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Optical thickness measurement with multi-wavelength THz interferometry

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ABSTRACT

We report unambiguous thickness measurement with an all-optical THz source. The optical thickness variation of a test target was measured in a Mach–Zehnder interferometer to approximately 0.5% of the illumination wavelength using an optical parametric THz laser. The frequency of the laser was continuously tuneable, enabling a synthetic wavelength to be produced by sequential illumination at discrete frequencies, thus extending the unambiguous measurement range to half the synthetic wavelength. The all-optical source provides some advantages with respect to opto-electronic and electronic sources, particularly measurement speed and resolution.

1. Introduction

The potential advantages of metrology at terahertz (THz) wavelengths are well-known. Many common non-polar materials are semi-transparent at THz wavelengths, including silicon, plastics, paper, cardboard, wood and clothing. The wavelength is long enough to avoid harmful ionizing effects but short enough to provide sub-millimetre spatial resolution for many imaging applications. Strong characteristic absorption wavelengths enable spectroscopic identification of specimens, whilst polar liquids (e.g. water) and metals are opaque.

THz time domain spectroscopy (THz-TDS) is the most widely used technique at THz wavelengths [1–3], and has been adapted for imaging [4]. Typically the opto-electronic source is a photoconducting dipole antenna illuminated with a femtosecond (fs) laser pulse that produces a broadband THz pulse with a sub-picosecond rise time [5], [6]. Coherent detection (i.e. the amplitude and phase) of the THz pulse is achieved with a second antenna that is gated by the same fs laser via a mechanically scanned delay line. THz-TDS has been implemented for optical thickness [7] and range [8] measurements, where detection of the zero crossing of the electric field of the single-cycle pulse enables ~1 μm depth resolution, determined by the positioning accuracy of the delay line. The mean output power of such systems is relatively low, making the imaging speed slow when the spectroscopic information is not required.

The delay line can be eliminated whilst retaining coherent detection in THz-TDS. Wilk et al. [9] measured the thickness variation of a polyethylene wedge using a continuous wave THz spectrometer operating at 300 GHz based on two dipole antennas (generation and detection) both driven by two laser diodes. The detected photocurrent was recorded with and without the sample, and the thickness determined from the measured phase delay and the known refractive index. In order to measure thicknesses greater than the THz wavelength, Scheller et al. [10] used a similar system but with the dipole antennas illuminated simultaneously at three wavelengths to generate and detect three THz signals. The phase delay introduced by a polyethylene stepped wedge sample was measured for the synthetic difference frequency between the two highest THz frequencies (380 and 447 GHz). Simultaneous illumination by THz frequencies avoided the need to perform sequential phase measurements at each frequency. Zhang et al. [11] had previously introduced a synthetic difference frequency approach with traditional pulsed, broadband THz-TDS. The optical thickness of a polyethylene wedge was measured and it was noted that the noise at the composite wavelength was reduced because the noise at the constituent wavelengths was recorded simultaneously and was therefore correlated.

Electronic THz sources generally provide higher output powers. They have been used with direct detection for thickness and profile measurements, with the ultimate goal of achieving real-time imaging, although this has still not been realised to date. Hils et al. [12] used a hybrid system comprising a microelectronic Gunn oscillator operating at 600 GHz and dual coherent electro-optic

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detectors. The two detectors directly compared the phase of the THz reference beam with the phase of the THz wave scattered from an object. The concept is analogous to heterodyne detection, with the fs laser triggering the electro-optic detectors acting as a local oscillator, obviating the need for mechanical scanning. Lu et al. [13] used a Gunn oscillator operating at 320 GHz launched into sub-wavelength diameter polyethylene fibre and recorded the Fabry–Perot interference intensity between a test surface and the fibre end with a Golay cell. Wang et al. [14] used two THz sources simultaneously, a Gunn diode operating at 100 GHz and a backward wave oscillator operating at 120 GHz, with a pyroelectric detector in a Michelson interferometer. One arm of the interferometer was scanned and the phase change at the synthetic wavelength with and without the target was used to calculate the optical thickness of polyethylene wedge and stepped targets.

