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Shell-side boiling of water at sub-atmospheric pressures

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Abstract

Experimental data are reported for water boiling at pressures of 850 and 50 mbar absolute on the shell-side of a model industrial boiler slice. The boiler test section was 1 m high, 0.75 m wide and contained 36 electrically heated tubes. The tubes were 28.5 mm in diameter and 98 mm long. The design of the boiler ensured that the tubes were submerged in a liquid pool. The height of the liquid pool could be varied. The pool height was set to approximately 0.8 m for the tests carried out at a pressure of 850 mbar, submerging the top of the tube bundle by about 200 mm. Two pool heights were used in the tests carried out at a pressure of 50 mbar, one at approximately 0.8 m and another at approximately 2 m. The later submerged the top of the tube bundle by about 1.6 m. The heat flux was varied within the range 10-70 kW/m\textsuperscript{2}. A near-symmetrical half of the tube bundle contained wall thermocouples. An additional 29 thermocouples were located throughout the liquid pool.

The liquid temperature in the pool was found to be reasonably uniform and controlled by the pressure at the free surface. This led to a small amount of subcooling at a pressure of 850 mbar, up to 3 K, and a significant amount of subcooling at a pressure of 50 mbar, up to 16 K for the smaller pool height and up to 31 K for the larger pool height. The reasonably uniform pool temperature suggests that the liquid re-circulates within it.

Boiling was found to occur at all heat fluxes at a pressure of 850 mbar, with the measured heat-transfer coefficients shown to be in broad agreement with nucleate boiling correlations available in the open literature. However, they were also consistent with a flow boiling process involving natural convection and nucleation, where the convection was driven by variations in liquid temperature on the walls of the tubes. This natural convection relies on an interaction between the tubes that produces mass fluxes in the range 46-87 kg/m\textsuperscript{2}s, based on the approach area to the tube bundle. Boiling occurred only at the higher heat fluxes during the low level tests.
at a pressure of 50 mbar, with interactive natural convection being the dominant heat-transfer mechanism. The mass fluxes produced were in the range 28-70 kg/m²s. Boiling also occurred only at the higher heat fluxes during the high level tests at a pressure of 50 mbar. However, the convective heat transfer was more compatible with little interaction between the tubes, although some evidence suggests that the evaporator oscillates between interactive and isolated tube behaviour.

1. **Introduction**

Some evaporators, like those used to process nuclear waste, boil fluids that are highly corrosive. The corrosion rate of the materials used to construct these evaporators depends on their temperature. Thus, the life of the evaporator can be extended if the wall temperatures of the evaporator are kept low. One way of achieving this is to boil the fluid at a low pressure, and hence a low saturation temperature. This study was initiated to investigate the changes that occur in evaporator operation as the pressure is reduced. The investigation was carried out on a one-quarter scale, thin slice model of a bespoke industrial evaporator that is used in the processing of nuclear waste. The model evaporator could operate at any pressure from close to total vacuum to atmospheric and had a glass front to allow visual information to be obtained. The pressures investigated were 850 mbar, the HP series, and 50 mbar absolute, the LP series. The latter tests were carried out at two pool heights, one at approximately 0.8 m, the low level tests, and one at approximately 2 m, the high level tests.

The geometry of this thin slice model is similar to those used in the study of kettle reboilers. A review of these studies is given by McNeil et al [1]. Most of these studies were carried out using pentane and refrigerant R113. This study used water.

The kettle reboiler is one of the most commonly used shell and tube heat exchangers in the process industry and consists of a tube bundle placed in a shell. The liquid level above the tube bundle is set by a weir. The heating fluid flows inside the tubes while the heated fluid boils on the outside surfaces of the tubes. The difference in densities between the two-phase mixture flowing shell side within the tube bundle and the liquid flowing between the tube bundle and the shell is known to cause natural circulation. The circulation flow rate is needed in the estimation of the local heat-transfer coefficient.
The simplest analytical approach available is the one-dimensional model, see for example Brisbane et al [2] or Jensen [3]. This model assumes that liquid enters each column of the tube bundle from the bottom and evaporates as it moves vertically upwards. The two-phase pressure drop in a column is assumed to balance with its static head of liquid. This is based on reasonably static liquid being present between the tube bundle and the shell wall and could therefore be relevant to the model evaporator used in this study. The two-phase pressure drop has gravity, acceleration and friction components. A void fraction and a two-phase friction multiplier correlation are therefore required to complete the model. For conditions typical of kettle reboilers, Bamardouf and McNeil [4] have shown that the void fraction correlation of Feenstra et al [5] and the two-phase multiplier correlations of Ishihara et al [6] give the best agreement with the available experimental data, which was mostly obtained from one-dimensional flow experiments. The two-phase multiplier approach requires the liquid only pressure drop to be found. ESDU [7] was used by Bamardouf and McNeil [4]. When applied to thin kettle slices, the one-dimensional model has been shown to be consistent with its inherent assumptions at heat fluxes lower than 20 kW/m² by Burnside et al [8].

Boiling at low (vacuum level) pressures has not had very much attention in the literature. Some pool boiling data has been reported. A reasonable summary is given by Feldmann and Luke [9]. The reduced pressure is the ratio of the pressure to the critical pressure and has a value of 0.00023 in this study. Only two data sets are reported in the literature to go that low, one by Minchenko [10] and another by Gorodov et al [11]. The Minchenko [10] heat-transfer coefficient data is shown to be more than twice the magnitude of that reported by Gorodov et al [11], with the later reasonably predicted by the Gorenflo [12] correlation and the former by the Cooper [13] correlation, with the correlation’s coefficient increased in line with the recommendation for a copper horizontal cylinder.

