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On the groundstate of octonionic matrix models in a ball

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A B S T R A C T

In this work we examine the existence and uniqueness of the groundstate of a SU(N) × G2 octonionic matrix model on a bounded domain of R N. The existence and uniqueness argument of the groundstate wavefunction follows from the Lax–Milgram theorem. Uniqueness is shown by means of an explicit argument which is drafted in some detail.

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1. Introduction

Matrix model groundstate wavefunctions have been investigated by means of different approaches in the past. In the quest for a better understanding of M-theory, this has been carried out either from the point of view of Supermembrane Theory [1–3]; or from the point of view of matrix models [4]; or from the point of view of Yang–Mills theories described in the slow mode regime [5]. The existence of a groundstate has been tested indirectly by means of a unique normalizable zero-energy wavefunction invariant under SO(9) × SU(N) [6–8]. Direct attempts to characterize it include those reported in [1,2]. Regimes near the origin were examined in [9] and asymptotic regimes were considered in [5] for different supersymmetric matrix models. The interest in a better understanding of these theories relies in part on the AdS/CFT conjecture [10].

The N = 16 supersymmetric SU(N) matrix model is dual to the decoupling limit of the D0-brane geometry in the type IIA String Theory [11,12]. The spectrum of this theory is continuous from zero to infinity as it was established in [13]. In the BFSS interpretation, the nonzero energy eigenstates of the model correspond to scattering states that form a continuum. In the bulk picture, black holes can decay into radiating D0-branes that can escape to infinity [14]. The D0-branes are long-life metastable states associated in the bulk description to the microstates of a black hole at finite temperatures.

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onic twists and a gauge $G_2 \times U(1) \times SU(N)$ symmetry was considered in [22]. The symmetry in the latter was obtained by performing a deformation of the original 11D supermembrane matrix model described in a 11D Minkowski spacetime theory [1].

2. The truncated octonionic $D = 11$ supermembrane

This example of truncation of the supermembrane in the L.C.G. was originally formulated [1] in a Minkowski spacetime $M_9$. Although it is possible to obtain explicitly two solutions for the ground state problem, it was shown in [1] that they fail to be square-integrable so the truncated model has no massless states. In this paper we examine the ground state wave function, but now for the restricted case when the spatial part of the target space is compact.

The model may be formulated in terms of a pure imaginary octonion with coefficients valued on the $su(N)$ algebra:

$$X = X^A_i e^i X_A,$$

where $X^A_i$ are real $(0 + 1)$ fields which only depend on the time coordinate, $X_A$ are the generators of the $su(N)$ algebra, $A = 1, \ldots, N^2 - 1$ and $e_i$ denote the pure imaginary basis of the octonionic non-associative division algebra.

The other object involved in the supersymmetric model is a pure imaginary octonion with coefficients valued on the $su(N)$ algebra

$$\lambda = \lambda^A_i e^i X_A,$$

where $\lambda^A_i$ are $(1 + 0)$ fields valued on an odd part of a Grassmannian algebra.

The quantum hamiltonian is given by

$$H = H_b + H_f.$$

The bosonic part of the hamiltonian is a Schrödinger operator

$$H_b = -\frac{1}{2} \Delta + V(X),$$

where

$$V(X) = \frac{1}{4} f_{ABC} f_{DEF} X^A_i X^B_j X^C_k X^D_l,$$

and $f_{ABC}$ are the structure constants of $su(N)$ algebra

$$[T_A, T_B] = f_{ABC} T_C.$$

The fermionic part of the hamiltonian is given by

$$H_f = -f_{ABC} X^A_i X^B_j \lambda^C_l \frac{\partial}{\partial \lambda^D_l},$$

where $\lambda^C_l$ are the structure constants $[e^i, e^j] = c^{ijk} e^k$ of the octonian algebra. The bosonic potential $V(X)$ is a quartic polynomial in $X$, while the fermionic hamiltonian is linear in the $X$ variable.

