Characterization of LPGs Via Correlation Analysis of an Analytical Solution with Observed Transmission Spectra

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Abstract—In this paper, we describe a technique to characterize long period fiber gratings (LPG) by use of a correlation between the measured transmission spectrum and a theoretical model. The model is robust enough to require only a priori knowledge of the fiber design and the inscription period. We demonstrate the application of this technique on an example LPG resulting in the calculation of the dispersive core index, length and strength of the grating to better than experimental error. The technique is further expanded to allow for the modeling of metal jacketed long period gratings by use of an extended version of Erdogan’s model.

Index Terms— Fiber Gratings, Hydrogen, Optical Fiber Sensors, Palladium

I. INTRODUCTION

There has been considerable interest in recent years in the development of reliable, repeatable long period grating (LPG) structures for use as band pass filters [1] as well as fiber optic sensors [2]. Much of this work has concentrated on LPGs jacketed in dielectrics of various forms [3-5], often in an attempt to tailor the sensitivity to external environment. Some work utilized metallic jacketed fibers, both for band pass [6] and as sensors, often when coated with Palladium (Pd) as a hydrogen sensor [7]. An LPG, like an FBG, is fundamentally a mode coupling structure. Whereas in an FBG radiation is coupled from the core mode into the counter propagating core mode in an LPG the core mode is coupled to the co-propagating radially symmetric LP01 cladding modes. This process is resonant and as such each cladding mode is coupled at a discrete wavelength. The resonant wavelength at which this coupling occurs has been demonstrated to be dependent on temperature [2, 8, 9], strain [2, 8, 10], bend [2], and perhaps most importantly on external refractive index [2, 11].

Fig. 1. Illustration of the parameters necessary to model an LPG: a) the fiber, b) and example of a top hat type envelope grating

Whereas there have been a number of papers published, illustrating proof of principle systems [2, 5, 7, 12] the development of a more robust reliable LPG has been hampered by limited characterization of both the written LPGs and of the LPG writing system. Recent work to characterize LPGs after inscription and by extension, the inscription process, has been accomplished [13]. However the implementation of this process requires specialized microscopy which may not be available in all cases. In the absence of such a system it is possible to use theoretical modeling to fit the transmission spectrum to the LPG parameters. To date this has only been applied to determining the fiber parameters rather than the LPG parameters [14, 15]. Presented in this paper is a simple and robust method for characterizing an LPG by fitting the transmission spectrum to theory via correlation analysis and the extension of this analysis to cover LPGs with a metallic jacket (or coating).
having a complex refractive index. The presented method provides an improvement in simplicity and potential application over previously published models evaluating fiber parameters though the LPG transmission spectrum [14, 15]. Although there exist several models of analytical descriptions of LPGs, we have selected Erdogan model [16-18] because it provides a complete solution to for a fiber configuration and can be extended to fibers with external coatings. The following analysis is however general, the fitting functions are based on the physical properties of the fiber and as such should be applicable to any model. The extended analysis presented here is based on the model published by Erdogan and Erdogan model is reported here in sufficient detail only to explain the extension to the model.

The parameters necessary to fully characterize an LPG are shown in Fig. 1. These can be split into fiber and grating parameters. The fiber consists of a core and cladding of certain radii \((a_1, a_2)\), having dispersive refractive indices \((n_1, n_2)\). In addition there is a refractive index associated with the external environment \(n_i\). A commercially available single mode fiber will generally be well characterized in terms of dispersion, mode field diameter and physical dimensions however commercial confidentiality limits the accuracy and finer details of various optical and dimensional fiber parameters. For an arbitrary fiber it is assumed that the physical dimensions as well as the cladding index are either known (for example many fibers use a fused silica cladding) or can be determined. Since these parameters are not affected by LPG inscription they shall be considered well defined for the purposes of this paper.

The inscription process introduces further uncertainty in the core refractive index \(n_c\), even where this is well known from the onset. A DC component is generally generated in addition to the ac grating. For accurate modeling this DC component needs to be included in the core index used for modeling \(n_c\).

For a truly arbitrary LPG, no parameters for the LPG are known; (strength \(\Delta n\), length \(l\), period \(\Lambda\) and envelope). However for a real LPG it is usual to know the period and potentially the length, envelope or strength of the grating depending on the inscription technique.

