td>1/-146°</td>
</tr>
</tbody>
</table>

III. EXAMPLES

Following the theoretical derivation in Section II-B, the input impedances of a four-element coupled antenna array driven by Doherty PAs are computed for random array excitation vectors, i.e. random \( \alpha_i e^{j\beta_i x} (i = 1, 2, 3, 4) \). A two-by-two antenna array, operating at 3.5 GHz, was designed and fabricated (see in Fig. 2b), and its measured S-matrix (given in Table 1) is used in the calculation. The polynomial order \( N \) of 6 for knee voltage \( k(V_i) \) is selected as it is a typical value for GaN PA transistors [12]. As an example of array excitations, see Case 1 in Table 2, the calculated loads of carrier and peaking PAs, as well as the antenna array input impedance are plotted in Fig. 2. It can be observed that \( Z_{1,c} \) and \( Z_{in,1} \) experience negative impedances, and both of them are non-linearly related to the IBO point which is 2 \( \times \log_{10}(x/2) \) in dB. In Table 2, we also list two other array excitation conditions which lead to negative impedance. Due to page limitation, the results are omitted.

When isolators are inserted, using (1), the normalized real part \( R_0 \) of input impedance of the four antenna elements can be calculated. They are 0.09, 0.81, 1.18, and 1.11, respectively, when the same excitation vector of Case 1 in Table 2 is used. Different to the impedances shown in Fig. 2, they are independent to \( x \).

IV. EXPERIMENTAL RESULTS

Experiment was conducted in order to reveal the above discussed issue facing MIMO applications when PAs and coupled antenna arrays are not isolated. Since the ‘negative impedance’ is not the direct consequence of choosing Doherty PA architecture, it can occur when any non-linear PAs are used. Thus, for simplicity the experiment, with the block diagram shown in Fig. 3, was set up, where two Class-AB biased PAs (Mini-Circuit ZX60-V63+) for 3.5 GHz operation were used to drive two coupled antenna elements (antennas 1 and 2, see Fig. 4). In the experiment, we fixed the power of input signal \( S_1 \) (see Fig. 3) at -1.9 dBm, and swept the power of \( S_2 \). Fig. 4 depicts the PA2 reflection coefficient at the input port in dB (obtained by taking the ratio of measured power at port 6 over that at port 5) as the function of the power of \( S_2 \). It can be observed that there is an abrupt change of the input return loss.
In this example, the power cannot be effectively injected into the PA2 when the power of $S_2$ is below around $-9$ dBm, and no power can be injected into PA2 at all when it is smaller than $-20$ dBm (i.e. the PA2 essentially shut down). The power at port 7 and port 8 were also measured. It is noted that when PA2 behaves abnormally (i.e. input power of $S_2$ below $-9$ dBm) the signal measured at port 7 comprises both PA2 output and the reflected signals (at the PA2 output) coupled from the antenna element 1. Thus, in this situation, the PA2 loads (or antenna load impedance) cannot be calculated with powers and phases in Port 7 and Port 8.

![Experiment block diagram](image)

Fig. 3. Experiment block diagram

![Measured reflection coefficient at the input port of PA2 as a function of the input power of $S_2$ while the input power of $S_1$ is fixed to $-1.9$ dBm.](image)

Fig. 4. Measured reflection coefficient at the input port of PA2 as a function of the input power of $S_2$ while the input power of $S_1$ is fixed to $-1.9$ dBm.

![Relationship between the power of $S_2$ and the power of $S_1$ when the magnitude of PA2 input reflection coefficient reaches $-0.5$ dB.](image)

Fig. 5. Relationship between the power of $S_2$ and the power of $S_1$ when the magnitude of PA2 input reflection coefficient reaches $-0.5$ dB.

Fig. 5 shows the power of $S_2$, at which the PA2 shuts down (we use the magnitude of PA2 input reflection coefficient $-0.5$ dB as the threshold), for various power of $S_1$. It can be concluded that the negative impedance conditions are non-linearly related to the array excitation signals, especially in high power region where PAs are compressed. It has also been observed from the ADS harmonic-balance simulations that this issue can be more severe when the number of array elements becomes large.

V. CONCLUSION

In this paper, we have studied the effect of antenna coupling in an active array on the integrated PAs. Without isolators between PAs and antenna elements, it was observed that for MIMO applications some antenna elements inputs can experience negative impedance, which was non-linearly related to array excitation vectors. This non-linear problem was studied theoretically when a coupled array is driven by Doherty PAs. In addition, the experimental results validated the existence of the negative impedance and revealed its relationship with the array excitation and PA non-linearity.

REFERENCES


