An investigation into flow boiling heat transfer and pressure drop in a pin-finned heat sink

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Abstract

Heat-transfer coefficient and pressure drop measurements are reported for a heat sink comprising pin-fins with a cross section of 1 mm by 1 mm and a height of 1 mm. The pin-fins were manufactured on a 50 mm square base plate in a square, in-line arrangement with a pitch of 2 mm. The data were produced while boiling de-ionised water at atmospheric pressure. The mass flux range was 40 - 200 kg/m²s and the heat flux range was 30-470 kW/m².

The test section was heated from below by an electrical heating method that is normally associated with a constant heat flux boundary condition. However, because of the variation in single-phase liquid temperature in the entrance zone and the variation in the heat-transfer coefficient, the interceding aluminium and copper material is shown to produce a non-uniform heat flux with a near isothermal wall boundary condition. The heat conduction effect in the wall is taken into account in the analysis of the data and in the calculation of the heat-transfer coefficients.

Heat-transfer coefficients and pressure drops are reported for single-phase and boiling flows, with subcooled and saturated boiling data obtained. The single-phase results are shown to be reasonably independent of position. The measured boiling heat-transfer coefficients are shown to be associated with flow pattern and to be confined, as the capillary length for water is much larger than the flow passage dimensions. This is true of the saturated and subcooled boiling data, as slugs were observed in subcooled liquid flows. The criterion for the onset of slug flow is not compatible with that associated with macro-scale flows. All of the boiling data were obtained at wall superheats...
substantially above the values associated with nucleate boiling. They therefore have a nucleate and convective component. The convective component is shown to be the most dominant.

The water data are compared with R113 data obtained in a previous study. The R113 data are shown to be associated with slug and annular flow, and are, in the main, not confined, with its capillary length similar to the channel flow dimensions. The R113 flow pattern transition criteria are comparable to macro-scale values. Some consistency in behaviour is shown to exist in the slug heat-transfer coefficients obtained with water and R113.

1. Introduction

Flow boiling in mini- and micro-flow passages can be used for cooling many high power density devices such as Micro Electro Mechanical Systems (MEMS), microprocessors, laser diode arrays and Light Emitting Diodes (LEDs). This has led to an abundance of research into flow boiling in small-scale flow passages. Many researchers have investigated flow boiling heat transfer phenomena with the objective of developing reliable design models for mini and micro-flow passages. Nevertheless, there is still a lack of understanding of the phenomena involved. Hence there is some doubt about the reliability of the new models and when they should be applied.

Boiling studies of complex geometries have been undertaken at the macro-scale, for example Ishahara et al (1980), Shrage et al (1988), Dowlati et al (1990), Dowlati et al (1992) and Feenstra et al (2000). These studies have investigated flow patterns, heat transfer, pressure drop, and void fraction characteristics for flow across a bank of conventional size tubes. A state-of-the-art review of research on two-phase flow and flow boiling across conventional horizontal and small tube bundles has been reported by Ribatski and Thome (2007).

Recent development in micro-fabrication techniques have allowed small scale complex geometries such as mini- and micro-pin fin surfaces, drilled cavities, re-entrant cavities and alumina sprayed particles to be fabricated and explored. A review of various surface enhancement methods has been reported by Honda and Wei (2004) for nucleate boiling heat transfer. Honda and Wei (2004) reported that the heat transfer and critical heat flux were found to improve with enhanced area, but not in direct proportion. Critical heat flux was found to increase with increased subcooling. The most effective enhancement method was found to be pin fins. This gave
enhanced heat transfer because vapour trapped between the fins provided additional nucleation sites. Additionally, these spaces retained vapour for longer, giving enhanced heat transfer. The optimum spacing of the fins depended on liquid subcooling. Thus, mini- and micro-pin fin arrays are a promising structure. There are a number of studies of single-phase convective heat transfer in micro-pin fin arrays, such as Peles et al. (2005) and Qu and Siu-Ho (2008a, 2008b). However, fewer studies have been undertaken on flow boiling heat transfer in micro-pin-fin arrays.

Honda et al. (2002) undertook nucleate boiling studies on an in-line array of 50 $\mu$m square pin fins using FC-72 at atmospheric pressure as the working fluid. The pin fins were 60 $\mu$m high with a 100 $\mu$m square pitch, located on a 10 mm square surface. Heat fluxes of up to 620 kW/m$^2$ were used with liquid sub-coolings of up to 45$^\circ$C. The additional fin area was not included in the calculation of the heat-transfer coefficient. In the free convection region, for the same wall-to-fluid temperature difference, the pin fins had a lower heat flux than a flat surface, despite having an area enhancement of 2.2. This was because the pin fins lay below the thermal boundary layer. At higher temperature differences, the pin fins produced heat fluxes up to 1.8 times larger with a greater critical heat flux. They reported that high wall-to-fluid temperature differences, caused by trapped vapour between the pin fins, led to a rapid growth in heat flux.

Lie et al. (2007) investigated flow boiling of FC-72 on plain and pin finned surfaces at atmospheric pressure. Two pin-fin surfaces were tested, one with fins 200 $\mu$m square and 70 $\mu$m high on a square in-line arrangement with a 400 $\mu$m pitch, and one with fins 100 $\mu$m square and 70 $\mu$m high on a square in-line arrangement with a 200 $\mu$m pitch. The mass fluxes used were within the range 287 – 431 kg/m$^2$s and the heat fluxes within 1 – 100 kW/m$^2$. Single-phase and flow boiling heat-transfer coefficients were reported. The mass flux was found to have only a slight effect on the boiling heat-transfer coefficient. The temperature at the onset of nucleate boiling was found to increase with increasing mass flux and, for the same wall-to-fluid temperature difference, the largest heat transfer occurred with the 100 $\mu$m fins.

Kosar and Peles (2007) investigated flow boiling using micro pin fins manufactured as NACA 66-021 hydrofoils. The fins were in a staggered arrangement with 12 or 13 fins lateral to the flow on a lateral pitch of 150 $\mu$m and 20
fins in the flow direction with a pitch of 500 μm. Each fin had a chord length of 100 μm, a length of 500 μm, a wetted perimeter of 1.03 mm and a height of 243 μm. The working fluid was R-123 at pressures between 486 and 539 kPa. Single-phase, partial boiling and fully-developed boiling ranges were identified. The boiling heat-transfer coefficient was found to increase in the assumed nucleate boiling region and decrease in the assumed convective boiling region to critical heat flux. From their visual observations, bubbly, wavy intermittent and spray-annular flow patterns were identified depending on the heat flux and mass velocity. They constructed a flow map to specify the transition boundaries between each flow pattern.

Krishnamurthy and Peles (2008) reported flow boiling in circular micro-pin fins having a diameter of 100 μm and a height of 250 μm, using water as the working fluid at atmospheric pressure. The fins were in a staggered arrangement with 11 or 12 columns and 68 rows. Tests were carried out by setting a constant mass flux between 346 and 794 kg/m²s and increasing the voltage to the heater in steps of 0.5 V until critical heat flux occurred. The single-phase heat-transfer coefficient was found to be proportional to the mass flux, while the two-phase heat-transfer coefficient was found to vary moderately with mass flux and to be independent of heat flux, suggesting convective boiling domination. Only slug and annular flow patterns were observed.

Krishnamurthy and Peles investigated water (2007) and Krishnamurthy and Peles (2009) investigated nitrogen-ethanol flows across an array of circular pin fins, 10 μm diameter and 10 μm high. The fins were configured in a staggered array with a longitudinal and transverse pitch of 150 μm on a surface of 1.5 mm wide and 10 mm long. Surface tension effects were deduced by comparing the results of two studies. Water has a similar density and viscosity to ethanol but a much higher surface tension. The flow pattern transition boundaries were found to depend on surface tension. The pressure drop was also affected because of its flow map dependence. Except for a small range of gas-mass fractions, the void fraction was found to be reasonably independent of surface tension.

Qu and Siu-Ho (2009a) investigated water boiling at atmospheric pressure in a square pin-fin array. The test piece was 33.8 mm long and 10 mm wide and contained 1950 fins 200 μm wide and 670 μm high on a staggered configuration with a lateral and transverse pitch of 400 μm. The ratio of the total to the base area was 3.9. Fin efficiency was used to include the pin-fin areas in the heat-transfer analysis. Water with mass fluxes of 183 –
420 kg/m²s and with subcoolings of 10, 40 and 70 K were supplied to the test piece. Heat fluxes ranged from 23.7 to 248.5 W/cm². Three in wall thermocouples allowed local heat-transfer coefficients to be deduced. Partitioning the test piece into sub-cooled and saturated liquid portions from a heat balance allowed single and two-phase heat transfer to be identified. Only the saturated heat-transfer coefficients obtained at exit gas-mass fractions greater than 0.01 were reported. The saturated boiling values had unusual characteristics, being insensitive to mass flux and heat flux, but not exit quality. The observed flow pattern was said to be annular which suggested that the boiling heat-transfer mechanism was convective. The lack of mass flux dependency was explained in terms of a constant liquid film induced by the pin fins, and sub-cooled entrained liquid. The data were correlated in terms of an ‘equilibrium’ heat-transfer coefficient and the exit gas-mass fraction.