We have selected an example LPG to illustrate this characterization method and its transmission spectrum from 1.2 to 1.6 µm is presented in Fig. 2. This LPG was fabricated within a Corning SMF-28 fiber which was hydrogen loaded prior to inscription at 100 bar, 100 °C for 24 hrs. A 248 nm KrF excimer laser was used in the inscription, illuminating the fiber through a 200 µm slit producing a 400 µm period with a 50% duty cycle. In the case of this LPG, periods were written point by point through a slit of width one half of the periodicity until the desired transmission spectrum was achieved. The number of periods and hence the length was not recorded. These illumination conditions will provide a square wave like index variation, for simplicity this is approximated as a Sine. In a more general case it is more likely that the grating envelope is unknown while the grating length is well defined depending on the inscription technique. Since the envelop and the length of the grating have a similar effect on the transmission spectrum it is possible to apply the same fitting procedure for the envelope as it is for the length. Following the LPG inscription the fiber has been annealed during which any remaining residual hydrogen will have been driven off. SMF 28 can be modeled as having radii: \(a_1 = 4.1\) µm, \(a_2 = 62.5\) µm (c.f. Fig. 1) with a cladding of fused silica, the Sellmeier coefficients of which are taken from Malitson [19]. The grating is known to have a period of 400 µm in a top hat envelope of length of ~24 mm and the UV inscription technique is expected to provide approximately \(3 \times 10^{-4} \pm 0.75\) Refractive Index Units (RIU) of index change. The unknown parameters are therefore \(l, n_1\) and \(\Delta n\).

II. INITIAL MODE ORDER IDENTIFICATION

Each loss band within an LPG transmission spectrum is associated with a particular mode order but for a general LPG this association is unknown, primarily due to uncertainty in the core refractive index after inscription. Due to the poor mode field overlap there is essentially zero coupling between the even order LP_{0\nu} modes the core mode up to the \(\sim 23^{rd}\) order. These modes are therefore often ignored for modeling purposes and many mode order indexing schemes in use negate these even orders. In this paper however the indexing system retains the even order modes.

The first step in modeling such a transmission spectrum must therefore be the correct identification of the mode orders of each loss band. This process can be best done by considering the phase matching condition (1).

\[
\lambda_c = \Lambda(n_{rod} - n_{i,c})
\]  

(1)

Where: \(\Lambda\) is the period of the grating, \(n_{rod}\) \(n_{i,c}\) are the core and \(v^{th}\) cladding mode effective refractive indices and \(\lambda_c\) is the \(v^{th}\) cladding mode resonant wavelength.

The only unknown quantity within (1) is the DC component of the refractive index of the core, which is necessary to calculate all the modal effective refractive indices. It is however possible to estimate the range of permitted values
that this can take. Crucially, this DC offset will, due to the nature of the UV induced index change, always be positive and smaller than the maximum refractive index change in the LPG. $\Delta n$ can therefore be described by the inequality (2):

$$0 < \Delta n < \Delta n$$

(2)

Where: $\Delta n$ is the DC index change and $\Delta n$ is the maximum refractive index change in the core, (i.e. the strength of the grating).

The core refractive index $n_c$ may be well characterized for standard fibers or may be estimated as a refractive index step from the cladding ($n_2$) index. In either case, it is necessary to estimate the maximum plausible LPG strength, a figure which can be determined from the inscription technique. With the example LPG, $n_c$, can be estimated as between $n_2 + 0.005$ and $n_2 + 0.0055$ [20]. These values can then be used to plot phase matching curves via (1) for the maximum and minimum plausible core refractive index Fig. 3.

By considering the measured mode coupling wavelengths and the bands of plausibility the mode orders can be assigned unambiguously. In this case the loss band centered around 1.5 µm can be labeled as the 9th order mode. This then indexes the remaining modes within Fig. 2 as the (left to right) odd numbered modes from the 1st to 9th order, with the even orders having insufficient coupling strength to be visible in the transmission spectrum.

In a more general case identification of the correct mode order will be dependent on the uncertainty (and therefore the width of the plausibility boundaries) in the core refractive index. For particularly high uncertainties the correct identification of the mode order may be difficult, especially with shorter period LPGs. It will in most cases be possible to assign the mode orders unambiguously by expanding the range of wavelengths within the transmission spectrum. Moving to shorter wavelengths will result in more closely spaced cladding mode resonances but will eventually allow for the identification of the 1st order mode. Alternatively moving to higher wavelengths will increase the separation of cladding mode resonances until it is possible to identify the mode order unambiguously even with high uncertainty in core index.