All-optical THz sources can also provide high output powers, but at shorter wavelengths thus increasing the resolution. Each type of optical THz source has particular advantages and disadvantages, concerning output power, room-temperature operation, cost and wavelength tunability [15]. The only all-optical THz source used for thickness measurement to date was reported by Wang et al. [16] who used an optically-pumped far-infrared laser operating at 2.52 THz. The interference intensity from a Michelson interferometer was recorded with a Golay cell and the optical thickness measured at discrete points for a polyethylene wedge. The depth was determined from the known refractive index, but the absolute depth of step heights that exceeded the THz wavelength could not be calculated.

In this paper we demonstrate the first, to the best of our knowledge, unambiguous thickness measurement with an all-optical THz source. We used an optical parametric THz laser that is continuously tuneable over a large frequency range at room temperature and has a reasonably narrow-bandwidth. The all-optical source enabled direct detection that reduces the measurement time compared to opto-electronic sources, and does not require the specimen to be placed in a chamber with a dry atmosphere. It operates at a shorter wavelength than electronic sources thus increasing the resolution. Tuning this source is considerably easier than for a far-infrared laser, enabling the absorption lines of atmospheric water vapour to be avoided and sequential illumination at discrete THz frequencies to produce a synthetic wavelength for unambiguous thickness measurement.

2. Theory

The intensity I recorded by a detector at position (x) in a typical two-beam interferogram can be represented by

$$I(x) = I_O(1+\gamma_O \cos \phi) \tag{1}$$

where $I_O(x)$ is the bias intensity and $\gamma_O(x)$ is the visibility that depends on the relative intensities of the two beams, their coherence, and the state of polarization. When light from one of the interferometer beams undergoes a single pass through a transparent object with constant refractive index, the interference phase $\phi(x)$ between the beams changes by an amount

$$\Delta\phi_1(x) = k_1 t \Delta n_1 \tag{2}$$

where $k_1 = 2\pi/\lambda_1$ is the wave number at wavelength λ_1, t is the geometrical thickness of the object and Δn_1 is the difference in refractive indices of the object material and the surrounding medium at wavelength λ_1. The optical thickness $t\Delta n_1$ is ambiguous to multiples of $\lambda_1/2$. When the geometrical thickness of the object is known, measurements made with and without the object enable its refractive index to be determined from the optical thickness [17–18]. Similarly, when the refractive index is known, the geometrical thickness can be recovered from the measured optical thickness [19]. Simultaneous measurement of the geometrical thickness and the refractive index generally require two independent techniques to be applied, for example interferometry and confocal imaging [20–21] or the use of multiple reflections within the specimen [22] and is not considered in this paper.

The change in phase produced when the object is measured at a second wavelength, λ_2, is given by

$$\Delta\phi_2(x) = k_2 t \Delta n_2 \tag{3}$$

The phase difference between measurements at two different wavelengths can be used to extend the unambiguous range of the thickness measurement:

$$\begin{aligned}\Delta\phi_2 - \Delta\phi_1 &= 2\pi\frac{\lambda_1-\lambda_2}{\lambda_1\lambda_2}t\frac{\Delta n_1+\Delta n_2}{2}\left\{1-\frac{\lambda_1+\lambda_2}{\Delta n_1+\Delta n_2}\frac{\Delta n_1-\Delta n_2}{\lambda_1-\lambda_2}\right\}\\ &= 2\pi\frac{\lambda_1-\lambda_2}{\lambda_1\lambda_2}t\Delta n_g\end{aligned} \tag{4}$$

For dispersive media, eq. (4) contains a linear approximation to the difference of the group refractive indices Δn_g of the general form:

$$\Delta n_g = \Delta n\left\{1-\frac{\lambda}{\Delta n}\frac{d(\Delta n)}{d\lambda}\right\} \tag{5}$$

For non-dispersive media $d(\Delta n)/d\lambda = 0$, in which case the group and the phase refractive index differences are equal, $\Delta n_g = \Delta n$. Then eq. (4) simplifies to

$$\Delta\phi_2 - \Delta\phi_1 = 2\pi\frac{\lambda_1-\lambda_2}{\lambda_1\lambda_2}t\Delta n \tag{6}$$

In both cases, the ambiguity in optical thickness $t\Delta n_g$ or $t\Delta n$ is increased to multiples of $\lambda_s/2$ where the synthetic wavelength, $\lambda_s = \lambda_1\lambda_2/(\lambda_1-\lambda_2)$.