2. Description of the test facility

The test facility is shown in Figure 1. To fill the rig with deionised water, both drain valves and the vent condenser valve were closed and the vacuum pump control valve was opened, Figure 1. The vacuum pump was switched on until the test section pressure was reduced to 500 mbar. The drain valve to the test section was opened, allowing water to flow from the storage tank to the evaporator. The drain valve was closed when the desired water level was achieved in the evaporator. The drain valve from the hot well was opened, allowing water to flow from the storage tank to the hot well. The drain valve was closed when the water reached the desired height in the hot-well sight glass. The circulating pump, water control valves and the evaporator entry
Shut-off valve were opened, allowing water to flow from the hot well to the vessel, purging any air from the pipe work. The shut-off valve was closed when a steady flow of water was evident in the evaporator.

To operate the rig, the vacuum pump was switched on and adjusted until the required test section pressure was achieved. Heat was supplied to the evaporator by Joule heating of rod heaters contained within the tubes. Initially, the tube heaters were switched on at 90% of full power. After some time, steam was generated. This pushed any remaining air into the hot well before it was expelled to the atmosphere. When condensate began to accumulate in the hot well, the liquid entry shut-off valve was opened and the circulating pump was started. The flow rate was set by adjusting the pump control valves until a steady level was obtained in the hot-well sight glass. Water from the hot-well was pumped by the circulating pump into the test section via the pre-heater. The inlet temperature can be set by adjustment of the pre-heater. However, this was not used in these tests. Vapour from the vessel was condensed and subcooled before being returned to the hot-well. Steady conditions were achieved in about 3 hours, whence the power controllers were set to produce the required heat flux for the test. Test conditions were achieved in a further 30 minutes.

The test section was manufactured from stainless steel and is shown in Figure 2. It represents a one-quarter scale, thin slice model of an industrial evaporator. The industrial heat exchanger has six coils. These are represented in the model evaporator by the two tube bundles, left and right, offset by 32 mm to mimic the slope of the coils. The top three inner tubes, left and right, represents coil 1 and the corresponding middle and outer tubes represent coil 2 and 3 respectively. The lower tubes similarly represent coils 4 to 6. The applied heat load to the industrial evaporator can be varied by changing the number of coils used. The main vessel was 1 m high, 0.75 m wide and 98 mm deep. The two smaller vessels were 0.6 m high and were used to vary the pool height from quarter scale up to the level attained in the actual evaporator. The pressure in the test-section vapour space was measured by an absolute pressure transducer, accurate to 0.25% of range. With this uncertainty in the pressure at the free surface, the uncertainty in the corresponding calculated saturation temperature is ±0.9 K for the 50 mbar tests and ±0.1 K for the 850 mbar tests. A second, similar pressure transducer measured the pool pressure. This transducer was connected to *p* _pool_ in Figure 2 and allowed the pool height to be estimated. The tube bundles contained tubes 28.5 mm in diameter in an in-line configuration, with a horizontal tube pitch of 69 mm and a vertical tube pitch of 62.5 mm. The heaters in each coil were on a single power controller. This allowed each coil to operate independently, as is possible on the actual evaporator. A uniform heat flux was applied to the tubes in
this study. The power to each left (three tubes) and right (three tubes) for each coil was measured by a power meter. The power meter was accurate to ±1% of reading.

Temperatures within the evaporator were measured by k-type thermocouples. The thermocouple locations in the test section are shown in Figure 2. NB: only the right-hand-side of the evaporator contained thermocouples.

The thermocouples were classified into 3 groups. The first group are referred to as the stream thermocouples. The ‘stream’ was considered to start at the free surface, flow down the centre line and on to the base of the test section, move across the base and up the side walls, before returning to the free surface. These thermocouples were numbered TS1-TS18 and are shown in Figure 2. The low level pool height was near TS5 and the high level near TS1. The thermocouples closest to the shell wall were 5 mm from the surface. The second group are the tube thermocouples. The tubes were made of brass, had an outside diameter of 28.5 mm, a length of 98 mm and were 5 mm thick. The tube thermocouples were located within the brass tube walls and were numbered from right to left going top to bottom as TT1-TT18. The third and final group are the fluid thermocouples. These were the thermocouples located in the fluid between the tubes and were also numbered right to left going from top to bottom as TF1-TF11 in Figure 2.

All instruments, except the power meter, were connected to data logging equipment that was linked to a PC and controlled through commercial software. The software allowed monitoring of the instruments during operation and logging of the data when required. The power meter data was entered into a computer file manually. A camera was used to monitor the test section. Recordings of each test condition were made.

Prior to their installation in the evaporator, all of the thermocouples were calibrated in a water bath. The water bath contained a heater, a stirrer and a resistance thermometer accurate to ±0.1°C. The thermocouples, with the necessary compensation cable attached, were connected to the data logger system. The same system was used in the calibrations and in the tests.

To obtain data set a, each instrument was read once per second over a 2 minute period. The readings were obtained a second time, ten minutes later, to obtain data set b. Thereafter, the power controllers were set to the next condition and the procedure repeated until the necessary heat-flux range had been achieved.
A heat balance was obtained by measuring the volume of condensate collected in the hotwell and comparing the associated cooling power with the power supplied to the tubes. The cooling power was found to be 96% of the heating power.

3. Data processing

The heat-transfer area, $A$, was taken as the outside surface area of a tube. The heat flow, $Q$, to the tubes on the right hand side of a coil, i.e. 3 tubes, was measured. The tube heat flux, $q$, was therefore found from

$$q = \frac{Q}{3A}$$

(1)

The surface temperature, $T_w$, of each tube wall was found from

$$T_w = T_{tc} - \frac{qD}{2k_B} \ln \left( \frac{D}{D-2L_{tc}} \right)$$

(3)

where $T_{tc}$ is the measured wall temperature, $D$ is the outside diameter of a tube, $k_B$ is the thermal conductivity of brass, 190 W/mK, and $L_{tc}$ is the depth of the thermocouple from the tube surface, i.e. 2.5 mm. The thermocouple holes were located at better than ±0.5 mm on their pitch circle radius. The uncertainty in the wall temperature therefore varied from ±0.1 K to ±0.2 K as the heat flux increased from 10 to 70 kW/m². The saturation temperature was evaluated from the local pressure, $p$, found from

$$p = p_s + \rho_L gH$$

(4)

where $p_s$ is the measured shell pressure, $\rho_L$ is the density of liquid, $g$ is the gravitational constant and $H$ the depth of the tube centre line from the free surface. The height of the free surface above the pool measurement location was found from

$$H_{pool} = \frac{(p_{pool} - p_s)}{\rho_L g}$$

(5)

where $p_{pool}$ was the measured pool pressure. The uncertainties in the pressure at the free surface and in the pool combined to give an uncertainty in the calculated pool height of ±51 mm.