Qu and Siu-Ho (2009b) investigated flow boiling pressure drop of water in the same micro-pin-fin heat sink with similar operating conditions as their previous study, Qu and Siu-Ho (2009a). They reported that, the total two-phase pressure drop in the micro-pin fin surface was affected strongly by the rate of vapour generation, with the two-phase pressure drop significantly increased in the saturated boiling regime. The two-phase pressure drops were dominated by friction. Ten existing two-phase pressure drop correlations were compared to the flow boiling pressure drop across the micro-pin fins. The Lockhart–Martinelli correlation for laminar liquid–laminar vapor combination in conjunction with a previous single-phase friction factor correlation provided the best agreement with the data.

Kosar et al. (2010) studied flow boiling pressure drop of water and R-123 across three different micro-pin fin heat sinks under unstable boiling conditions. Three different pin fin arrangements were examined; (a) circular pin fins having a diameter of 100 μm and a height of 243 μm, were placed in a staggered configuration with a pitch of 150 μm, using R-123 as the working fluid, (b) similar circular pin fins were placed in a staggered configuration with longitudinal and transverse pitches of 350 μm and 150 μm respectively, using water as working fluid, and finally (c) Hydrofoil pin fins having a chord thickness of 100 μm, length of 500 μm and a height of 243 μm, were placed in a staggered configuration with longitudinal and transverse pitches of 500 μm and 300 μm respectively, using water as the working fluid. They used flow images and Fast-Fourier Transforms (FFT) of the pressure signals during flow boiling to investigate the experimental results. They reported that flow instabilities during flow boiling in micro-pin fin surfaces occur in a manner similar to those in parallel micro-channel surfaces. The magnitude of the
pressure drop fluctuations for water tests was lower than that for R-123 tests. Unlike the R-123 tests, the water tests showed no significant change in the FFT profiles with unstable boiling. The high frequency of the oscillations for the R123 tests was in agreement with the frequency of the rapid bubble growth instabilities.

Chang et al. (2010) investigated sub-cooled flow boiling heat transfer and associated bubble characteristics of FC-72 on a heated micro-pin-finned silicon chip. The test sections used were the same as Lie et al. (2007). The test piece was a rectangular channel, 20 mm wide, 5 mm high and 150 mm long, containing a 10 mm square silicon chip, located at the central bottom surface. Three different geometries of micro-structures on the silicon chips were examined; a smooth surface and two micro-structured surfaces in the form of square micro-pin fins. The two pin-finned surfaces contained micro-pin fins 100 and 200 μm square, 70 μm high, with a fin spacing equal to the width of the fins. The mass flux used was within the range 287 – 431 kg/m²s, the heat flux was within the range 1 – 100 kW/m² and the inlet sub-cooling varied from 2.3 to 4.3 °C. Single-phase and sub-cooled flow boiling heat transfer coefficients were reported. The sub-cooled flow boiling heat transfer coefficient was reduced at increasing inlet liquid sub-cooling. The mass fluxes were found to have only a slight effect on the sub-cooled flow boiling heat transfer coefficient. The single-phase convection and flow boiling heat transfer coefficients were significantly higher for the pin-fin surfaces than for the smooth one. The mean bubble departure diameter and active nucleation site density reduced with increasing mass flux and inlet liquid sub-cooling. The authors proposed empirical correlations for the heat transfer and bubble characteristics.

Xue et al. (2011) studied heat transfer for FC-72 on a silicon chip with micro-pin-fins in three different systems, including pool boiling, natural circulation flow boiling, and forced flow boiling. The test piece was a square P-doped N-type silicon chip, 10 mm wide, 10 mm long and 0.5 mm high. Two surfaces were tested, one a smooth chip, chip S, and one with micro-pin-fins 30 μm square and 60 μm high, chip PF30-60. For the forced flow boiling system, FC-72 was supplied to the test section at velocities of 0.5 and 1 m/s. The liquid sub-coolings were 25 and 35 K. It was reported that, in all three test systems, the micro-pin-fin surface significantly increased the heat transfer relative to the smooth surface. Wall superheat for forced flow boiling tests was smaller than pool boiling and natural circulation flow boiling tests. At the fluid velocity of 1 m/s, the forced flow boiling critical heat flux was the highest.
The previous flow boiling heat transfer studies in micro-pin fin arrangements investigated fins and wires in the range 0.2 – 200 μm in diameter, while other relevant studies in heat exchangers investigated tube bundles containing tubes with diameters of up to 20 mm. To investigate the gap in the geometry parameters between these studies, McNeil et al. (2010) undertook a study in a pin-fin geometry of an intermediate size. Flow boiling heat transfer and pressure drop data were reported for R113 flowing in an array of square pin-fins with a 1 × 1 mm cross-section and a 1 mm height. The pin-fins were placed on a 2 mm square pitch in-line array. They found that the saturated boiling heat transfer coefficient was dependent on heat flux and reasonably independent of mass flux and that the two-phase pressure drop could be predicted by tube bundle methods. However, they did not analyse their subcooled boiling data and they assumed, as is assumed in most studies, that electrical heating led to uniform heat fluxes. McNeil et al. (2013) analysed similar flows in similarly scaled parallel channels placed on the same substrate and found that heat conduction in the heating substrate was significant. In fact, substrate conduction has recently been identified as one of the phenomena that should be taken into account, Szczukiewicz et al. (2014). This study was undertaken to expand the analysis of McNeil et al. (2010) to include substrate conduction, subcooled boiling and to include the effect of fluid properties. R113, when boiling near atmospheric pressure, has a capillary length of about 1 mm, which is close to the width of the square flow channel. The later objective was therefore achieved by performing similar tests with deionised water, a fluid that has very different fluid properties from R113 and with a capillary length of about 2.5 mm when boiling near atmospheric pressure, significantly larger than the channel width.

2. Description of the test facility

The flow loop is shown schematically in Figure 1. Liquid water was drawn from the accumulator by the Micro-motion magnetic drive gear pump. Valves in the by-pass and main lines allowed the desired mass flow rate to be set. Liquid passed from the pump to a filter. The coarse filter was used during commissioning to remove large debris. The finer filter was used during testing. Liquid passed from the filter to a metric series rotameter, where the liquid flow rate was measured to within ±2% of reading. From the flow meter, the liquid passed through a pre-heater, where its temperature was set to obtain a subcooling of typically 3-13 °C, and a sight glass, where visual observation confirmed a liquid entry to the test section. A second sight glass and valve were used for fine de-gassing control. Boiling occurred in the test section. Sheathed, K-type thermocouples
were located upstream and downstream to allow the inlet and outlet temperatures to be measured. Fluid from the test section passed to a t-piece, where the liquid and vapour were separated. The vapour moved vertically upwards and was condensed in the condenser before being re-united with the separated liquid that had moved vertically downwards. The re-united liquid flow passed through a sub-cooler before returning to the accumulator.

The test section is shown in Figure 2. Liquid entered the inlet plenum of the test-section through the two inlet ports set at 90° to the direction of flow through the heat sink. The plenum chamber dimensions were set to reduce the liquid velocity to close to zero before it accelerated into the settling length upstream of the heat sink. Pressure tappings were located just upstream and downstream of the heat sink. The liquid pressure was measured at the upstream tapping with a 0-12 bar SENSIT OEM absolute pressure transducer, accurate to 0.2% of reading. The pressure difference across the tappings was measured with a Rosemount differential pressure transducer, model 3051C. This was a smart transducer that allowed the pressure drop range to be set prior to testing. The transducer was accurate to ±0.25% of range. The fluid boiled as it passed across the heat sink. Heat was supplied to the heat sink from a Watlow Ultramic ceramic heater. The heater was 50 mm square and was placed below the test piece. The heater was fixed to the test section by a securing plate. A PTFE block was located between the securing plate and the heater to minimize heat transfer from the lower surface of the heater. Power to the heater was adjusted to give the required heat flux. The heat load was measured with a RS wattmeter to within ±1% of reading.

The heat sink is shown in Figure 3. It was constructed from a piece of copper, 50 mm wide by 50 mm long and 6 mm high. The channels were formed by cutting slots 1 mm wide and 1 mm deep in the longitudinal and translational directions. The slots were 1 mm apart. Two holes, 0.6 mm in diameter and 12.5 mm long, were drilled into the heat sink at the inlet and outlet ends. The holes were located 2.5 mm from the top of the boiling surface and 11 and 39 mm from an edge. These holes allowed four sheathed K-type thermocouples, 0.5 mm in diameter, to be located below the boiling surface. All thermocouples were calibrated in a water bath and were accurate to ±0.1 K.
The thermocouples were connected to a NI 9211 thermocouple differential unit that was connected to a NI 9172 data acquisition system. The pressure transducers were connected to an NI 9205 system. Both systems were connected to a PC and controlled by Labview software. The thermocouple, pressure and pressure-drop readings were obtained over a 40 s period, during which 2000 readings of each were obtained. This gave reproducible average values for all. The mass flow readings were read and entered into the PC manually.