Note that this technique is sufficiently robust to operate with an uncertainty in $\Lambda$ much larger than a typical experimental error in LPG inscription, i.e. <1 µm.

### III. Determination of Core Index

With the mode orders correctly identified it is straightforward to calculate $n_c$. The resonant coupling wavelengths of each mode can be directly measured from the transmission spectrum. This, combined with the corresponding mode order, allows for the calculation of the core index including DC offset via the phase matching condition, (1), at the resonant wavelengths. This calculation provides a series of data points allowing for the refitting of the $n_c$ Sellmeier coefficients, shown in Fig. 4. For SMF-28 fiber, the 1st order mode is below the cut-off wavelength for single mode operation [20] and the low mode multi mode characteristics will affect the $n_e$ of this order. For this reason, the 1st order has been ignored for fitting purposes. The fit has been made by a non linear regression starting from the Sellmeier coefficients of the cladding refractive index. This is not a unique solution; however the small difference between core and cladding index ensures that a local minimum is found with satisfactory fitting over the wavelength range of interest. In the case of the sample LPG, the Sellmeier coefficients of the core are identified as: $A_1 = 0.7040$, $A_2 = 0.4144$, $A_3 = 0.7724$, $B_1 = 0.0736$, $B_2 = 0.1154$, $B_3 = 9.3795$. The accuracy of this process is dependent on the resolution and accuracy of the transmission spectrum but also on the error in the assumed value of $\Lambda$. Errors in calculated core index corresponding to ± 5% in $\Lambda$ (4 µm) are show in Fig. 4. Note that this technique can also be used to identify unknown $\Lambda$ given a known $\Delta n$ e.g. if $\Delta n = 0$.
IV. DETERMINATION OF LENGTH (L) AND STRENGTH (Δn) OF GRATING

With the core refractive index now defined, the only remaining potential unknown properties of the grating are the envelope, \( l \) and \( Δn \). Whereas it is, in principle, possible to fit both of these simultaneously to the transmission spectrum the result is inadequate without sufficient weighting of the various parts of the transmission spectrum. To adequately determine these separately it is necessary to investigate the effect of each parameter on the transmission spectrum.

Fig. 5 shows the effect of independently varying the length and strength of the grating. A change in length is seen to affect not only the coupling efficiency (the depth and form of the resonant peak) but also the formation of the side lobes of the resonant coupling band, particularly when overcoupled. By contrast \( Δn \) has an effect confined almost entirely to the resonant peak. In this example the envelope is well defined as a top hat however the analysis can be applied to the envelope in the same manner as the length, i.e. it primarily affects the wings of the cladding mode resonance. These two regimes, the resonant peak and the side lobes, provide the means to independently fit \( l \) and \( Δn \) to the transmission spectrum by separating the spectrum as shown in Fig. 6.

This then forms the basis for a correlation analysis to fit the length and strength of the grating. A series of theoretical transmission calculations are carried out and a correlation performed between the theoretical and experimental data over the separate regions. The result of the correlation may then be plotted as a function of the length and strength of the grating for the two regions separately, Fig. 7.

Fig. 7a illustrates the dependence of the resonant peak primarily on the strength of the grating whereas Fig. 7b shows the dependence of the side lobes primarily on the length of the LPG. By combining these two correlations it is possible to
identify where the two intersect and thus fit the length and strength of the LPG. This can be done by summing the two sets of data, Fig. 8. This fitting procedure then provides the LPG length as 24.18 ± 0.4 mm (within a period of the LPG) and the grating strength is determined to be 3.2 ± 0.05 x 10⁻⁴ RIU. This then provides all the necessary parameters to plot a theoretical transmission spectrum of the example LPG. Fig. 9 plots the fitted theoretical plot and the measured transmission data for the example LPG.

There is excellent agreement between these data, with the exception of the first order mode, which is below the cut-off wavelength and therefore in a region where the fiber is no longer single mode. This is to be expected since the theoretical model has been fitted to the data however it is also in good agreement with results determined via a matrix modeling method by Nandini et al. (CGCRI Kolkata), which will be published in the near future.

V. METAL JACKETED LPG

In a metal jacketed fiber the external refractive index (n₂) has a complex value. To model this it is necessary to carefully analyze Erdogan’s equations to understand the implications of this property on the modeling. His model is a two stage process; first available cladding modes are determined from the dispersion equation then coupled mode theory is used to model the transmission spectrum.