The liquid temperatures associated with each tube were obtained from interpolations using the nearest thermocouples, fluid and stream. Liquid properties were evaluated at the liquid temperature with vapour properties evaluated at the local saturation conditions deduced from the local pressure.

4. Experimental results
Three tests series were undertaken, low level tests at pressures near 850 mbar, the HP series, low level tests at pressures near 50 mbar, the LL LP series, and high level tests at pressures near 50 mbar, the HL LP series. All pressures were absolute.

4.1 Stream temperatures
Tests with the tube heat flux set to 65 kW/m² for the LL LP and HL LP series and 70 kW/m² for the HP series produced the stream temperatures shown in Figure 3. The values shown are the averages of data sets a and b. Other heat fluxes produced similar results. Included in Figure 3 are the saturation temperatures corresponding to the pressure at the free surface and the evaporator base. The HP tests show that the saturation temperature does not change much as the pressure increases, because the pool depth is a small fraction of the total pressure. The LL LP and HL LP tests are different, with the saturation temperature varying from 32 °C at the free surface to 49 °C at the evaporator base for the LL LP series and 32 °C to 64 °C for the HL LP series. In all cases the stream temperature is reasonably uniform and close to the free surface saturation temperature. The stream temperatures are well distributed throughout the pool, Figure 2. These results are therefore indicative of fluid recirculation taking place, with fluid flashing to the saturation temperature at the free surface before the liquid is returned to the depth of the pool.

4.2 Liquid temperatures
The liquid temperatures in close proximity to the tubes, Figure 2, are shown in Figure 4. They are arranged in terms of tube rows and columns. The tube rows run from bottom to top with the columns running from left to right. Only rows 1 and 6 are shown. The other rows behave similarly. Included in the figure is the local saturation temperature. The HP data have a reasonably small degree and range of subcooling, with the average row value varying from 2.8 K on row 1 to 2.1 K on row 6, with all subcoolings within ±0.5 K of the averages. The HP pool temperatures are probably suppressed slightly by the low liquid inlet temperature of about 11 °C. The LP data have a much larger degree of subcooling, with the average row value varying from 14 K on row 1 to 7.9 K on row 6 for the LL series and form 27.8 K to 24.2 K for the HL series. All subcoolings are within ±3 K of the averages. Therefore, saturated boiling is not possible as all liquid temperatures are below the saturation temperature.

4.3 Tube wall temperatures
The measured tube wall temperatures are shown in Figure 5. Included in the figure is the local saturation temperature and the boiling onset temperature. The boiling onset temperature was taken to occur when the heat flux was related to the wall temperature through, [14],

$$q = \frac{k_L h_{LG} (T_w - T_{sat})^2}{\sigma \Delta T_{sat}}$$

(6)

where $k_L$ is the liquid thermal conductivity, $h_{LG}$ is the enthalpy of evaporation, $\rho_v$ is the vapour density, $T_{sat}$ is the saturation temperature and $\sigma$ is the liquid surface tension. The boiling onset temperature in Figure 5 was found from the applied heat flux so that it represents the maximum temperature that can occur without boiling happening. The saturation and boiling onset temperatures increase with pool pressure (depth). All of tube wall temperatures for the HP data series are shown to be above the boiling onset temperature and therefore all tubes are in the subcooled boiling regime. At the lowest heat flux, most of the LL LP data are below the saturation temperature and are therefore in the single-phase convective heat transfer regime. As the heat flux increases, the LL LP data series are close to, and usually below, the boiling onset temperature, so that the heat-transfer regime is not easily specified. At heat fluxes below 40 kW/m$^2$, the HL LP data are below the saturation temperature and are therefore in the single-phase convective heat transfer regime. At heat fluxes above 40 kW/m$^2$, they are above the boiling onset temperature and are therefore in the subcooled boiling heat transfer regime. The transition between single-phase convection and subcooled boiling is shown to in the vicinity of 40 kW/m$^2$.

4.4 Visual evidence

Photographs taken of the tube bundle at a pressure of 850 mbar for various heat fluxes are shown in Figure 6. At a heat flux of 10 kW/m$^2$, small bubbles are evident towards the top of the tube bundle. As the heat flux increases to 20 kW/m$^2$, bubbles are evident further down in to the tube bundle. For heat fluxes greater than 20 kW/m$^2$, bubbles are present from row 1 upwards, confirming that subcooled boiling is occurring. However, what cannot be deduced is the presence or absence of convective heat transfer. It is noticeable that, even at a heat flux of 70 kW/m$^2$, the void fraction is low.

Photographs taken of the tube bundle at a pressure of 50 mbar during the LL LP series for various heat fluxes are shown in Figure 7. At a heat flux of 10 kW/m$^2$, bubbles were not evident so the photograph is excluded. Bubbles are evident as the heat flux increases. However, the bubbles are relatively large and attached to the tube wall. Larger heat fluxes led to a more frequent appearance of the bubbles. These photographs indicate that subcooled boiling happens at heat fluxes greater than 10 kW/m$^2$ but the information obtained does not allow any evidence
of the presence or absence of convection to be confirmed. The HL LP series looked similar to the LL LP series. However, bubbles were not observed until a heat flux of 40 kW/m².