The fluid was boiled vigorously for 2 hours before a test series. During this period the vent valve above the condenser was periodically opened to allow dissolved gases to escape to the atmosphere. This also set the test pressure to near atmospheric.

The pre-heater was attached to a controller so that the required inlet temperature could be set and controlled. Tests were conducted by setting the required liquid flow rate, setting the required inlet temperature and circulation the liquid through the flow loop until the required entry temperature was achieved. This took approximately two hours. The test section was then set to the desired heat flux. Steady state conditions were achieved when the fluid outlet and the aluminium body temperatures were shown to be stable. This took approximately 35-40 minutes. All of the required readings were obtained before the heat flux was re-set to the next desired value and the process repeated.

3. Heat losses in the test-section

The heat flow from the heater to the test piece is proportional to the difference between the heater temperature, \( T_h \), and the test-piece surface temperature, \( T_w \). The relationship between this temperature difference and the heat loss was established by obtaining data taken when the wall superheat was below that required for the onset of nucleate boiling, which was assumed to be that given by Sato and Matsumura (1963), i.e.

\[
q_L = \frac{k_L h_{fg} \rho_v (T_w - T_{sat})^2}{8 \sigma_{sat}}
\]  

(1)

where \( k_L \) is the liquid thermal conductivity, \( h_{fg} \) is the enthalpy of evaporation, \( \rho_v \) is the vapour density, \( \sigma \) is the surface tension, \( T_{sat} \) is the saturation temperature and \( q_s \) is the local surface heat flux. The average base heat flux, \( q_{Bh} \), the heat flux based on the base area of the heat sink, was determined from the ratio of the increase in sensible heat of the liquid to the base area of the test-piece and was correlated by
\( q_b = 8.0(T_h - T_w) \)  

(2)

where the heat flux is in kW/m\(^2\) and the temperatures are in Kelvin. The sensible heats were estimated for mass fluxes in the range 1.5-4 g/s that gave temperature rises of 3-42 K. If the transmission efficiency is taken as the ratio of the base heat flux to the heat flux applied by the heater, Equation (2) led to transmission efficiencies of 86-99 %, depending on the mass flux. The rms difference between Equation (2) and the measured heat fluxes was 4.6%, with a maximum difference of 9% occurring at the lower heat fluxes.

4. Data reduction

The working fluid was delivered to the test piece in a sub-cooled state. The local heat-transfer coefficients could therefore fall into one of three categories, single-phase convection, subcooled boiling or saturated boiling. Irrespective of the mechanism, the heat-transfer coefficient was determined in a similar way. Two heat flow paths from the heater to the fluid were identified, Figure 4. The first was through the base of the heat sink and the second was through the walls of the pin-fins. These heat paths allowed the fin efficiency approach to be used. A one-dimensional heat balance through a repeating pitch of flow length at the base of the channel gave

\[
q_b(W_{ch} + W_w)^2 = \left[ \alpha_c(T_w - T_L) + \alpha_{nb}(T_w - T_{sat}) \right] \left[ (W_{ch} + W_w)^2 - W_w^2 + 4\eta W_w H_{ch} \right]
\]

(3)

where \( q_b \) is the local base heat flux, \( \alpha_c \) is the convective heat-transfer coefficient, \( \alpha_{nb} \) is the nucleate boiling heat-transfer coefficient, \( \eta \) is the fin efficiency, \( W_{ch} \) is the channel width between the pin fins, \( W_w \) is the width of a pin-fin, \( H_{ch} \) is the height of a pin-fin and \( T_L \) is the liquid temperature.

The fin efficiency was found by assuming that the fins could be treated as rods with no heat transfer from their tips, i.e.

\[
\eta = \frac{\tanh(\lambda H_{ch})}{\lambda H_{ch}}
\]

(4)

where \( \lambda \) was the fin parameter, given by

\[
\lambda = \sqrt{\frac{4(\alpha_c + \alpha_{nb})}{kW_w}}
\]

(5)

10
in which \( k \) is the thermal conductivity of copper. The derivation of Equation (4) assumes that the nucleate boiling, heat-transfer coefficient is constant along the length of a pin. This is an approximation as it varies with wall superheat and therefore with position on the pin-fin. The nucleate boiling heat-transfer coefficient was evaluated at the base wall superheat.

The geometry of the test section has two plates sandwiched between the fluid and the ceramic heater, an aluminium plate and the copper heat sink, both 50 mm square and 5 mm thick, Figure 4. This geometry will only produce a uniform heat flux distribution at the solid-fluid interface if the heat-transfer coefficient and temperature are reasonably constant. This was not the case. The heat-conduction through the walls was therefore incorporated into the analysis.

The temperature variation across the width of the heat sink was found to be insignificant. The dominant wall conduction effect was therefore assumed to be two-dimensional, with one-dimensional parallel to the fluid flow and the other perpendicular to it, Figure 4. The heat-conduction equation is

\[
\frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0
\]

where \( T \) is the temperature in the copper or aluminium walls and \( y \) and \( z \) are the ordinates perpendicular and parallel to the flow respectively. McNeil et al (2013) analysed data from the same test section for a parallel channel heat sink. They found that Equation (6) could be solved to sufficient accuracy by sub-dividing the area into 1 mm square cells. For each cell, Figure 4, an energy balance gave

\[
T_{ij} = \frac{\Delta y^2(T_{i+1,j}+T_{i-1,j}) + \Delta z^2(T_{i,j+1}+T_{i,j-1})}{2(\Delta y^2+\Delta z^2)}
\]

where \( \Delta y \) and \( \Delta z \) were the cell dimensions. Equation (7), and the variants of it necessary to implement the boundary conditions shown in Figure 4, were solved iteratively until the relative temperature in each cell was the same as the previous estimate to within a relative error of 0.0003. The wall temperatures were solved relative to the liquid inlet temperature so that temperature differences could be obtained accurately. Heat-flux continuity was enforced at the aluminium-copper interface. Note that the base heat load boundary condition was set to the value given by Equation (2). Heat losses from the heater to the atmosphere were not included in the analysis.
Four thermocouples were located within the test piece, two 12.5 mm from the inlet and two 12.5 mm from the outlet, hereafter referred to as the inlet and outlet locations. The variation in the measured inlet and outlet wall temperatures was typically less than 0.4 and 0.2 K respectively, but could, on occasion, be as large as 1.1 and 1.2 K respectively. The local wall temperature was therefore obtained by averaging the readings from the two relevant thermocouples to obtain \( T_{tc} \), which was then corrected for depth from the pin-fin base. The measurements were made 1.5 mm from the surface so that the thermocouples were located at the centre of a cell in the heat conduction mesh, Figure 4. The wall temperature was therefore estimated from

\[
T_w = T_{tc} - \frac{\Delta y}{k} \left( \frac{q_c + q_k}{2} \right)
\]

where \( q_c \) was the y-direction heat flux from the measurement location to the cell above.

The data reduction process was iterative in two variables, the heat-transfer coefficient and the surface heat-flux distribution, i.e. the heat-flux distribution at the fluid-solid interface. To begin the process, a uniform heat flux was assumed. The energy equation was used to estimate the liquid temperature and the gas-mass fraction at the inlet and outlet thermocouple locations. This allowed estimates of the heat-transfer coefficients to be made from the measured wall temperatures. These heat-transfer coefficients were used in the boundary conditions of Equation (6), which was solved to update the surface heat flux distribution. With this better estimate of the surface heat flux, improved estimates of the liquid temperature and the gas-mass fraction were obtained at the inlet and outlet thermocouple locations from the energy equation, which, in turn, allowed better estimates of the heat-transfer coefficient to be obtained from the measured wall temperatures. This processes were repeated until the best estimates of surface heat flux and heat transfer coefficient were obtained. More details are given in the section describing each specific test.

5. Results and analysis of single-phase flow data

Inlet, outlet and wall temperatures, as well as pressure drops, were measured for single-phase liquid water near atmospheric pressure and temperature. Data were obtained for heat loads in the range 200-735 W. These heat loads gave rises in fluid temperature of at least 35 K. This gave heat fluxes in the range 76-295 kW/m², based on the base area of the test piece. Mass flow rates between 1 g/s and 4 g/s were used. This gave mass fluxes in the...
range 40-160 kg/m²s, based on the flow area between the pins in the heat sink channels. Pressure drops were obtained under isothermal conditions for the same mass fluxes.

