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Continuum Damage Interactions Between Tension and Compression in Osteonal Bone

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Abstract

Skeletal diseases such as osteoporosis impose a severe socio-economic burden to ageing societies. Decreasing mechanical competence causes a rise in bone fracture incidence and mortality especially after the age of 65 y. The mechanisms of how bone damage is accumulated under different loading modes and its impact on bone strength are unclear. We hypothesise that damage accumulated in one loading mode increases the fracture risk in another.

This study aimed at identifying continuum damage interactions between tensile and compressive loading modes. We propose and identify the material constants of a novel piecewise 1D constitutive model capable of describing the mechanical response of bone in combined tensile and compressive loading histories. We performed several sets of loading – reloading experiments to compute stiffness, plastic strains, and stress-strain curves.

For tensile overloading, a stiffness reduction (damage) of 60 % at 0.65 % accumulated plastic strain was detectable as stiffness reduction of 20 % under compression. For compressive overloading, 60 % damage at 0.75 % plastic strain was detectable as a stiffness reduction of 50 % in tension. Plastic strain at ultimate stress was the same in tension and compression. Compression showed softening and tension exponential hardening in the post-yield regime. The hardening behaviour in compression is unaffected by a previous overload in tension but the hardening behaviour in tension is affected by a previous overload in compression as tensile reloading strength is significantly reduced.

This paper demonstrates how damage accumulated under one loading mode affects the mechanical behaviour in another loading mode. To explain this and to illustrate a possible implementation we proposed a theoretical model. Including such loading mode dependent damage and plasticity behaviour in finite element models will help to improve fracture risk analysis of whole bones and bone implant structures.

Keywords: cortical bone, bone damage, conewise elasto-plastic damage law, osteoporosis, bone strength,
1. Introduction

Skeletal diseases such as osteoporosis expose a severe socio-economic burden to ageing societies (World Health Organization, 2003; Burge et al., 2007; Mithal et al., 2009). Osteoporosis is a progressive skeletal disease characterised by a loss of bone mass and impaired cellular repair mechanisms (Metcalf, 2008; Teti, 2011). Both induce a loss of structural integrity and mechanical competence in whole bones. This decreasing mechanical competence causes a rise in bone fracture incidence and mortality (Marcus and Majumdar, 2001; Karlsson et al., 2005).

In case of the proximal femur, where the most detrimental fractures are observed, it is widely accepted that the load direction during a fall is distinct from the loading of everyday activities (Keyak, 2000). Regarding the elderly, even small loads in non-habitual load situations during daily activities such as collisions on the edge of a table, falling or simply lifting daily shopping off axis could damage the bone and, thus, increase the risk of fracture (Pinilla et al., 1996; US Department of Health and Human Services and Others, 2004). In fact, 90% of all hip fractures are related to falls and a third of all people over 65 years fall annually (Tinetti, 2003). It should be noted that every hip fracture is initiated by emerging microcracks (Figure 1). The mechanisms how bone damage is accumulated under different loading modes and coupled into another loading mode and its impact on bone strength are unclear.

Figure 1: Basic Fuchsin stained, transverse histological sections of a sample overloaded in uniaxial tension (+) or uniaxial compression (−) are shown. Note, that small or perpendicular microcracks may close upon unloading (Sun et al., 2010) and are, thus, difficult to detect histologically. The images of the samples overloaded in tension are taken close to a macrocrack of which some effects are partially visible. The cracks in this modes were smaller and more diffuse than those found under compression. The scale bar is 0.1 mm long.

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Pre-failure microdamage in bone is considered to be the most detrimental factor in defining its strength and toughness with respect to health and disease (Gupta and Zioupos, 2008). It accumulates as microcracks (Figures 1) due to isolated, non-physiological overloading events in a quasi-static mode or after suffering a large number of physiological loading cycles in a fatigue mode (Schaffler et al., 1989; Moore and Gibson, 2003; Fleck and Eifler, 2003; Rapillard et al., 2006; Wolfram et al., 2011). Microdamage is not detectable using clinical imaging modalities but decreases bone’s stiffness, strength, and toughness and eventually leads to collapse of whole bones (Fyhrie and Schaffler, 1994; Burr et al., 1997; Kopperdahl et al., 2000; Nalla et al., 2005; Ritchie et al., 2005; Ritchie, 2011).