5. Heat-transfer regime investigation

Three analyses were undertaken in an attempt to establish the heat-transfer mechanisms existing on the tube surfaces, the isolated tube analysis, the non-equilibrium one-dimensional column analysis and the equilibrium one-dimensional column analysis.

5.1 The isolated tube analysis.

This analysis assumed that the tubes acted independent of each other in either natural convection or subcooled nucleate boiling. For a horizontal cylinder, Churchill and Chu [15] gave the natural convection, heat-transfer coefficient, $\alpha_{nc}$, as

$$
\alpha_{nc} = \frac{Nuk_L}{D} = \frac{k_L}{D} \left\{ 0.6 + \frac{0.387Ra^0.1}{\left[ 1 + \left( \frac{0.559}{Pr} \right)^{9/16} \right]^{9/22}} \right\}^2 
$$

where $Pr$ is the Prandtl number, given by

$$
Pr = \frac{\mu_L c_p L}{k_L} 
$$

in which $\mu_L$ is the liquid viscosity, $c_p L$ is specific heat capacity at constant pressure and $Ra$ is the Rayleigh number, given by

$$
Ra = GrPr = \frac{\rho g (T_w - T_L) D^3}{\mu_L^2} Pr
$$

in which $T_L$ is the liquid temperature. The thermal expansion coefficient, $\beta$, was obtained from a curve fit to the liquid density across the required temperature range. The fluid properties were evaluated at the film temperature, $T_f$, given by

$$
T_f = \frac{(T_w + T_L)}{2}
$$

Natural convection took place with increasing heat flux until the onset of nucleate boiling. This occurred when the wall superheat from natural convection balanced with the onset condition, Equation (6), i.e.
with nucleate boiling continuing thereafter. The correlations considered for the boiling heat-transfer coefficient, $\alpha_{nb}$, were of the form

$$q_{omn} = \frac{k_l h_f \rho_l (T_w - T_{sat})^2}{\beta r_{sat}} = \alpha_{nc} (T_w - T_L) \quad (11)$$

Two correlations were used, the Cooper [13] and Gorenflo [12] correlations, as they had been identified by Feldman and Luke [9] as having some success at these reduced pressures.

At a pressure of 850 mbar, Equation (11) gave onset wall superheats of typically 0.7 K, so that all of the HP data set is predicted to be boiling. A comparison between the measured and predicted wall superheats for the Cooper correlation is shown in Figure 8. Data sets a and b, taken at the same conditions ten minutes apart, are both shown. The wall temperature difference is the difference between the wall and liquid temperatures. This temperature difference is used for the statistical comparisons for all of the models. Figure 8 shows that the data behaviour is consistent with pool boiling in that the wall superheat increases with increasing heat flux. There are differences between the behaviour of the columns, with columns 1 and 2 behaving similarly and column 3 running hotter. The upper and lower limits shown in Figure 8d are set at ±30%, with the data for columns 1, 2 and 3 shown in black, red and blue respectively. The Cooper correlation [13] under-predicts the data with an average difference of 2.8%, 4.8% and 15.2% for columns 1, 2 and 3 respectively. The corresponding root mean square differences (rms) are 15%, 14.8% and 20.8%. Thus, the Cooper correlation predicts the wall temperatures in columns 1 and 2 better than those in column 3. Overall, the average and rms differences are 7.6% and 17.1% respectively. The Gorenflo correlation [12] over-predicts the data with an average difference of -18.1%, -16.5% and -8.2% for columns 1, 2 and 3 respectively. The corresponding root mean square differences (rms) are 21.7%, 20% and 14.4%. Thus, the Gorenflo correlation [12] predicts the wall temperatures in column 3 better than those in columns 1 and 2. Overall, the average and rms differences are -14.3% and 18.9% respectively. Thus, the Cooper correlation [13] predicts the data marginally better than the Gorenflo correlation [12], with the Cooper correlation [13] under-predicting to a similar degree to Gorenfo’s [12] over-predictions.

At a pressure of 50 mbar, Equation (11) gave onset wall superheats of typically 4-6 K for the LL LP data series. This translates to the data taken at a heat flux of 10 kW/m$^2$ being in the natural convection regime with the others in the subcooled nucleate boiling regime. A comparison between the measured and predicted wall superheats using the Cooper correlation [13] in the subcooled boiling regime is shown in Figure 9. There are differences between the behaviour of the columns, with columns 1 running cooler than column 2, which, in turn, is running
cooler than column 3. The upper and lower limits shown in Figure 9d are set at ±30%. The Cooper-based approach over-predicts the data with an average difference of -23.4%, -16.1% and -9.24% for columns 1, 2 and 3 respectively. The corresponding rms differences are 28.1%, 19.2% and 16.3%. Thus, the Cooper-based approach predicts the wall temperatures in columns 3 better than those in columns 1 and 2. Overall, the average and rms differences are -16.3% and 21.8% respectively. When the Gorenflo correlation [12] is used in the subcooled nucleate boiling regime, over-predictions of the data occur with average differences of -35.5%, -29% and -23% for columns 1, 2 and 3 respectively. The corresponding rms differences are 37.6%, 30.2% and 26.8%. Thus, the Gorenflo-based approach predicts the wall temperatures in column 3 better than those in columns 1 and 2. Overall, the average and rms differences are -29.2% and 31.8% respectively. Thus, the Cooper-based approach predicts the data significantly better than the Gorenflo-based approach.