3. Experimental set-up

An intra-cavity optical parametric THz laser was used, shown schematically in Fig. 1. A Q-switched, Nd:YAG laser provided the pump-wave for the parametric oscillator. The Nd:YAG laser gain crystal was end-pumped by a fibre-coupled, quasi-continuous-wave diode laser operating at a wavelength of 808 nm. Standard quarter-wave Q-switching using an electro-optic Q-switch (QS) in combination with a quarter-wave plate ($\lambda/4$) and a linear polarizer (P) produced pump-wave pulses of typical duration 10–20 ns. The Nd:YAG laser cavity is defined by mirrors M1 and M2. The nonlinear medium Mg:LiNO$_3$ providing the parametric gain was located within the pump laser cavity [23], and the parametric oscillator cavity is defined by mirrors M3 and M4. A silicon prism array attached to one side of the nonlinear medium coupled the terahertz output into free-space. The laser was operated at 52 Hz pulse repetition rate, with pulses of duration 10 ns and energy of 10 nJ per pulse (peak power ~1 W and 0.5 μW mean power). The spectral bandwidth was approximately 100 GHz, corresponding to a coherence length of approximately 3 mm. The THz output frequency was adjusted by rotating mirrors M3 and M4 together to modify the angle between the idler-wave cavity and the pump-wave cavity: continuous wavelength selectivity in the range 0.7–2.5 THz was achieved by changing the angle of the pump beam in

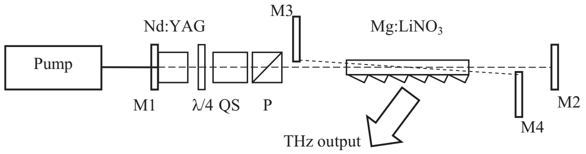

Fig. 1. Schematic of the intra-cavity optical parametric THz laser.

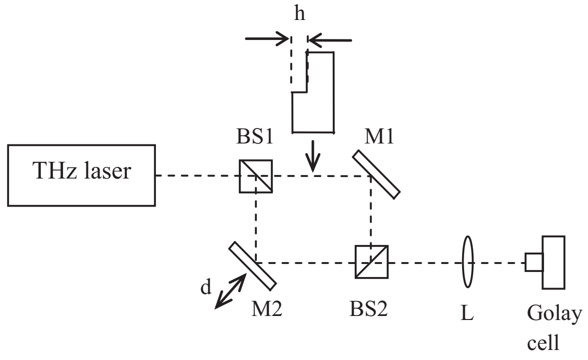

Fig. 2. Schematic of the interferometer used for thickness measurement. The test object with a step height of $h=235\ \mu\text{m}$ is shown.

the non-linear crystal. The intra-cavity position of the nonlinear medium leads to an order of magnitude reduction in the pump power required to reach oscillation threshold, accompanied by greater than an order of magnitude increase in the energies of THz pulses generated compared to conventional extra-cavity devices.

A Mach–Zehnder interferometer was constructed using the THz laser, Fig. 2. The interferometer was aligned initially with a HeNe laser (not shown), after which the THz beam was introduced. The interferometer comprised two 23 μm thick Mylar sheet beam splitters (BS1 and BS2) and gold-coated mirrors (M1 and M2). The THz beam diameter in the interferometer was approximately 10 mm, which was reduced to 2 mm on a Golay cell detector (Tydex GC-1 P). The detector output voltage was filtered and amplified by a lock-in amplifier (Stanford Research Systems SR530). The reference beam mirror M2 was mounted on a linear translation stage (positional accuracy ~1 μm) to change the optical path length of the reference beam. Moving M2 by distance d produced a path length change ($2d\cos\theta$), where θ is the angle of incidence of the laser beam on mirror M2. The object tested was $66\times108\ \text{mm}^2$ polytetrafluoroethylene (PTFE or Teflon) block of nominal thickness 5 mm. Material was milled from one-half of the block to produce a step height of 235 μm (measured with a micrometre gauge).