Bone microdamage can be classified into microcracks and diffuse damage which are microcracks on a lower length scale (Fazzalari et al., 1998; Vashishth, 2007). Microcracks appear linear and spatially organised in 2D histological sections with a pertinent length of 10 μm to 70 μm (Reilly and Currey, 1999; Vashishth, 2007; Arlot et al., 2008; Zhang et al., 2010). In 3D, microcracks appear in approximately elliptical shape with an aspect ratio of 4:1 to 5:1 (Zioupos and Currey, 1994; O’Brien et al., 2000; Larrue et al., 2011). Their thickness is one to two orders of magnitude smaller than their length. Microcracks are associated and guided by micro-structural and ultra-structural features of bone (Rho et al., 1998; Jepsen et al., 1999; O’Brien et al., 2005, 2007). They appear at highly mineralised zones in bone tissue, between interstitial lamellae, along osteonal cement lines, at the boundaries of trabecular packages, at resorption cavities in trabecular bone, and, in case of sub-lamellar microcracking, along the canaliculi in cortical bone (Zioupos and Currey, 1994; Jepsen et al., 1999; O’Brien et al., 2005, 2007; Vashishth, 2007; Peterlik et al., 2006; Slyfield et al., 2012; Ebacher et al., 2012).

It was noted that microdamage is loading mode dependent as it appears to be different in bone regions loaded primarily in tension compared to regions loaded primarily in compression (Jepsen and Davy, 1997; Reilly and Currey, 1999). In histology studies, tensile microdamage appears to be more diffuse while compressive microdamage is rather expressed as microcrack (Boyce et al., 1998; Wang and Gupta, 2011) due to the different yielding process for tension and compression in bone (Reilly and Burstein, 1975; Currey, 2002). Microcracks open approximately normal to the loading direction under tension and show a cross-hatched pattern under compressive loading (Figure 1, Chamay (1970); Boyce et al. (1998)). This loading mode dependent accumulation of microdamage eventually leads to the earlier noted difference in the post-elastic behaviour of cortical bone (Reilly and Burstein, 1975; Currey, 2002; Gupta and Zioupos, 2008; Nyman et al., 2009; Li et al., 2013).

Although important for understanding fragility of bones and bone-implant systems loaded along different modes, there is no substantial data on the mutual influence and mechanical consequences of bone microdamage accumulated in such different modes.

Continuum damage mechanics (CDM) is a well developed framework to describe the
impact of emerging microcracks on the mechanical behaviour of a material. An extensive
body of work has evolved to describe brittle, quasi-brittle, ductile, creep, ratcheting, and fa-
tigue failure in different materials such as concrete, metal alloys, ceramics, etc. (Kachanov,
1958; Horii and Nemat-Nasser, 1983; Bažant and Oh, 1983; Lemaitre, 1984; Bažant and Oh,
1985; Bažant and Prat, 1988a,b; Chaboche, 1992; Lemaitre and Chaboche, 2000; Lemaitre
et al., 2000; Lemaitre and Desmorat, 2005; Chaboche, 2008). Capturing the “unilateral
effect” present for instance in concrete (Mazars et al., 1990; Halm and Dragon, 1996; Wele-
mane and Cormery, 2003; Challamel et al., 2005; Cormery and Welemane, 2010; Goidescu
et al., 2013) where damage under tension evolves differently than damage under com-
pression or more generally loading mode dependent damage mechanisms remains an open
question. For initially anisotropic materials such as bone, the asymmetric behaviour is for
instance modelled using spectral decomposition (Schreyer, 1995; Biegler and Mehrabadi,
1995) or by multi-mechanism approaches (Saï and Cailletaud, 2007; Saï et al., 2011; Saï,
2011). Crack closure in orthotropic materials was modelled recently in 2D (Goidescu et al.,
2013). Furthermore, micromechanics based approaches were used to model the closure of
microcracks that appear as elliptic voids emerging in a matrix along predefined failure
planes (Ladeveze, 1995; Guéry et al., 2008; Flatscher and Pettermann, 2011). In case of
fibre reinforced composites different damage modes were associated with fibre and matrix
damage (Schuecker and Pettermann, 2006; Maimí et al., 2007a,b; Flatscher et al., 2009;
Flatscher and Pettermann, 2011). In case of bone tissue, a time dependent damage model
was proposed for the tensile case in 1D (Fondrk et al., 1988). In an effort to describe the
mechanical behaviour of bone tissue in 3D several CDM models were proposed that feature
an initially anisotropic stiffness tensor and one irreversible process with a scalar damage
variable (Zysset and Curnier, 1996; Charlebois et al., 2010; Schwiedrzik and Zysset, 2013).
Garcia et al. (2010) proposed a 1D model capable of capturing the asymmetric damage
behaviour of cortical bone. The model features a full coupling between tensile and com-
pressive damage and a decoupled plastic process. Since damage and plasticity are tightly
coupled in bone tissue the mutual influence of microdamage accumulated under different
loading modes as well as a model that captures this influence remain unclear.