At a pressure of 50 mbar, Equation (11) gave onset wall superheats of typically 6 K for the HL LP data series. This translates to the data taken at heat fluxes of 10 and 25 kW/m² being in the natural convection regime with those obtained at 55 and 65 kW/m² in the subcooled nucleate boiling regime. Some of the data obtained at a heat flux of 40 kW/m² are in the natural convection regime while others were in the subcooled boiling regime. A comparison between the measured and predicted wall superheats using the Cooper correlation [13] for the subcooled nucleate boiling regime is shown in Figure 10. There is little difference in the behaviour of the columns. The upper and lower limits shown in Figure 10d are set to ±30%. The Cooper-based approach under-predicts the data with an average difference of 0.4%, 1.2% and 3.1% for columns 1, 2 and 3 respectively. The corresponding rms differences are 14.8%, 13.7% and 14.5%. Thus, the Cooper-based approach predicts the wall temperatures in all columns equally well. Overall, the average and rms differences are 1.6% and 14.3% respectively. Using the Gorenflo correlation [12] in the subcooled nucleate boiling regime also shows little difference between the columns, with average differences of -4.5%, -3.8% and -1.9% for columns 1, 2 and 3 respectively. The corresponding rms differences are 13.1%, 11.4% and 12.1%. Thus, the Gorenflo-based approach also predicts the wall temperatures in all columns equally well. Overall, the average and rms differences are -3.4% and 12.2% respectively. Thus, the Cooper-based approach and the Gorenflo-based approach predict the data equally well. However, a significant proportion of the good agreement comes from the natural convection predictions, which masks the boiling element. Boiling occurs at heat fluxes of 55 and 65 kW/m², for which the Gorenflo correlation [12] has an rms difference of 4% while the Cooper [13] correlation has an rms difference of 12.5%. Thus, the Gorenflo correlation [13] predicts the data much better than the Cooper correlation [12]. It is worth noting the disparity between data sets a and b at heat fluxes of 25 and 40 kW/m². These are discussed later.
5.2 The non-equilibrium, one-dimensional column model

The one-dimensional model, see for example [1 or 2], normally assumes that the liquid enters a column of tubes in a saturated state and evaporates as the fluid moves upwards across the tubes. The mass flux upwards through the column is the value that balances the pressure drops in the tube column with the static liquid value outside of it. The pressure gradient, \(\frac{dp}{dz}\), in the column has three components, \(\frac{dp}{dz}_A\), the pressure gradient due to acceleration, \(\frac{dp}{dz}_F\), the pressure gradient due to friction and, \(\frac{dp}{dz}_G\), the pressure gradient due gravity, i.e.

\[
\frac{dp}{dz} = \left( \frac{dp}{dz}_A \right) + \left( \frac{dp}{dz}_F \right) + \left( \frac{dp}{dz}_G \right)
\]

(13)

The acceleration pressure gradient was determined from the separated flow model, i.e.

\[
\left( \frac{dp}{dz}_A \right) = -m^2 \frac{d}{dx} \left( \frac{\nu_G}{\varepsilon} + \frac{(1-x)\nu_L}{(1-\varepsilon)} \right)
\]

(14)

where \(m\) is the column mass flux based on the horizontal tube pitch, \(P_H\), \(x\) is the gas-mass fraction, \(\nu_G\) is the gas specific volume and \(\nu_L\) is the liquid specific volume. The void fraction, \(\varepsilon\), was found from,

\[
\varepsilon = \frac{\nu_G}{(x\nu_G + k(1-x)\nu_L)}
\]

(15)

where \(k\) is the slip ratio obtained from the Feenstra [5] correlation.

The frictional pressure gradient was obtained from the two-phase multiplier method, i.e.

\[
\left( \frac{dp}{dz}_F \right) = -\frac{f_L}{2D} (1-x)^2 m^2 \nu_L \phi_L^2
\]

(16)

The single-phase friction factor, \(f_L\), was found from the method of ESDU [7], using the liquid component of the mass flow rate, and the two-phase multiplier, \(\phi_L^2\), was given by the method of Ishihara et al [6], i.e.

\[
\phi_L^2 = 1 + \frac{8}{X} + \frac{1}{X^2}
\]

(17)

where the Martinelli parameter, \(X\), was found from

\[
X = \left( \frac{1-x}{x} \right)^{0.9} \left( \frac{\mu_L}{\nu_G} \right)^{0.1} \left( \frac{\mu_L}{\mu_G} \right)^{0.1}
\]

(18)

in which \(\mu_G\) is the vapour viscosity.
This model can be implemented in this application by assuming that all of the heat added causes phase change, allowing the liquid temperature to remain constant as observed. The gas-mass fraction at the exit of a tube pitch, $x_E$, is related to the inlet value, $x_I$, through

$$x_E = \frac{x_I (c_pL(T_{sat} - T_L) + h_{L,i}) + \frac{dq}{m F_H}}{c_pL(T_{sat} - T_L) + h_{L,exit}}$$  \hspace{1cm} (19)

Subscripts $I$ and $E$ refer to the inlet and exit values respectively. The gas-mass fraction at the centre of a tube is taken as the average of the inlet and outlet values.

The heat flux was assumed to partition into nucleate, $q_{nb}$, and single-phase, $q_{sp}$, components thus

$$q = q_{nb} + q_{sp}$$  \hspace{1cm} (20)

The tube wall superheat, $\Delta T_{sup}$, was therefore found from the solution of

$$q = (\gamma \Delta T_{sup})^{\frac{1}{\sigma - m}} + (\phi_k^2)^{0.36} \alpha_{sp} (\Delta T_{sup} + \Delta T_{sub})$$  \hspace{1cm} (21)

where $\Delta T_{sub}$ is the liquid subcooling and $\alpha_{sp}$ is the single-phase, heat-transfer coefficient. This was based on the liquid-only mass flux and was obtained from the ESDU [16] correlation.

The onset of nucleate boiling was obtained from Equation (11) with the single-phase heat transfer coefficient, evaluated at the column mass flux, replacing the natural convection value.