For a liquid with a specific heat capacity of $c_p$, the liquid temperature was obtained from

$$T_L = T_{in} + \frac{W}{M} \int_0^x \frac{q_b}{c_p} \, dz \quad (9)$$

where $T_{in}$ is the liquid inlet temperature, $M$ is the mass flow rate of fluid and $W$ is the width of the test piece. In conventionally-sized tube bundles, the heat-transfer coefficient attains a reasonably constant value after a few rows of tubes. It was therefore assumed to be independent of heat-sink position. Copeland (1995) reported that the local, laminar-flow, Nusselt number, $Nu$, varied with the product of the liquid Prandtl and Reynolds numbers, $Pr_L$ and $Re_L$ respectively. The Reynolds number is based on the equivalent thermal diameter with three heated sides. The assessment of the single-phase, heat-transfer coefficients therefore assumed that

$$Nu = \gamma (Re_L Pr_L)^m \quad (10)$$

McNeil et al (2010) used the same heat sink to obtain single-phase data with R113 at atmospheric pressure as the working fluid. These data were included with the water data in the analysis to obtain values for $\gamma$ and $m$ in Equation (10). In the first instance, a uniform heat flux was assumed for $q_b$. This allowed the local liquid temperature to be found at the inlet and outlet thermocouple locations from Equation (9), with the corresponding heat-transfer coefficients obtained by using the measured wall temperatures in Equation (3). This was done for each heat and mass flux setting tested. These heat-transfer coefficients were used in a least squares analysis to find values of $\gamma$ and $m$ in Equation (10). These heat-transfer coefficients were imposed on the wall conduction model, Equation (6), through the boundary conditions, allowing the heat flux distribution to be updated. Equation (9) was subsequently used to update the liquid temperature at the inlet and outlet thermocouple locations, which produced update inlet and outlet heat-transfer coefficients, Equation (3), from the measured wall temperatures. The process was repeated until $\gamma$ and $m$ converged. The values obtained were 0.0592 and 0.726 respectively.

A comparison between the ‘measured’ heat-transfer coefficients and Equation (10) is shown in Figure 5. These data are compared to the ESDU (1973) correlation for conventionally sized tube bundles and to the micro-method of Short et al (2002). This method was recommended by Krishnamurthy and Peles (2008) for water in this Reynolds
number range. Correlations that contain a Reynolds number based on the tube diameter were evaluated with an equivalent diameter of four times the pin width divided by pi. The agreement with the ESDU correlation is very good. The micro method was verified for pins typically 100 μm in diameter with water flowing through them. It under-predicts the data. Fully-developed laminar flow in passages 1 mm square would produce Nusselt numbers of about 3.5. Thus, single-phase heat-transfer in these pin-fins is much more macro than micro in magnitude.

Comparing flows of this type with tube bundle methods brings in the concept of end effects, where the boundary layer on the base interacts with the flow across the pin fins. A lot of discussion is available on this topic; see for example Kosar and Peles (2006). There is a view that end effects reduce as the Reynolds number increases and that it is small for Reynolds numbers in excess of 50. These results are consistent with that view.

To demonstrate the magnitude of the effect of the variable heat flux, an example of the distributions of heat-transfer coefficient, heat flux, liquid temperature and wall temperature are shown in Figure 6. The heat-transfer coefficients, Figure 6a, are the values derived from Equation (10). They are constant as assumed in their derivation. These cause the heat flux to distribute as shown in Figure 6b, with values ranging from 24.9 – 52.1 kW/m² resulting from the applied value of 36.4 kW/m². The non-uniform heat-flux distribution leads to the non-linear temperature distribution shown in Figure 6c. This temperature distribution distorts from the uniform heat flux distribution that is included for comparison. These distortions are a result of the high thermal conductivities of copper (390 W/mK) and aluminium (250 W/mK) that forced the wall temperature to fall in between an isothermal and a uniform heat flux distribution, as shown in Figure 6d.

The single-phase heat-transfer coefficients are shown for the range of mass fluxes in Figure 7. The estimated error in the measured heat-transfer coefficients is ±10%. The values deducted from the constant heat flux assumption are included. Both data sets show the heat-transfer coefficients at the inlet and outlet positions of the heat sink to increase with increasing mass flux. However, the constant heat flux assumption produces outlet values that are significantly larger than the inlet values. The variable heat flux shows that, for any given mass flux, the correlating procedure significantly reduced the difference between them. This demonstrates that wall conduction effects significantly alter the results.
The variation of single-phase pressure drop with Reynolds number is shown for the heat-sink flows in Figure 8. Included in the figure is a prediction using the ESDU (1979) correlation for tube bundles and the micro method of reported by Krishnamurthy and Peles (2008) based on the data of Kosar and Peles (2006). The Reynolds number is again based on the equivalent diameter. The pressure drop data is shown to increase initially, stall for a range of Reynolds numbers before increasing again. The stall is associated with Reynolds number increases brought about by having the same mass flux at different temperatures. This is consistent with the ESDU correlation. The micro method shows the correct order of magnitude but its friction factor is more dependent on Reynolds number (viscosity) than the data indicates.

6. Analysis of Boiling flow data

Most heat-transfer investigations measure the heat-transfer coefficient before trying to understand the mechanisms that produced it. This approach requires the heat-flux distribution to be known. Solving the heat-conduction equation to obtain the heat-flux distribution requires the solid-liquid interface boundary condition to be specified. This either requires numerous wall temperatures to be known or it requires flow and heat-transfer models to be proposed and optimised. These models need to be linked and require the deduction of the heat-transfer mechanisms to be built into the data reduction process in an iterative manner. This latter approach is used in this study. Results are only credible if reasonable agreement between the measured and deduced wall temperatures is achieved. The order that the parts of the iterative process are presented in is chosen to aid understanding of the process.

6.1 Flow model

Two-phase flow was observed while subcooled boiling was taking place. The simplest way to mimic this behaviour, was to have a channel through the heat sink that could contain single-phase, growing slug, frozen slug and saturated flows placed above the conducting wall depicted in Figure 4. An energy balance for an element of the heat sink that allows for all of these possibilities gives

\[ q_b W = (1 - x)M c_p \frac{dT_b}{dz} + M \left[ c_p (T_{sat} - T_L) + h_f \right] \frac{dx}{dz} \]  

(11)
where \( x \) is the gas-mass fraction. Subcooled liquid entered the heat sink and remained liquid until the onset of nucleation occurred, as set by the criterion of Sato and Matsumura (1963), Equation (1). The single-phase liquid temperature was therefore found from Equation (11) for a constant gas-mass fraction of zero, i.e.

\[
q_bW = M c_p \frac{dT_L}{dz} \tag{12}
\]

The end of the single-phase region was concurrent with the onset of the growing slug regime. In this region it was assumed that all of the added energy resulted in slug growth, implying that the liquid temperature was constant and that the gas-mass fraction could be found from Equation (11) as

\[
q_bW = M (c_p(T_{sat} - T_L) + h_{fg}) \frac{dx}{dz} \tag{13}
\]

The growing slug regime was assumed to continue until bubble coalescence occurred at a critical void fraction. The frozen slug regime followed the growing slug regime and was characterized by a constant gas-mass fraction. In this regime the liquid temperature was the solution Equation (11) as

\[
q_bW = (1 - x_T) M c_p \frac{dT_L}{dz} \tag{14}
\]

where \( x_T \) is the gas-mass fraction corresponding to the critical void fraction. The frozen slug regime continued until the liquid temperature reached the saturation value, whence Equation (11) allowed the gas-mass fraction to be found from

\[
q_bW = M h_{fg} \frac{dx}{dz} \tag{15}
\]

The pressure distribution in a channel through the heat sink was obtained from the integration of the pressure gradient, \( dp/dz \), given by

\[
\frac{dp}{dz} = \left( \frac{dp}{dz} \right)_A + \left( \frac{dp}{dz} \right)_F \tag{16}
\]

where \( (dp/dz)_A \) is the pressure gradient due to acceleration and \( (dp/dz)_F \) is the pressure gradient due to friction. The acceleration pressure gradient was determined from the drift flux formulation, i.e.

\[
\left( \frac{dp}{dz} \right)_A = -\rho_v \left[ \frac{2 j_v}{\epsilon} \frac{d \rho_v}{dz} + \frac{(j_v)^2}{\epsilon} \frac{d \epsilon}{dz} \right] - \rho_l \left[ \frac{2 j_L}{(1-\epsilon)} \frac{d j_L}{dz} + \frac{(j_L)^2}{(1-\epsilon)} \frac{d \epsilon}{dz} \right] \tag{17}
\]

where \( j_v \) is the vapour superficial velocity, found from

\[
j_v = \frac{x M}{\rho_v \Lambda} \tag{18}
\]

and \( j_L \) is the liquid superficial velocity, found from
The void fraction, \( \varepsilon \), was determined from
\[
\varepsilon = \frac{j_V}{(C_\alpha j_m + j_V)}
\]  
(20)
in which \( C_\alpha \) is the distribution parameter, \( j_V \) is the drift velocity and \( j_m \) is the mixture velocity, found from
\[
j_m = j_V + j_L
\]  
(21)
The model was implemented by assuming homogeneous flow, i.e. \( C_\alpha = 1 \) and \( j_V = 0 \).

The frictional pressure gradient was obtained from the two-phase multiplier method, i.e.
\[
\left( \frac{dp}{dz} \right)_F = \frac{f_L}{2d^2 \rho_L} (1 - x)^2 \left( \frac{M}{X} \right)^2 \phi_L^2
\]  
(22)
The single-phase friction factor, \( f_L \), was found from the method of ESDU (1979) for tube bundles. The two-phase multiplier, \( \phi_L^2 \), was given by the method of Ishihara et al (1980), also for tube bundles, i.e.
\[
\phi_L^2 = 1 + \frac{C}{X} + \frac{1}{X^2}
\]  
(23)
where the Martinelli parameter, \( X \), was found from
\[
X = \left( \frac{1-x}{x} \right)^{0.9} \left( \frac{\mu_L}{\mu_v} \right)^{0.1}
\]  
(24)
in which \( \mu_L \) and \( \mu_v \) are the liquid and vapour viscosities respectively. The value of the constant \( C \) in Equation (29) was set to 8, in accordance with Ishihara et al (1980).