We therefore, aimed at investigating how damage is coupled between tension and com-
pression and how a damage state accumulated under one loading mode influences the post
elastic regime under another. Specifically we hypothesise: i) microdamage accumulated
under tension does not affect the elastic modulus under compression; ii) microdamage
accumulated under compression affects the elastic modulus in tension; iii) hardening in
one loading mode is affected by a previous overloading in a similar fashion then the mi-
crodamage; iv) cortical bone does not show kinematic hardening and Bauschinger’s effect
(Bauschinger, 1886) is missing.
2. Theoretical Model

To identify the damage and hardening interactions and to test the stated hypotheses, we formulate a 1D mechanical model that is capable not only of distinguishing different post-yield behaviour in tension and compression along the osteonal axis but also of describing the coupling mechanisms between the two modes. For the sake of simplicity, the constitutive model does not include the influence of torsion and does not include 3D stress and strain states. Tension and compression are physiologically interesting load cases for cortical bone. Furthermore, a 1D model is appropriate to illustrate our fundamental ideas. The rheological model features a damageable spring and a plastic slider in series. Both elements behave different in tension and compression (Figure 2). Since plasticity and damage are tightly coupled, the distinct damage processes are steered with distinct plastic mechanisms. Justified by experimental observation, we assume small strains and split the strain $\varepsilon$ according to Green and Naghdi (1965) into an elastic $\varepsilon^e$ and a plastic part $\varepsilon^p$, so that

$$\varepsilon = \varepsilon^e + \varepsilon^p.$$  \hspace{1cm} (1)

To account for different irreversible mechanical processes in tension (+) and compression (−), the accumulated plastic strain in the respective loading mode is defined as

$$\kappa^+(t) = \int_0^t H(\dot{\varepsilon}^p)|\dot{\varepsilon}^p| \, d\tau$$

$$\kappa^-(t) = \int_0^t H(-\dot{\varepsilon}^p)|\dot{\varepsilon}^p| \, d\tau$$ \hspace{1cm} (2)

with $H$ denoting the Heaviside step function, $t$ the time and $d\tau$ the infinitesimal time differential.
We define a piecewise damaged Helmholtz free energy

$$
\psi = \begin{cases} 
\frac{1}{2} (1 - D^-(\kappa^-, \kappa^+)) \epsilon_0 (\varepsilon - \varepsilon^p)^2 & \text{if } (\varepsilon - \varepsilon^p) < 0 \\
\frac{1}{2} (1 - D^+(\kappa^-, \kappa^+)) \epsilon_0 (\varepsilon - \varepsilon^p)^2 & \text{if } (\varepsilon - \varepsilon^p) \geq 0
\end{cases} 
$$

and use it to identify the state equations in the usual way. As known from the literature (Reilly and Burstein, 1975; Currey, 2002), the initial modulus $\epsilon_0$ is assumed to be equal in tension and compression. Following our hypotheses, a plastic process that occurred under one loading mode may have an influence on the other one. Furthermore, we observe that the occurrence of irreversible strains in bone is tightly coupled to the emergence of microcracks. These microcracks, in turn, reduce the carrying area and trigger a reduction of macroscopic stiffness. Therefore, we couple the evolution of damage on the accumulated plastic strains $\kappa^+$ and $\kappa^-$ in an exponential fashion (Zysset and Curnier, 1996; Wolfram et al., 2011) along the respective tensile and compressive loading modes:

$$
D^-(\kappa^-, \kappa^+) = 1 - e^{-(\beta^- - \kappa^+)} \\
D^+(\kappa^-, \kappa^+) = 1 - e^{-(\beta^+ + \kappa^+)}
$$

where $\kappa^-, \kappa^+$ are the driving accumulated plastic strains, $\beta^+$ and $\beta^-$ are material constants relating the decrease in stiffness with the plastic strain accumulated along the same loading mode, while $\beta^+$ and $\beta^-$ are interaction constants that represent the influence of plastic deformation previously accumulated in the other loading mode.