At a pressure of 850 mbar, Equation (11) gave onset wall superheats of typically 2.5-3.75 K for the HP data series. This model is so heavily dominated by convection that the choice of boiling correlation is not important. The Cooper [13] correlation was used in the predictions presented. A comparison between the measured and predicted wall superheats for column 2 is shown in Figure 11. Other columns behave similarly. Figure 11a shows that the data behaviour is inconsistent with this model at a pressure of 850 mbar. The model significantly under-predicts the data with an average difference of 207% and a rms difference of 234%. The model deduces mass fluxes of 320-560 kg/m$^2$s, which are clearly an order of magnitude too large. Figure 11b shows that the LL LP data behaviour is also inconsistent with this model at a pressure of 50 mbar. The model significantly under-predicts the data with an average difference of 362% and a rms difference of 399%. The model mass fluxes of 550-654 kg/m$^2$s are clearly at odds with those occurring. The HL LP data series predictions are even worse with an average difference of 551% and an rms difference of 569%.
5.3 The equilibrium, one-dimensional model

This model is similar to that described in Section 5.2 with two exceptions. Firstly, the liquid in the column flow is raised to the local saturation temperature before vaporisation occurs, and secondly, the driving force for the mass flux in the column is modified to include natural circulation caused by temperature differences in the liquid. Thus, for a subcooled liquid, the energy equation becomes

$$T_{LE} = T_{Li} + \frac{aq}{m_{PHEP}}$$  \hspace{1cm} (22)

and for a saturated mixture

$$x_L = \frac{x_L h_{Li} \varepsilon (T_{Li} - T_{LE}) + a q}{h_{LE}}$$  \hspace{1cm} (23)

The net pressure drop, $\Delta p_{net}$, that must be balanced by friction and acceleration in the column is given by

$$\Delta p_{net} = \sum_{i=1}^{N_{row}} g P_e \left[ (\rho_{li} - \rho_{Li}) + \varepsilon \left( \rho_{Li} - \rho_{li} \right) \right]$$  \hspace{1cm} (24)

where $N_{row}$ is the number of rows, $i$ is the row number, $P_e$ is the vertical tube pitch, $\rho_{li}$ is the bulk fluid density and $\rho_{li}$ is the row gas density. The average row liquid density, $\rho_{Li}$, is given by

$$\rho_{Li} = \rho_{li} + \left( \rho_{Li} + \rho_{le} \right) \left( \frac{P_e - D}{2P_e} \right)$$  \hspace{1cm} (25)

where $\rho_{li}$ is the liquid density at the wall temperature.

The onset of nucleate boiling was obtained from Equation (11) with the single-phase heat transfer coefficient, evaluated at the column mass flux, replacing the natural convection value.

At a pressure of 850 mbar, Equation (11) gave onset wall superheats of typically 1.2-1.5 K, so that all of the HP data set is predicted to be convecting and boiling. A comparison between the measured and predicted wall superheats, using the Gorenflo correlation [12] to describe boiling and the ESDU [16] correlations to describe convection, is shown in Figure 12. The upper and lower limits shown in Figure 12d are set at ±30%. The Gorenflo-ESDU combination under-predicts the data with an average difference of 14.6%, 16.7% and 27.42% for columns 1, 2 and 3 respectively. The corresponding rms differences are 25.9%, 26.4% and 31.9%. Thus, the Gorenflo-ESDU combination predicts the wall temperatures in columns 1 and 2 better than those in column 3. Overall, the average and rms differences are 19.6% and 28.2% respectively. The corresponding Cooper-ESDU combination also under-predicts the data with an average difference of 37.5%, 40.1% and 52.9% for columns 1,
2 and 3 respectively. The corresponding rms differences are 44.1%, 45.7% and 55.1%. Overall, the average and rms differences are 43.5% and 48.6% respectively. Thus, the Gorenflo-ESDU combination predicts the data significantly better than the Cooper-ESDU combination. The mass flux range required for this model is in the range 47-87 kg/m²s.

At a pressure of 50 mbar, Equation (11) gave onset wall superheats of typically 5.7-8.1 K for the LL LP data set, so that most of the data at heat fluxes of 10 and 25 kW/m² are in single-phase convection, with the remainder convecting and boiling. A comparison between the measured and predicted wall superheats for the Gorenflo-ESDU combination is shown in Figure 13. The upper and lower limits shown in Figure 13d are set at ±30%. The Gorenflo-ESDU combination over-predicts the data with an average difference of -12.4%, -3.3% and 5.7% for columns 1, 2 and 3 respectively. The corresponding rms differences are 17%, 8.6% and 21.6%. Thus, the Gorenflo-ESDU combination predicts the wall temperatures in columns 1 and 2 better than those in column 3. Overall, the average and rms differences are -3.3% and 16.6% respectively. The Cooper-ESDU combination under-predicts the data with an average difference of -1.5%, 8.2% and 17.6% for columns 1, 2 and 3 respectively. The corresponding rms differences are 16%, 12.3% and 25.1%. Overall, the average and rms differences are 8.1% and 18.6% respectively. Thus, the Gorenflo-ESDU combination predicts the low level data slightly better than the Cooper-ESDU combination. The mass flux range required for this model is in the range 28-70 kg/m²s.

At a pressure of 50 mbar, Equation (11) gave onset wall superheats of typically 6-8.7 K for the HL LP data set, so that virtually all of the data are in single-phase convection, with only row 1 data boiling at a heat flux of 65 kW/m², making the choice of boiling correlation somewhat irrelevant. A comparison between the measured and predicted wall superheats for the Gorenflo-ESDU combination is shown in Figure 14. The upper and lower limits shown in Figure 14d are set at ±30%. The Gorenflo-ESDU combination under-predicts the data with an average difference of 46.1%, 47% and 49.2% for columns 1, 2 and 3 respectively. The corresponding rms differences are 53.6%, 53.2% and 54%. Thus, the Gorenflo-ESDU combination predicts the wall temperatures in all columns similarly. Overall, the average and rms differences are 47.4% and 53.6% respectively. The Cooper-ESDU combination predicts the data similarly because virtually all of the predictions come from the ESDU correlation. The mass flux range required for this model is in the range 28-83 kg/m²s.