6.2 Heat-transfer model

Below the onset of nucleation, heat transfer is by single-phase convection. Equation (10) gives the heat-transfer coefficient as
\[
\alpha_{zp} = 0.0592 \frac{k_L}{D_l} (Re_L Pr_L)^{0.726}
\]  
(25)
As the local heat flux increases, the onset of nucleation occurs when
\[
q_s = \frac{k_{L,f,g}(T_w-T_{sat})^2}{\sigma d_{sat}} = \alpha_{zp} (\Delta T_{sup} + \Delta T_{sub})
\]  
(26)
where \( \Delta T_{sup} \) is the local wall superheat and \( \Delta T_{sub} \) is the local liquid subcooling. Equation (26) either locates the onset of nucleation somewhere between the inlet and outlet of the heat sink, or it demonstrates that nucleation occurs at the
heat-sink inlet. In either case a slug inception length was used. The slug inception length was used to move the heat-transfer mode from single-phase flow at its inlet to boiling slug flow at its outlet. This process involved rapid changes in heat-transfer coefficient that led to rapid changes in heat flux. These changes could take place at scales incompatible with the heat-conduction mesh, Figure 4. Values from the slug inception length were therefore ‘smoothed’ onto the conduction mesh to allow these rapid changes to interact with heat conduction in the copper substrate. Examples of the heat flux distribution within the slug inception lengths originating from the heat-sink inlet are shown in Figure 9 for R113 and water. To accommodate the growth of slugs in subcooled flow, slug inception took place at a constant liquid temperature. Figure 9a shows the heat-flux distribution for conditions close to those given by Equation (26). The nucleate boiling heat-flux for R113 is negligible relative to the applied heat flux, which is the value applied at the base, averaged over the surface and corrected for fin efficiency. Notice that the heat-sink heat flux averages to the applied value over the heat-sink length and not necessarily in the slug inception length. The dominant heat flux is through convection. This starts with the single-phase value at the plate inlet and increases as the vapour generated enhances it. The convective heat-transfer coefficient, $\alpha_{cv}$, was obtained from

$$\alpha_{cv} = E_{sf} \alpha_{sp}$$

Homogeneous flow was assumed in the inception length, allowing the slug flow enhancement factor, $E_{sf}$, to be given by

$$E_{sf} = \left( \frac{x L + (1-x) v_L}{v_L} \right)^{0.726}$$

(28)

where $v_L$ and $v_L$ are the specific volumes of vapour and liquid phases respectively. At the higher wall superheat, Figure 9b, which corresponds to the largest applied heat flux for R113, nucleate boiling and convection are similar in magnitude. The heat fluxes removed by convective and nucleate boiling were averaged over a surface element. The surface to fluid heat flux therefore balanced as,

$$\dot{q}_s = \dot{q}_{cv} + \dot{q}_{nb}$$

(29)

The nucleate boiling heat transfer coefficient was assumed to follow a power-law relationship with surface heat flux, i.e.

$$\dot{q}_{nb} = \alpha_{nb} \Delta T_{sup} = \beta \dot{q}_{nb}^{\frac{n}{n+\beta}} \Delta T_{sup}$$

(30)

This allows Equation (29) to be re-cast as

$$\alpha_{cv} (\Delta T_{sup} + \Delta T_{sub}) + (\beta \Delta T_{sup})^{\frac{1}{n+\beta}} - \dot{q}_s = 0$$

(31)
Lazarek and Black (1982) found that the Cooper correlation (1984) could be applied to nucleate boiling in small channels and was therefore used in this study. This correlation has an index, $n$, of 0.67 and a coefficient, $\beta$, given by

$$
\beta = \frac{55}{M_w} P_r^{0.12 - 0.4343 \ln(R_p)} \left( -0.4343 \ln(P_r) \right)^{-0.55}
$$

where $P_r$ is the reduced pressure, $R_p$ is the surface roughness in µm and $M_w$ is the molecular weight. The surface roughness parameter was chosen to remove surface roughness dependency. Water heat fluxes were always dominated by convection, with the heat flux distribution in Figures 9c obtained at conditions similar to those associated with Equation (26) and Figure 9d obtained at the largest applied heat flux for water.

Notice that the slug initiation lengths in Figure 9 vary from 1.1 to 0.2 mm for R113 and from 3.2 to 1.8 mm for water, decreasing with increasing wall superheat.

Following the slug inception length, slug flow existed in a subcooled liquid with the slug flow heat-transfer coefficient evaluated at the gas-mass fraction corresponding to the critical void fraction. For water, the heat-transfer coefficient downstream of the slug inception length remained constant. For R113, it remained constant until saturated flow occurred, whence it moved from the slug to the annular flow value. In all cases Equation (31) was used to describe the heat-transfer processes.

The physical properties of the fluids were evaluated at the temperature and pressure at the centre of each of the fifty segments required by the heat conduction model, Figure 4, and were assumed constant throughout the segment.

The solution was sought by firstly determining the applied heat flux necessary to produce a single-phase solution with the onset of nucleation at the heat sink exit. If the actual applied heat flux was less than this, the solution was single-phase. The minimum applied heat flux required to produce fully-developed boiling from the heat sink inlet was then determined. If the actual applied heat flux was greater than this, a solution with fully-developed boiling from the heat sink inlet was obtained. If the actual applied heat flux was less than this, a solution involving single-phase and two-phase flow on the heat sink was obtained.

7. Analysis of boiling R113 flows
Data for boiling R113 at near atmospheric conditions, using the same test arrangements, were reported by McNeil et al (2010). They produced data in the single-phase, subcooled boiling and saturated boiling regimes. However, their analysis focused on the saturated boiling data, analysed by assuming a uniform heat flux. Their data are re-analysed in this study using the approach outlined in Section 6. The approach requires a critical void fraction to be set for the bubbly to slug flow transition. A value of 0.25 was used to be consistent with the flow maps of Taitel et al (1976). Additionally, the Taitel et al (1976) critical Martinelli parameter of 1.6 was used for the transition from intermittent to annular flow. These flow pattern transitions separated the convective heat-transfer into three regimes, bubbly flow, intermittent flow and annular flow. In the bubbly flow regime, Equation (27) was used for the heat-transfer coefficient. In the annular flow regime it was determined from

\[
\alpha_{cv} = E_{af} \alpha_{spo}
\]  

(33)

where \( \alpha_{spo} \) is the single-phase value, determined from Equation (25) using the liquid-only flow. The annular flow enhancement factor, \( E_{af} \), is given by

\[
E_{af} = 1 + \frac{\gamma}{m}
\]  

(34)

where \( \gamma \) and \( m \) were obtained by minimising the difference between the measured and predicted wall superheats. An iterative process converged on values of 0.914 and 0.814 respectively. In the intermittent flow regime, the convective heat-transfer coefficient was obtained from a linear interpolation along the gas-mass fraction line between the slug and annular flow values. The slug heat-transfer coefficient corresponded to the bubbly regime value, evaluated at the critical void fraction.

A comparison between the ‘measured’ and the predicted heat-transfer parameters is given in Figure 10. Figure 10a shows the comparison of the measured and predicted heat-transfer coefficients for non-annular flows, referred to as slug flows. The predicted values follow from the single-phase value and the assumption of homogeneous flow, and to a limited extent, the annular flow values. There are no adjustable parameters. The level of agreement is therefore very good, with only the data taken at a mass flux of 100 kg/m²s showing any significant deviation. This is discussed later. Figure 10b shows a comparison of the measured and correlated enhancement factors. With the exception of some of the data taken at a mass flux of 100 kg/m²s, the data collapse on to a single curve.
The variations of the wall superheat with the base heat fluxes are shown for each mass flux in Figure 11. In general, all three heat-transfer mechanisms occurred. This, and subsequent figures, uses squares covering a data point to denote single-phase heat transfer and circles covering a data point to denote subcooled boiling. Saturated boiling has no covering symbol. If only one symbol is shown when an inlet and outlet point are very close to each other, the symbol is covering the inlet point. The trends in the wall superheat are well represented by the model, with the data obtained at a mass flux of 100 kg/m²s again being the exception. Predictions from the Cooper (1984) correlation are included in Figure 11. This prediction represents the wall superheat that would have been achieved had only nucleate boiling been present. Thus, the difference between the data and the Cooper (1984) correlation demonstrates the significant level of surface cooling that is attributable to forced convective heat transfer. The larger the mass flux, the larger the effect of the slug flow on the distribution shape. This is again reasonably well predicted by the model.

Since the measured and predicted wall superheats are in reasonable agreement, Figure 11f, the data reduction method is acceptable.