4. Results and discussion

Fig. 3(a) shows phase-stepped intensities recorded at 0.9520 THz (wavelength 315.13 μm) for the 'thin' section of the object, i.e. milled half of the object. The phase was stepped through approximately π radians by moving mirror M2 in 10 μm steps. Fig. 3 (b) shows the numerical differences between intensity measurements recorded at adjacent mirror positions, around the maximum and minimum fringe peaks of Fig. 3(a). The purpose of calculating the intensity differences was to determine positions of the maximum and minimum intensity, where the local intensity gradient is zero. As shown in Fig. 3(b) for the thin section of the object, linear fits were applied to the intensity differences and the fringe maximum and minimum positions were identified from zero crossing positions. The movement of mirror M2 between the fringe maximum and minimum for the thin section was Δd=213.0–110.6 μm=102.4 μm corresponding to a measured wavelength of $\lambda_1=313.8\ \mu\text{m}$ for $\theta=40^\circ$, i.e. an error of 0.4% of the laser wavelength. For an ideal cosine, this linear fit method to the intensity difference of $\pm\pi/4$ rad around a maximum (or minimum) is accurate to <0.1% for the position of the maximum (or minimum).

The object was then moved to illuminate the 'thick' section of the block, i.e. the non-machined half of the object. Fig. 3(a) shows the phase-stepped intensities recorded in the region of the fringe minimum only. Fig. 3(b) shows the intensity differences and the position of the minimum identified from the linear fit. As shown in the figure, the measured change in the mirror position at the fringe minima between the thick and thin sections of the object was Δd_1=213.0–145.3 μm=67.7 μm, corresponding to an optical thickness $t\Delta n_1$ of 103.6 μm. Teflon is non-dispersive and has a refractive index of $n_g=n=1.445$ that varies by less than 6% over 0.2–3 THz range [24]. Therefore using $\Delta n_1=\Delta n=0.445$ for the object in air, the measured geometrical thickness of the step height was 232.9 μm, i.e. difference of 0.7% of the laser wavelength compared to the micrometre measurement. Clearly the measurement is ambiguous to multiples of half the THz wavelength.

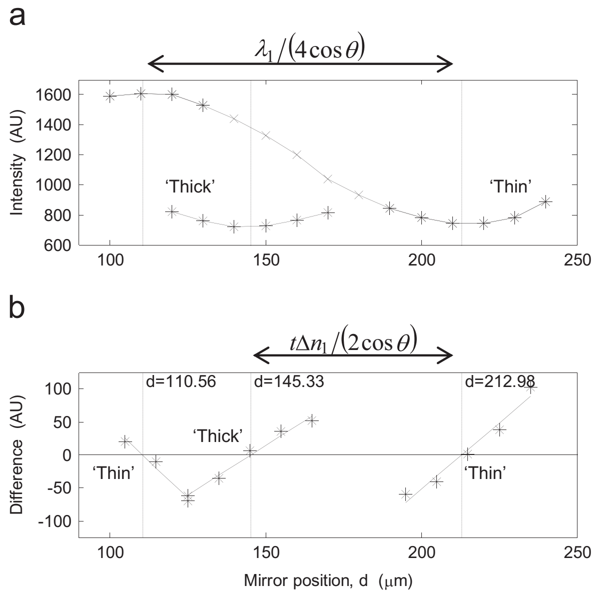

Fig. 3. (a) Phase-stepped intensity plotted against mirror position at 0.9520 THz for 'thick' and 'thin' sections of object. (b) Intensity difference between adjacent measurements and linear least squares fit in order to determine the position of fringe maximum and minima.