6. Discussion and conclusion

The visual evidence presented in Figures 6 and 7 show changes to the liquid pool behaviour as the pressure is reduced. At a pressure of 850 mbar the flow contains many bubbles whereas few bubbles are evident at a
pressure of 50 mbar. This is caused by the increase in the subcooling that occurs at the lower pressure, a subcooling that is even larger when the pool height is increased. The increase in subcooling is produced by liquid recirculating within the pool, as evidenced by the stream temperatures, Figure 3. The pool temperature is similar to the saturation temperature corresponding to the pressure at the free surface. This produces small liquid subcoolings at a pressure of 850 mbar and large liquid subcoolings at a pressure of 50 mbar. However, recirculation requires a liquid velocity which means that convection and/or subcooled boiling could be present. What heat-transfer mechanism is dominating is therefore less clear. Three analyses methods were used to help deduce what the heat-transfer mechanisms were. The statistics summarising the comparison between the data and the models is given in Table 1.

The current test section looks similar to those used to model kettle reboilers, which has been successfully modelled by an approach similar to the non-equilibrium model. McNeil et al [17] have suggested that this model is a good approximation for bubbly flows, where the void fraction is typically less than 0.3, as it is in this case. Subcoolings generated by the pool head have traditionally been ignored with this approach so that its divergence from the experimental data, particularly at a pressure of 850 mbar, was unexpected. However, as is clear from the statistics presented in Table 1, this type of flow does not happen irrespective of the pressure and pool height. This only leaves two possible modes of operation, either the tubes operate independent of each other, as described by the isolated tube model, or they interact, as described by the equilibrium model.

The model analysis approaches described in Section 5, when applied to the high level data at a pressure of 50 mbar, produced rms differences of 14.3% and 53.7% when the Cooper correlation [13] was used to describe the boiling element of the isolated tube and equilibrium models respectively. The corresponding figures achieved when the Gorenflo correlation [12] was used were 12.2% and 53.6%. These results suggest that isolated tube behaviour is the most likely. The isolated tube model suggests that boiling only occurs at heat fluxes of 55 and 65 kW/m², for which the rms differences are 12.5 % and 4% for the Cooper [13] and Gorenflo [12] correlations respectively. Thus, the Gorenflo correlation is probably the more accurate at these low pressures.

Figure 10 shows that the two data readings, a and b, obtained at the same nominal conditions. At heat fluxes of 25 and 40 kW/m², these are significantly different. Reading b was taken 10 minutes after reading a. At a heat flux of 25 kW/m², reading b is much lower than reading a, ruling out the possibility that the data were taken before ‘steady’ conditions were achieved. These data are reproduced in Figure 15a and compared to three
predictions, the natural convection correlation, Equation (7), the equilibrium model and the Gorenflo [12] correlation. The two data sets are shown to correspond to the equilibrium and natural convection values, indicating that the test facility is oscillating between these regimes. The data obtained at a heat flux of 40 kW/m$^2$ are shown in Figure 15b. Some of these data can also be interpreted in this way. However, other data can be interpreted as including boiling in the oscillation.

The low pressure, high level data are therefore consistent with the buoyancy force being insufficient to drive an interactive flow when the heat flux is 10 kW/m$^2$. The data at this heat flux are therefore described by natural convection, Equation (7). The buoyancy driving force increases as the heat flux is increased, allowing tube interaction to occur, as described by the equilibrium model. However, as the interaction gets established, the tube temperatures are reduced by the increasing liquid velocity, reducing the velocity again. Thus, at a heat flux of 25 kW/m$^2$ the flow oscillates between the isolated and interactive tube behaviours. Similar behaviour is observed at a heat flux of 40 kW/m$^2$. However, at this heat flux, the wall temperatures can exceed the value required for the onset of boiling so that boiling may also be present in this oscillation. Boiling is established at heat fluxes of 55 and 65 kW/m$^2$. Nucleate boiling rather than flow boiling is more likely to have occurred since the equilibrium model predictions are poor at these heat fluxes, Figure 14. Thus, boiling probably follows on from isolated tube conditions because they produce the higher wall temperature and flow boiling did not occur.

The visual evidence at a pressure of 50 mbar, Figure 7, does not show bubbles rising up the columns. To get an indication of the flow patterns, neutrally buoyant particles 1 mm in diameter were added to the flow. These particles were observed to move chaotically, with streams of particles observed to move from vertically upwards through horizontally to vertically downwards in different parts of the ‘cycle’. The mean ‘cycle’ motion could not be identified, but ‘time of flight’ estimates from these particles indicated that the liquid velocities were consistent with the predictions from the equilibrium model. The motion of the particles is supportive of a convective component of heat transfer.

The model analysis approaches described in Section 5, when applied to the low level data at a pressure of 50 mbar, produced rms differences of 21.8% and 18.6% when the Cooper correlation [13] was used to describe the boiling element of the isolated tube and equilibrium models respectively. The corresponding figures achieved when the Gorenflo correlation [12] was used were 31.8% and 16.6%. These results suggest that isolated tube behaviour is less likely than interactive tube behaviour because the high level data show that the Gorenflo [12]
correlation is probably more reliable at these pressures. Velocity magnitudes supportive of the equilibrium model have been observed. Thus, at a pressure of 50 mbar, the equilibrium model with the Gorenflo correlation [12] used to describe boiling gives the more probable results. However, at a heat flux of 10 kW/m², the isolated tube model, Figure 9, does show good agreement with some of the data, while others agree with the equilibrium model, Figure 13. This is further evidence that a minimum heat flux may be required to fully-establish tube interaction, as described by the equilibrium model. Boiling is apparent at heat fluxes of 25 kW/m² and above. In the low-level case, boiling is achieved at these heat fluxes after tube interaction is established. Thus, flow boiling occurred at these heat fluxes, as described by the equilibrium model, Figure 13. NB: This equilibrium model does not predict vaporisation to occur at any of the test conditions.