A comparison between the ‘measured’ and predicted heat-transfer coefficients is shown in Figure 12. The values obtained at a mass flux of 100 kg/m²s are inconsistent with the modelled values, Figure 12a. At larger mass fluxes, Figures 12b-12e, the inlet heat-transfer coefficients are shown to be reasonably constant, with the outlet values increasing with increasing heat flux. These trends are predicted reasonably well by the model, with most modelled and measured values falling within 15% of each other, Figure 12f. Notice that the convective heat-transfer coefficients are reasonably well predicted for subcooled and saturated flows and that no attempt has been made to optimise these for flow pattern, i.e. established criteria were used.

Examples of the distributions across the heat sink predicted by the model are shown in Figure 13. At the lowest heat flux of 6.2 kW/m², the flow is single-phase throughout, resulting in a constant heat-transfer coefficient, Figure 13a, a declining heat flux, Figure 13b, near constant wall temperature, Figure 13c and zero void fraction, Figure 13d. With increasing heat flux, slugs develop in the first few millimetres of the heat sink, as depicted in the increase in void fraction from 0 to 0.25 in Figure 13d. These slugs generate constant heat-
transfer coefficients in subcooled liquid with increasing values in the saturated region, Figures 13a. The wall temperatures remain reasonably constant at heat-fluxes below 137 kW/m², Figure 13c, with the temperature difference across the heat sink being less than 1 K. This gives the relatively minor, but still significant, distortions to the heat flux distributions shown in Figure 13b. These distortions arise mainly from the onset of nucleation. At a heat flux of 137 kW/m², a major distortion occurs in the heat-flux distribution, Figure 13b, as heat is drawn to the heat-sink outlet by the high heat-transfer coefficients, Figure 13a. Figure 13b also shows the average applied heat flux, corrected for actual area and fin efficiency. The lines are of the same type as the actual distributions and are in grey. At larger heat fluxes, the wall temperature variation becomes significant, Figure 13c, emphasising the effect of lateral conduction in the substrate.

8. Experimental results for boiling water flows

The two-phase tests were performed by supplying liquid water near atmospheric pressure to the test section with subcooling of 3-13 K. Heat-transfer and pressure drop data were obtained for heat loads in the range 100-1210 W. These were applied to each mass flow rate in steps of 100 W. This gave pre-heat loss heat fluxes in the range 40-484 kW/m². The five mass flow rates used were 1, 2, 3, 4 and 5 g/s, giving mass fluxes, based on the flow area between the pin-fins, of 40-200 kg/m²s. These heat and mass inputs gave gas-mass fractions of 0-0.48 at the smallest mass flux and 0-0.09 at the largest.

8.1 Heat-transfer data

An attempt was made to repeat the analysis carried out for the R113 data described in Section 7. The results are shown in Figure 14. Clearly, this method of analysis is not capable of describing the boiling water data. Firstly, Figure 14a, the slug heat-transfer coefficient is poorly predicted. Secondly, the annular flow enhancement data, Figure 14b, contain a mass flux and position dependency that prevents a single curve from being formed. The capillary length of saturated R113 at atmospheric pressure is 1 mm, which is comparable to the size to the passages between the pin-fins. However, the capillary length of water at the same condition is 2.5 mm, which is significantly larger than the size of the passage between the pin-fins. Thus, it seems probable that the water flows were confined whereas the R113 flows were not.
The failure of the model used with R113 caused a difficulty in obtaining the heat-transfer coefficients because the data reduction method required a mechanistic model and the data did not conform to the known mechanisms. For each condition tested, investigations of the experimental data suggested that the heat-transfer coefficient was reasonably constant across the length of the heat sink. For this to be the case, the heat-transfer coefficient had to be reasonably insensitive to local conditions. This is an unusual assertion as the heat-transfer coefficient depends on local flow structures. It was thought possible that the flow structures produced in the slug inception length had characteristics that dominated the heat-transfer coefficient and that these characteristics changed little with changing local flow conditions. This being the case, the heat-transfer coefficients measured at the inlet and outlet thermocouple locations did not depend on the local flow conditions, but on the conditions in the slug inception length.

The heat-transfer coefficients were found from an iterative process. To start the iteration, a uniform heat flux was assumed. The flow model described in Section 6 used an estimated critical void fraction to establish the local conditions at the inlet and outlet thermocouple locations. Equation (31) was used to establish the convective heat-transfer coefficients from the measured wall superheats. The average of these coefficients was used in the boundary conditions of the heat-conduction equation, Equation (6), to update the heat-flux distribution. To update the critical void fraction for the slug inception length, the slug model described by Equations (27) and (28) was invoked. This allowed the implied critical gas-mass fraction, and by implication the critical void fraction, to be obtained and used as a proxy for the heat-transfer coefficients and kept the heat-transfer model compatible with the flow model described in Section 6.1. The updated critical void fraction and heat-flux distribution allowed the process to be repeated until convergence. This procedure was applied to each heat and mass flux combination tested. The converged values of critical void and critical gas-mass fraction are shown in Figure 15.

The critical void fraction variation with heat flux is shown in Figure 15a. Clearly the critical void fraction varies with heat flux and mass flux. The critical void fraction variation with Boiling number is shown in Figure 15b. The Boiling number, $B_N$, is defined as

$$B_N = \frac{q_B}{m_{hfg}}$$

(35)
For low Boiling numbers, this shows that the critical void fraction no longer has a mass flux dependency and that the critical void fraction tends to values associated with macro-scale systems. It is possible that the critical void fraction depended on local conditions. Figure 15c shows the critical void fraction variation with a local Boiling number. This Boiling number used the heat flux at the location of the onset of slug flow in its evaluation. The magnitude of the Boiling number obviously changes but the similarity in general trends between Figures 15b and 15c suggest that little is gained by using local values. The variation in critical gas-mass fraction with the global Boiling number, Equation (35), is shown in Figure 15d. The critical gas-mass fraction changes less dramatically with Boiling number than the critical void fraction and is therefore easier to correlate. The data are shown as three types, low mixed and high. The low data are shown as solid symbols. These data are independent of mass flux and are correlated by

\[ x_f = 433B_n^{1.54} \]  

(36)

The high data are shown as open symbols and are independent of Boiling number but vary with mass flux. The horizontal lines in Figure 15d correspond to a gas superficial velocity of 1.21 m/s. The ‘best fit’ values for each mass flux are within ±15% of this. The mixed data are shown as closed symbols with a grey filling. These data diverge from the low data at a critical void fraction of about 0.83 ±3%, corresponding to a critical gas-mass fraction of 0.0031, before arriving at the high values corresponding to the critical superficial gas velocity. They therefore vary with Boiling number and mass flux. However, for each mass flux, these data can be correlated by

\[ x_f = \gamma B_n^m \]  

(37)

The mixed flow curve shown in Figure 15d represents a mean value, with \( \gamma = 3.1 \) and \( m = 0.92 \), included for demonstrative purposes only. The characteristics shown in Figure 15 are consistent with slug flow at low Boiling numbers, annular flow at high Boiling numbers and elongated slug flow in between these extremes. These critical void fractions are significantly larger than those found in unconfined flows.

The finding illustrated in Figure 15d were implemented as three-part curve fits to the critical gas-mass fractions for each mass flux. These were used in the slug model, Equations (27) and (28), to predict the wall superheats shown in Figure 16. Predictions from the Cooper (1984) correlation are included. The significant
deviation between the Cooper-based predictions and the experimental data indicates that a significant convective heat-transfer component results from the large critical void fractions.

The variation for the lowest mass flux of 40 kg/m²s is shown in Figure 16a. At the inlet location, single-phase heat-transfer occurs at the lowest heat flux, subcooled nucleate boiling occurs at the second lowest with saturated boiling occurring at all of the other heat fluxes. At the outlet location, saturated boiling occurs at all heat fluxes. The inlet and outlet wall superheats are similar at each heat flux. At larger mass fluxes, Figures 16b-16e, the number of data points in the single-phase and subcooled boiling regimes increases. This can be explained by the increase in sensible heat required to reach the saturation temperature for a given heat flux as the mass flux increases. The increasing differences between the inlet and outlet wall superheats at each heat flux also become more evident as the mass flux increases. Reasonable agreement is shown between the measured and predicted wall superheats, Figure 16f.