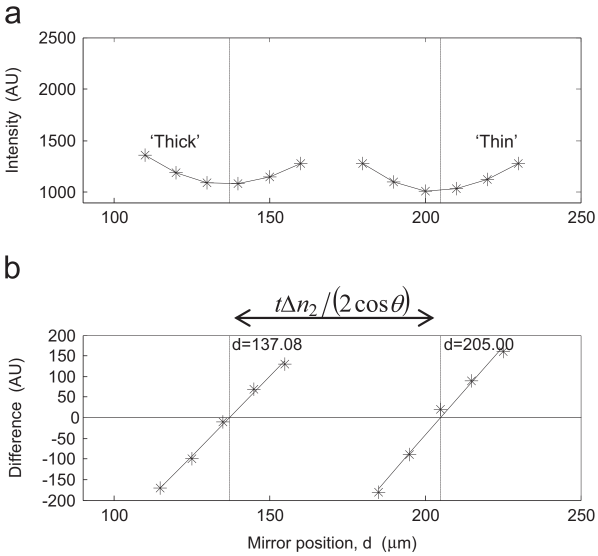

Fig. 4. (a) Phase-stepped intensity plotted against mirror position at 1.1063 THz for 'thick' and 'thin' sections of object. (b) Intensity difference between adjacent measurements and linear least squares fit in order to determine position of fringe minima.

The measurement was repeated with the laser tuned to 1.1063 THz ($\lambda_2=271.14\ \mu\text{m}$). Fig. 4(a) shows the phase-stepped intensities recorded in the region of the fringe minima for the thick and thin sections of the object. Fig. 4(b) shows the positions of the fringe minima determined from the intensity difference. As shown in the figure, the measured change in the mirror position at the fringe minimum between the thick and thin sections of the object was $\Delta d_2 = 67.9\ \mu m$, corresponding to an optical thickness $t\Delta n_2$ of 104.1 μm. Therefore the measured geometrical thickness of the step height was 233.8 μm, i.e. difference of 0.4% of the THz wavelength compared to the micrometre measurement. Again, this measurement is ambiguous to multiples of half the THz wavelength.

The phase differences $\Delta\phi_1$ and $\Delta\phi_2$ due to the step height can be calculated from the optical thicknesses $t\Delta n_1$ and $t\Delta n_2$ at each illumination wavelength, respectively, eqs. (2) and (3). Hence eq. (6) can be used to determine the measured step height from the synthetic wavelength ($\lambda_s = 1942.4\ \mu m$) to be 239.8 μm. The synthetic wavelength in measurement is 'coarse' because it magnifies the noise in the phase measured at each wavelength by the same factor as the magnification of the wavelengths. It is more accurate to divide the synthetic wavelength step height into integer multiples of one of the wavelengths, for example λ_1, and add that number of integer wavelengths to the λ_1 measurement. For the current example, the synthetic wavelength demonstrates unambiguously that no correction is required to the measured optical thickness at either of the individual wavelengths. The unambiguous measurement range of the optical thickness has been extended to half the synthetic wavelength.

5. Conclusions

An intra-cavity optical parametric THz laser was used to measure the optical thickness variation of a test target in the Mach–Zehnder interferometer. The output frequency of the laser was tuned to 0.9520 and 1.1063 THz in order to generate a synthetic wavelength and the unambiguous measurement range was extended to seven wavelengths. An accuracy of approximately 0.5% of the THz wavelength was achieved over the extended measurement range for the step height. These are the first unambiguous thickness measurements with an all-optical THz source, to the best of our knowledge. The optical parametric THz laser enabled direct detection to reduce the measurement time compared to opto-electronic sources, and does not require the specimen to be placed in a chamber with a dry atmosphere. It operates at a shorter wavelength than electronic sources thus increasing the resolution. Tuning the optical parametric THz laser was straightforward enabling the absorption lines of atmospheric water vapour to be avoided. Having demonstrated the suitability of the source for interferometric measurements with direct detection, we are currently implementing a system for full-field measurements.

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