The visual evidence at a pressure of 850 mbar, Figure 6, shows bubbles rising up the columns. These bubbles exist in a subcooled liquid pool. The fluid thermocouples are located at a reasonably large distance from the tube centres, typically 35 mm horizontally and 31 mm vertically, Figure 2. It is possible that these thermocouples are not detecting rises in fluid temperature from fluid nearer the top of each tube. However, the bubble locations suggest that this is unlikely. The presence of a moving bubble stream is supportive of a convective component of heat transfer.

The model analysis approaches described in Section 5, when applied at a pressure of 850 mbar, produced rms differences of 17.1% and 48.6% when the Cooper correlation [13] was used to describe boiling heat transfer in the isolated tube and equilibrium models respectively. The corresponding figures achieved when the Gorenflo correlation [12] was used were 18.9% and 28.2%. The low rms differences are obtained with the isolated tube model by a systematic change from over prediction at low heat flux to under-prediction at high heat flux, Figure 8. This is not the case for the equilibrium model, Figure 12, which does give reasonable results when the Gorenflo correlation [12] is used to describe boiling. The measured wall superheats for the high level, low pressure tests, Figure 10 and Table 1, show little difference between the columns. These data are consistent with the isolated tube model. In other words, when tubes at a similar vertical position are subjected to the same conditions, the wall superheats are similar. This is not the case for the low level results at 850 mbar, Figure 8 and Table 1, where the columns behave differently. This suggests that a significant convective element is present. Also, the visual evidence, Figure 6, is suggestive of a convective contribution. Thus, it is probable that the low level, high pressure data are interactive, as described by the equilibrium model. The equilibrium model predictions suggest that two-phase flow does not occur. However, bubbles were observed, Figure 6. It is possible
that this approach could be made more accurate if it were modified to allow void fractions to exist in the subcooled pool. These void fractions would need to be small because increasing the void fraction leads to larger mass fluxes and the non-equilibrium approach demonstrates that large mass fluxes do not occur, Figure 11a.

The three test series, HL LP, LL LP and LL HP, demonstrate that two modes of behaviour are possible in the heat exchanger, the isolated tube and interactive modes. Interactive behaviour is described reasonably well by the equilibrium model. This model demonstrates that liquid subcooling allows a much lower mass flux to exist than would have been inferred from knowledge gained from kettle reboilers, as represented by the non-equilibrium model. The mass flux in the current evaporator is driven mainly by natural convection and not void fraction. The corresponding gas-mass fractions, when they exist, are very small so that the two-phase multiplier effect is negligible, Equation (21). Thus, tube cooling by convection is due to the magnitude of the mass flux. There are two opposing effects at play within the heat exchanger. The void fraction will only increase if bubbles are generated. However, increasing the void fraction will lead to much larger mass fluxes that will cool the tube wall and prevent bubbles from being generated. Mass fluxes consistent with the non-equilibrium model will only occur when the mass flux induced permits the tube surfaces to remain hot enough to generate vapour, as was the case for kettle reboilers boiling refrigerants.

The high level, low pressure data were best described by the isolated tube model, in which heat transfer is by convection or nucleation. Both low level data sets are reasonably described by the equilibrium model, which can have both boiling and nucleation occurring simultaneously. The ratio of the convective to the total heat flux predicted by the equilibrium model is shown in Figure 16. The Gorenflo correlation [12] was used to evaluate the nucleate boiling component. At a pressure of 850 mbar, Figure 16a, the heat flux is convective at a heat flux of 10 kW/m². The convective fraction reduces with increasing heat flux, reaching about 50% at a heat flux of 70 kW/m². At a pressure of 50 mbar, Figure 16b, the heat flux is convective at a heat flux of 10 kW/m². The convective fraction reduces with increasing heat flux, reaching about 80% at a heat flux of 65 kW/m². Thus, even when boiling is present, low level flows are dominated by convection.

**Acknowledgement**

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**References**
Figure 2: Thermocouple locations
Figure 3: Variation of stream temperature with rig location

Figure 4a: Variation of liquid temperature with heat flux on Row 1

Figure 4b: Variation of liquid temperature with heat flux on Row 6
Figure 5: Wall temperature variation with heat flux

- HP column 1
- HP column 2
- HP column 3
- LL LP column 1
- LL LP column 2
- LL LP column 3
- HL LP column 1
- HL LP column 2
- HL LP column 3
- HP boiling onset temperature
- LL LP boiling onset temperature
- HL LP boiling onset temperature
- HP saturation temperature
- LL LP saturation temperature
- HL LP saturation temperature

Figure 5: Wall temperature variation with heat flux
Figure 6: Boiling visualisations at 850 mbar

Figure 7: Boiling visualisations at 50 mbar
Figure 8: Wall superheat comparisons with the isolated tube method at 850 mbar

Figure 9: Wall superheat comparisons with the isolated tube method at 50 mbar (low level)
Figure 10: Wall superheat comparisons with the isolated tube method at 50 mbar (high level)

Figure 10a: Column 1

Figure 10b: Column 2

Figure 10c: Column 3

Figure 10d: All columns

Figure 11a: Wall superheat comparisons with the non-equilibrium method at 850 mbar

Figure 11b: Wall superheat comparisons with the non-equilibrium method at 90 mbar
Figure 12: Wall superheat comparisons with the equilibrium method at 850 mbar

Figure 13: Wall superheat comparisons with the equilibrium method at 50 mbar (low level)
Figure 14: Wall superheat comparisons with the equilibrium method at 50 mbar (high level)

- 10 kW/m² a
- 25 kW/m² a
- 40 kW/m² a
- 55 kW/m² a
- 65 kW/m² a

- 10 kW/m² b
- 25 kW/m² b
- 40 kW/m² b
- 55 kW/m² b
- 65 kW/m² b

Figure 15a: Methods comparison at a heat flux of 25 kW/m² (high level)

Figure 15b: Methods comparison at a heat flux of 40 kW/m² (high level)

Figure 16a: Comparison of heat-flux ratio with position

Figure 16b: Comparison of heat-flux ratio with position
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Table 1: Summary of methods statistics