The ‘measured’ heat-transfer coefficients are compared to the modelled values in Figure 17. In the main, these convective coefficients are shown to increase with increasing heat flux. The agreement between the ‘measured’ and the modelled values is reasonable at the lowest mass flux, Figure 17a. For larger mass fluxes, Figures 17b-17e, the assumed constant value is shown to produce a discrepancy, with the outlet values systematically lower than the inlet values. The physical mechanism for this is not clear. Since the slugs were generated in a subcooled liquid, direct contact condensation could lead to a subsequent reduction in gas-mass fraction and hence in the convective heat-transfer coefficient. However, Szczukiewicz et al (2014) have reported that, in micro-channels, a decline also occurs and is due to the move from slug to annular flow. Most of the data presented have inlet and outlet heat-transfer coefficient that are within ±10% of each other. Inaccuracies in the data reduction method can be of this magnitude and, overall, the agreement is good, Figure 17f, with most data reproduced to within ±15%. Reducing the discrepancy between the inlet and outlet values will require more detailed studies of the flows. However, this data reduction method is giving a reasonable estimate of what occurred. Notice that enhanced heat-transfer coefficients exist in subcooled boiling conditions.
Comparison of the ‘measured’ water heat-transfer coefficients with existing methods is difficult to do because the ‘measured’ values depend on the conditions in the slug inception length, whereas methods available in the literature depend on local conditions. Local conditions on the heat-sink surface are available from the data analysis. Thus, the local conditions at the inlet and outlet thermocouple locations were used to evaluate some existing methods for Convective Enhancement. The values for the water data were obtained by dividing the ‘measured’ heat-transfer coefficient with the liquid-only values obtained by assuming that only the liquid component was flowing at the measurement locations. The results are shown varying with the local gas-mass fraction in Figure 18. Included in Figure 18 are the Jensen and Hsu (1988) method for tube bundles, termed the macro method, and the micro-method of Krishnamurthy and Peles (2008). The Jensen and Hsu (1988) method uses the ESDU (1973,1979) Reynolds number indices for friction and heat transfer. Also included is the correlation obtained for R113, Equation (34). Equation (34) and the ESDU method are in reasonable agreement, confirming the R113 data to be in the macro heat-transfer regime. The micro method is below the macro method, whereas the water data are above it. Thus, these water data are considerably more enhanced than those obtained for water boiling on pin-fins with a diameter of 100 μm and are significantly more enhanced than those obtained for flow in tube bundles.

The distributions of the key heat-transfer and flow parameters that result from the data reduction method are shown for a mass flux of 160 kg/m²s and base heat fluxes in the range 35-192 kW/m² in Figure 19. At a base heat flux of 35 kW/m², the inlet location, 12.5 mm from the inlet, is in single-phase flow with boiling initiated just before the outlet location, 37.5 mm from the inlet. This is evidenced by the significant increase in the convective heat-transfer coefficient, Figure 19a, the heat flux, Figure 19b, and the void fraction, Figure 19d. Notice that the critical void fraction is close to macro-scale value in this instance and that the heat flux near the onset of nucleation increases and then decreases. The increase in heat flux is caused by the increase in the heat-transfer coefficient with the decrease caused by the subsequent increase in liquid temperature as the liquid moves towards saturation. As the base heat flux increases, the onset of nucleation moves towards the heat-sink inlet and the slug critical void fraction increases. The void fraction remains at the critical value in subcooled boiling before increasing during saturated flow, Figure 19d, as assumed in the model formulation. The heat-transfer coefficient, Figure 19a, does not respond to the later increase in local void fraction, Figure 19d, as assumed in the model. Notice that the slug inception length, the length required for the void fraction
to increase from 0 to the critical value, is of the order of a few millimetres, Figure 19d. At a base heat flux of 192 kW/m², the onset of nucleation begins at the heat-sink inlet. The large increase in the heat-transfer coefficient causes a significant distortion to the heat-flux distribution, Figure 19b, and the wall temperature, Figure 19c. The heat flux distributions in the saturated flow regions are reasonably constant, Figure 19b. In fact, for heat fluxes of 192 kW/m² and above, the wall inlet and outlet thermocouple locations are in a region of constant heat flux. Figure 19b also shows the average applied heat flux, corrected for actual area and fin efficiency. The lines are of the same type as the actual distributions but are drawn in grey. The magnitude of the actual heat flux approaches the average value towards the heat-sink exit. In effect, the slug inception in the subcooled liquid produces an ‘entrance’ effect which is damped out with heat-sink length.

8.2 Two-phase pressure drop data
The variation of two-phase pressure drop with base heat flux is shown for a range of mass fluxes in Figure 20. The pressure drop is shown to increase with increasing heat flux and with increasing mass flux. The total pressure drop predictions, Figure 20a, are significantly above the measurements. This method was shown to be reasonably good for R113, McNeil et al (2010), but has produced results that are typically twice the measured values for water. Surprisingly, the predicted acceleration pressure drop is in very good agreement with the measurements, Figure 20b. Krishnamurthy and Peles (2008) used the Kawahara et al (2002) two-phase multiplier to describe their water data through 100 μm pin fins. This multiplier uses the laminar-laminar Martinelli parameter in Equation (23) with a constant C of 0.24, giving much smaller pressure drops, which is consistent with the measured data. The frictional pressure drop was neglected in the determination of the heat-transfer measurements. This ensured that a good estimate was made of the local saturation temperatures.

9. Discussion and conclusions
Heat-transfer and pressure drop data have been obtained for a pin-fin heat sink boiling water at atmospheric pressure. The pin fins had a 1 mm square base and were 1 mm high. They were placed on a 2 mm square pitch on a test piece that was 50 mm square and were heated from below by a uniform heat flux. Heat-transfer coefficient and pressure drop data were obtained for single-phase liquids and subcooled and saturated boiling flows.
Single-phase heat-transfer coefficients have been shown to be reasonably constant. This was assumed in the analysis described in Section 5. However, a length-wise power law variation was also considered. This led to a smallish power law index of about -0.1, and a greater rms difference between the correlation and the measured values, i.e. a poorer fit than those obtained from the constant heat-transfer coefficient assumption. The single-phase, heat-transfer coefficients correlation obtained, Equation (10), used water and R113 data in its formulation and therefore covers a wide range of fluid properties. However, the geometry of the pin-fins was not included in the correlation and the micro method, Figure 5, may be indicative of smaller values existing at smaller pin fin diameters. Data from different geometric configurations would therefore need to be obtained before Equation (10) could be generalised for other pin-fin surfaces. In these tests, the distribution of the liquid temperature caused significant distortions to the heat-flux distribution at the solid-fluid interface, Figure 6, that forced a wall temperature that lay somewhere between isothermal and constant heat-flux values.

The re-analysis of the R113 data show that this fluid in this geometry behaves similarly to conventional macro-scale flow, Figure 10. The re-analyses was very successful for mass fluxes of 200 kg/m² and above. It was not successful for a mass flux of 100 kg/m². Harirchian and Garimella (2012) have suggested that the confined-unconfined transition does not just depend on capillary length but also on liquid velocity. The confined analysis used for the water flows in Section 8 was applied to the R113 flows for mass fluxes of 100 and 200 kg/m². A comparison of the heat-transfer coefficients from the confined analysis is shown in Figure 21. The unconfined analysis for a mass flux of 100 kg/m², Figure 12a, shows a significant deviation between the measured and predicted heat-transfer coefficients. Much better agreement is achieved from the confined analysis, Figure 21a, where the critical void fraction was set to 0.82. Notice that the measured values at the inlet and outlet locations are separated in Figure 12a but are coincident in Figure 21a. This results from the assumption of a gas-mass fraction dependency in the heat-transfer coefficient for unconfined flows. This gives a variable heat-transfer coefficient across the heat-sink that distorts the heat-flux distribution. The confined analysis assumes that the slug heat-transfer coefficient is constant downstream of its inception point, producing a reasonably constant heat flux. The latter is shown to be more closely related to what actually occurred. The unconfined analysis for a mass flux of 200 kg/m², Figure 12c, shows good agreement between the measured and predicted heat-transfer coefficients, whereas, the confined analysis, Figure 21b, obtained with the critical void fraction set to 0.6, does not. This suggests that there is more to the confined-
unconfined transition than the capillary length and that the data obtained at a mass flux of 100 kg/m²·s were confined whereas those obtained at higher mass fluxes were not. Harirchian and Garimella (2012) suggested that the transition occurs when $\text{Re} \sqrt{\text{Bo}} = 160$. For R113 at 100 kg/m²·s this value is 200, and varies from 200-1000 for the data set. Thus, this criterion can only work for pin-fins if the transition lies between 200 and 400.

The heat-transfer coefficients shown in Figure 17 are unusual. The first thing to note is that the single-phase values are reasonably large, Figure 7, and that the variations in the liquid temperature along the length of the heat sink distorts the heat flux distribution, Figure 6. However, nucleation begins at the channel exit when the heat fluxes are small. The nucleate boiling coefficients are correspondingly small, Figure 9. Normally the onset of nucleate boiling is associated with an increase in the heat-transfer coefficient. This situation is the reverse in that the nucleate boiling heat-transfer coefficients are much smaller than the single-phase values. This is why the slug inception length demonstrated in Figure 9 was adopted. This length forces a sensible link between the wall superheat and the surface heat-flux during the transition from single-phase to flow boiling heat transfer. The inception length assumes that bubbles are produced at the onset of nucleation and that these bubbles produce slugs because they are confined in the small passages between the pin fins in the heat sink. Convection through relatively thin liquid films surrounding the slugs can lead to large heat-transfer coefficients, as reported by Thome et al (2004). As the heat flux is increased, the onset of nucleation moves upstream, Figure 19, until they reach the heat-sink inlet. Slugs were observed to persist in the subcooled boiling region. This has previously been reported by Celata et al (2012) for micro-channels and by McNeil et al (2013) for mini channels. Note that significant convective enhancement is shown to occur in subcooled flows for R113 and water, circled data points in Figures 12 and 17. This is why the flow model described in Section 6.1 was adopted. This is the simplest flow model that could be assumed that includes subcooled convective enhancement. It does seem to work reasonably well. As the heat flux is increased to values above 300 kW/m², the large heat-transfer coefficients caused by the slugs redistribute heat towards a more uniform flux downstream of the onset of saturated flow, Figure 19. For most mass fluxes the heat-transfer coefficients do not change if the flow is subcooled or saturated and they are independent of gas-mass fraction. However, their magnitude is significantly more than the values associated with nucleate boiling. Thus, two-phase convection and nucleate boiling are occurring simultaneously, with the convection component by far the most
dominant, Figure 9. The convective component probably results from the heat-transfer resistance of a thin liquid film surrounding elongated vapour bubbles, as described in Thome et al. (2004). However, the magnitude of the heat-transfer coefficient is controlled by the mechanism present at the onset of nucleation, Figure 15. The maximum slug heat-transfer coefficients are shown to be reasonably constant for base heat fluxes in excess of 300 kW/m², Figure 17. The value of average inlet and outlet convective heat-transfer coefficients is about 26 kW/m²K. All of the high heat-flux values lie within ±10% of this. In other words, they are reasonably independent of heat flux, mass flux and void fraction.