The equations required to solve the dispersion, in particular, are based on the work of Tsao [21] which is quite general in scope and valid for any circularly symmetric system, including complex refractive indices. The general solution is however rather difficult to process and Erdogan has applied a number of approximations. Care must therefore be taken to investigate each approximation in the presence of a metallic layer with a complex refractive index external to the fiber.

These approximations are best dealt with independently.

The calculation of the core effective index is dependent on a two layer approximation (infinite cladding), whereas the cladding indices are calculated from a three layer approximation (infinite metal layer). To maintain these it is necessary to assume that the metal layer does not induce stress in the fiber (therefore altering the core or cladding mode effective index) and is sufficiently thick that there is no interaction between the radiation and the metal-air interface. This is comparatively easy to achieve with a cold, slow deposition process e.g. RF sputter coating of at least 30 nm [22].

Secondly an approximation has been made in the choice of cladding modes to model. Due to the circular symmetry of the LPG it is necessary only to consider the LP₁₀ cladding modes. This is not altered by the presence of a stress free metal jacket.

Thirdly is the choice of the Bessel functions used in the solution of the cladding mode dispersion equations. These are determined by the sign of the layer phase parameters u₁ and u₂. The details of these parameters can be found in Erdogan [16-18] and Tsao’s [21] work and are not reproduced here. Suffice to say here, that in a general case these parameters can be positive or negative depending on if the effective cladding indices are larger (negative case) or smaller (positive case) than either the core (u₁) or cladding (u₂) index.

In a general case it is possible to determine if the sign of the layer phase parameters has changed by observing the change in the transmission spectrum. The phase matching condition (1) requires that a positive (red) shift in wavelength is the result in a decrease in the effective refractive index of the cladding mode (since the core mode is unaffected). A positive shift in wavelength (as is the case of our example LPG) corresponds to no change in the layer phase parameters whereas a negative (blue) wavelength shift requires a change in at least one layer phase parameter.

The final approximation is that there is no coupling between cladding modes. This is primarily due to the clear wavelength separation of the cladding mode coupling resonances. Again the transmission spectrum of metal jacketed cladding modes indicates clear wavelength separation of the modes and therefore no inter mode coupling.

These considerations allow for the modeling of an effective metal jacket by simply replacing n₂ with the dispersive refractive index of the metal layer. It is possible to
simplify this process by discarding the complex component of the effective modal indices once calculated since they are negligible. Care must be taken to ensure that the refractive index of the metal layer is known with sufficient accuracy as this will be a function not only of the jacket thickness but also the precise deposition technique. This refractive index needs to be determined separately. In the case of our example the complex refractive index was determined through ellipsometry \[22\]. This shows the same general form as the uncoated case, in particular the peak in coupling coefficients occurs over the 7\textsuperscript{th}–19\textsuperscript{th} order modes.

Fig. 10 shows the theoretically calculated coupling coefficients of the first 28 odd ordered cladding modes based on an infinite Pd jacket on SMF-28. The refractive index of Pd has been measured independently by ellipsometry \[22\]. This shows the same general form as the uncoated case, in particular the peak in coupling coefficients occurs over the 7\textsuperscript{th}–19\textsuperscript{th} order modes.

Finally Fig. 12 shows the theoretically calculated and experimentally measured transmission spectrum of our example LPG coated with a 40 nm thick layer of Pd. The resonant wavelength positions are in good agreement, supporting the usefulness and validity of the model. The coupling strengths however differ quite substantially. Recall that the accuracy of the model is based on the accuracy of the period of the LPG. This combined with the absorption within the metal layer, which has not been taken into account, can explain the differences between the coupling coefficients.

VI. CONCLUSIONS

We have presented a method to characterize an LPG based on a correlation analysis between a measured transmission spectrum and a theoretical model. This method is sufficiently robust to allow for the characterization of an arbitrary LPG requiring only knowledge of the period of the LPG from which the post inscription dispersive core refractive index can be derived. This process can also be applied in the reverse, if the dispersive core refractive index is known.

The veracity of the process has been demonstrated on an example LPG where the length, strength and core refractive indices have all been determined accurately, with errors smaller than experimental uncertainties.

The technique has been expanded to allow the modeling of LPGs with metallic jackets. The validity of an extension of Erdogan’s model for metal clad LPGs has been demonstrated. It has been used to model the transmission spectrum of a palladium coated LPG and there is good agreement between theoretical and experimental data. We believe that this technique provides an invaluable tool for the characterization of not only written LPGs but also the LPG writing technique itself.

References

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