These are the characteristic properties of the hamiltonian of the $D = 11$ supermembrane. The hamiltonian is invariant under the group $G_2$, the automorphisms of the octonions, and under $SU(N)$ associated to the regularized model. These are rigid symmetries. The hamiltonian is also invariant under $N = 1$ supersymmetry with the generators:

$$Q = \left( \frac{\partial}{\partial X^A_i} + \frac{1}{2} c^{ijk} f_{ABC} X^B_j X^C_k \right) \lambda^A_i,$$

$$Q^+ = \left( \frac{\partial}{\partial X^A_i} - \frac{1}{2} c^{ijk} f_{ABC} X^B_j X^C_k \right) \frac{\partial}{\partial \lambda^A_i}. \quad (1)$$

The corresponding anticommutation relations are:

$$\{ Q, Q \} = \{ Q^+, Q^+ \} = 0, \quad \{ Q, Q^+ \} = 2H.$$
When $(B, j) = (\hat{A}, i)$ there is no restriction on the corresponding wavefunction (from the anticommuting properties of $\lambda$). For the generic term we obtain
\[
\partial \Psi_{m_{B_1} \ldots m_{B_m}}^{j_1 \ldots j_m}(X) \bigg|_{\lambda_i} = 0
\] (4)
for the indices $(B_1, j_1), (B_2, j_2), \ldots, (B_m, j_m)$ different from $(\hat{A}, i)$. Without loss of generality we can assume $\Psi_{m_{B_1} \ldots m_{B_m}}^{j_1 \ldots j_m}(X)\bigg|_{\lambda_i}$ to be antisymmetric under the exchange of the $(B, j)$ indices.

Now, assume that $Q^1\Psi = 0$ in $\Omega$. Approach to a point on $\chi^\lambda_i = (\text{ctt})$. We have
\[
\partial \Psi_{\lambda_i} \bigg|_{\lambda_i} = 0.
\]

Hence the relevant restriction for $\Psi_{\lambda_1}$ is
\[
\partial \Psi_{\lambda_i} \bigg|_{\lambda_i} = 0.
\] (5)
In an explicit form we obtain
\[
\Psi_{\lambda_i} = 0.
\] (6)

From (3) and (6), we get
\[
\partial \Psi_{\lambda_i} \bigg|_{\lambda_i} = 0
\]
for all $(B, j)$. Consequently
\[
\partial \Psi_{\lambda_i} \bigg|_{\lambda_i} = 0
\]
for all $(B, j)$ and $(A, i)$ at the hypersurface $\chi^\lambda_i = (\text{ctt})$. For the generic term in the expansion we get
\[
\partial \Psi_{m_{B_1} \ldots m_{B_m}}^{j_1 \ldots j_m} \bigg|_{\lambda_i} = 0
\] (7)
which together with (4) yields
\[
\Psi_{m_{B_1} \ldots m_{B_m}}^{j_1 \ldots j_m} \bigg|_{\lambda_i} = 0
\]
on the hypersurface defined by $\chi^\lambda_i = (\text{ctt})$ for any set of indices.

Therefore, according to the arguments in the previous two paragraphs, the condition $H\Psi = 0$ in $\partial \Omega$, implies that
\[
\frac{\partial \Psi}{\partial X_i^\lambda} = 0 \quad \text{on} \quad \partial \Omega.
\]

Now, the equation
\[\psi = 0\]
is an elliptic system of partial differential equations on the components of $\Psi$: $\Psi_0, \Psi_1, \Psi_2, \ldots$. On $\partial \Omega$ we have
\[\Psi_0 = \Psi_1 = \Psi_2 = \ldots = 0\]
\[
\partial_n \Psi_0 = \partial_n \Psi_1 = \partial_n \Psi_2 = \ldots = 0.
\]
The partial differential system has analytic coefficients. Then, by virtue of the Cauchy–Kowalevski Theorem, it follows that $\Psi = 0$ on $\Omega$.

A detailed argument along these lines can be rigorously established and will be reported in due course.

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