Boiling water flows produced smaller heat-flux distortions downstream of the slug inception point, Figure 19b, than R113, Figure 13b. This is because the water heat transfer coefficients did not depend on the local conditions whereas R113 values did. Thus, a flow with a gas-mass fraction dependent heat-transfer coefficient will not produce uniform heat fluxes. About 80% of the water data reported was gathered under reasonably constant heat-flux conditions because the flow above the wall thermocouple was in the saturated flow region and the heat-transfer coefficients were reasonably constant. Upstream of the slug inception point water produces the greatest distortion to the heat-flux distribution because of the much larger slug heat-transfer coefficients generated.

The data reduction method has led to three regimes of flow, the low void fraction regime, where the critical gas-mass fraction is given by Equation (36), a high critical gas-mass fraction regime there the gas superficial velocity exceeds a critical value, \( j_{Gc} \), typically 1.21 m/s, and an intermediate regime. The critical superficial velocity gives the transition critical gas-mass fraction from

\[
x_f = \frac{\rho_{\text{lgc}}}{m}
\]

An approximate method can be applied by taking the critical gas fraction as the minimum of the low and high level regime values, i.e.,

\[
x_f = \min \left( 433B_1^{1.54}, \frac{\rho_{\text{lgc}}}{m} \right)
\]

The enhancement factor, based on all of the liquid flowing, follows from Equation (28). A comparison between the measured and predicted values, varying with the gas-mass fraction at the slug inception length outlet, is shown in Figure 22. The comparison is reasonable. Model deficiencies exist because the data reduction method does not discriminate sufficiently between the values at the inlet and outlet thermocouple locations, Section 8.1, and because
the intermediate flow region has not been properly defined. Future work will need to investigate these areas. Other confined fluids would also need to be investigated to obtain universal forms of Equation (36) and critical superficial gas velocity.

The water data are confined, as defined by the capillary length. The Harirchian and Garimella (2012) transition criteria is $Re\sqrt{Bo} = 160$. For the water tests the range was 60-280. Thus, this criterion does not identify confinement for pin-fins with water. However, this approach identifies confinement for pin-fins with water and R113 if the transition value is in the range 300-400, as this would make all water data and the lowest mass flux R113 data confined with the higher mass flux R113 data unconfined. NB: This implies that any method that purports to differentiate between confined and unconfined flows will need to take into account the flow diameter, the fluid properties and the fluid velocity.

The data reduction method used the concept of fin efficiency to account for the reduced wall temperature on the extended surface area. The ratio of actual to base area was 1.75. Typical values of the effective area ratios were 1.74 for R113 and 1.69 for water, a reduction of 0.5% and 3.6% respectively. The water value is lower because it had larger heat-transfer coefficients.

The analysis in this paper demonstrates that the conventional description of boundary conditions as constant wall temperature or constant wall heat flux is inadequate when dealing with flow in small scale channels under conditions which lead to spatially changing heat transfer mechanisms. The nature of the heating systems used, in both practice and in the current experimental analyses, is such that conduction in the heated substrate produces variations in the local heat flux, Figures 6b, 13b and 18b, that have a tendency to move the heat-sink wall towards isothermal conditions, Figures 6d, 13c and 18c. Uniform electrical heating of the substrate, which would conventionally be described as constant heat flux, produces near isothermal conditions on the boiling surface. The heat transfer performance of a heat sink must therefore be considered to be a function of the thermal conductivity and physical dimensions of the substrate as well as the pin-fin geometry and the transport properties of the fluid.

References


Fig. 1. Schematic of flow loop

Fig. 2. a: Test section construction, b: Longitudinal cross-section of the test section assembly with internal insulation, c: Test section open view with internal insulation
Fig. 3. In-line pin-fin test piece

Figure 4: Conduction mesh and heat flow paths
Figure 5: Variation of Nusselt number with correlating parameter

Figure 6a: Variation of heat-transfer coefficient with position in heat sink

mass flux = 200 kg/m²s

Figure 6b: Variation of heat flux with channel position

mass flux = 200 kg/m²s

Figure 6c: Variation of liquid temperature with position in heat sink

mass flux = 200 kg/m²s

Figure 6d: Variation of wall temperature with position in heat sink

mass flux = 200 kg/m²s

Figure 6e: Single-phase distributions

Figure 7: Variation of the single-phase water heat-transfer coefficient with mass flux

\[ \text{Nu} = 0.0592(Re \cdot Pr_L)^{0.726} \]

\[ R^2 = 0.965 \]
Figure 8: Variation of water pressure drop with Reynolds number

Figure 9: Slug inception examples
Figure 10a: Comparison of measured and slug heat-transfer coefficient

Figure 10b: Variation enhancement factor with Martinelli parameter

Figure 11: R113 wall superheat variations
Figure 12: R113 Heat-transfer coefficients

Figure 12a: Mass flux of 100 kg/m²s

Figure 12b: Mass flux of 200 kg/m²s

Figure 12c: Mass flux of 300 kg/m²s

Figure 12d: Mass flux of 400 kg/m²s

Figure 12e: Mass flux of 500 kg/m²s

Figure 12f: All mass fluxes
Figure 13: Boiling R113 distributions

Figure 13a: Variation of heat transfer coefficient with heat sink position

Figure 13b: Variation of heat flux with heat sink position

Figure 13c: Variation of wall temperature with heat sink position

Figure 13d: Variation of void fraction with heat sink position

Figure 14a: Variation measured with slug heat-transfer coefficient

Figure 14b: Variation enhancement factor with Martinelli parameter
Figure 15: Transition gas-mass fraction

- Figure 15a: Variation of critical void fraction with heat flux
- Figure 15b: Variation of critical void fraction with Boiling number
- Figure 15c: Variation of critical void fraction with local Boiling number
- Figure 15d: Variation of critical gas-mass fraction with Boiling number

Figure 16: Water wall superheat variations

- Figure 16a: mass flux 0.40 kg/m²
- Figure 16b: mass flux 0.80 kg/m²
- Figure 16c: mass flux 1.20 kg/m²
- Figure 16d: mass flux 1.60 kg/m²
- Figure 16e: mass flux 2.00 kg/m²

Figure 17: Water heat-transfer coefficient variations

Figure 17a: Mass flux of 40 kg/m²s

Figure 17b: Mass flux of 80 kg/m²s

Figure 17c: Mass flux of 120 kg/m²s

Figure 17d: Mass flux of 160 kg/m²s

Figure 17e: Mass flux of 200 kg/m²s

Figure 17f: All mass fluxes

Figure 18: Variation of enhancement factor with gas-mass fraction
Figure 19: Boiling water distributions

Figure 19a: Variation of heat transfer coefficient with heat sink position

Figure 19b: Variation of heat flux with heat sink position

Figure 19c: Variation of wall temperature with heat sink position

Figure 19d: Variation of void fraction with heat sink position

Figure 20a: Comparison of measured with predicted pressure drop

Figure 20b: Comparison of measured with acceleration pressure drop

Figure 21a: Mass flux of 100 kg/m²s confined R113

Figure 21b: Mass flux of 200 kg/m²s confined R113
Figure 22: Variation of enhancement factor with gas-mass fraction

Enhancement factor (\(\cdot\))

Critical gas-mass fraction (\(\cdot\))

M040 (inlet)  \[\triangle\] M040 (outlet)  \[\blacktriangleup\]
M080 (inlet)  \[\blacklozenge\] M080 (outlet)  \[\blacktriangle\]
M120 (inlet)  \[\blackcircle\] M120 (outlet)  \[\blacklozenge\]
M160 (inlet)  \[\blacktriangleleft\] M160 (outlet)  \[\blacklefttriangle\]
M200 (inlet)  \[\blacktriangleleftrightarrow\] M200 (outlet)  \[\blackrighttriangleleft\]

Lower bound (-30\%)  \[\blacktriangleleft\]
Upper bound (+30\%)  \[\blacktriangleright\]

Model  \[\blacktriangledown\]