The stress $\sigma$ is given as

$$
\sigma = \frac{\partial \psi}{\partial \varepsilon} = \begin{cases} 
(1 - D^-(\kappa^-, \kappa^+)) \epsilon_0 (\varepsilon - \varepsilon^p) & \text{if } (\varepsilon - \varepsilon^p) < 0 \\
(1 - D^+(\kappa^-, \kappa^+)) \epsilon_0 (\varepsilon - \varepsilon^p) & \text{if } (\varepsilon - \varepsilon^p) \geq 0
\end{cases}
$$

and the plastic stress as

$$
\sigma^p = -\frac{\partial \psi}{\partial \varepsilon^p} = \sigma.
$$

The conjugate damage energy densities are given as

$$
W^- = -\frac{\partial \psi}{\partial \kappa^-} = \begin{cases} 
\frac{1}{2} \frac{\partial D^-}{\partial \kappa^-} \epsilon_0 (\varepsilon - \varepsilon^p)^2 & \text{if } (\varepsilon - \varepsilon^p) < 0 \\
\frac{1}{2} \frac{\partial D^+}{\partial \kappa^-} \epsilon_0 (\varepsilon - \varepsilon^p)^2 & \text{if } (\varepsilon - \varepsilon^p) \geq 0
\end{cases}
$$
and

\[ W^+ = -\frac{\partial \psi}{\partial \kappa^+} = \begin{cases} \frac{1}{2} \frac{\partial D^-}{\partial \kappa^+} \epsilon_0 (\varepsilon - \varepsilon^p)^2 & \text{if } (\varepsilon - \varepsilon^p) < 0 \\ \frac{1}{2} \frac{\partial D^+}{\partial \kappa^+} \epsilon_0 (\varepsilon - \varepsilon^p)^2 & \text{if } (\varepsilon - \varepsilon^p) \geq 0 \end{cases} \]  

(8)

With these state variables we can express the isothermal dissipation \( \Phi \) that satisfies the Clausius-Duhem inequality

\[ \Phi = \sigma \dot{\varepsilon} - \psi = \sigma^p \dot{\varepsilon}^p + W^+ \dot{\kappa}^+ + W^- \dot{\kappa}^- \geq 0. \]  

(9)

Following previous work, the simultaneous flow of plastic strain and damage is modelled with a yield function of stress \( Y(\sigma) \) (Zysset and Curnier, 1996; Charlebois et al., 2010).

We hypothesise an associated flow rule featuring a plastic multiplier \( \dot{\lambda} \), so that

\[ \dot{\varepsilon}^p = \dot{\lambda} \frac{\partial Y}{\partial \sigma} \]  

(10)

It can be described with the classical Kuhn-Tucker conditions (e. g. Simo and Hughes (2000))

\[ Y \leq 0 \quad \dot{\lambda} \geq 0 \quad Y \dot{\lambda} = 0. \]  

(11)

Bone has distinct yield and ultimate stress amplitudes in compression and tension (Figure 3). To account for a possible coupling of the hardening behaviour, a yield criterion is proposed that has distinct hardening functions

\[ Y(\sigma) = \begin{cases} |\sigma| - \sigma_u^- (r_y^- + (1 - r_y^-) g^- (\kappa^-)) \leq 0 & \text{if } (\varepsilon - \varepsilon^p) < 0 \\ |\sigma| - \sigma_u^+ e^{-\delta \kappa^-} (r_y^+ + (1 - r_y^+) g^+ (\kappa^+; \kappa^-)) \leq 0 & \text{if } (\varepsilon - \varepsilon^p) \geq 0 \end{cases} \]  

(12)

Please note, that the composition of these functions anticipates already some experimental results. \( \sigma_u^- \) and \( \sigma_u^+ \) are the ultimate stresses, \( r_y^- \) and \( r_y^+ \) are the yield to ultimate stress ratios and \( g^- \) and \( g^+ \) are the hardening functions for each mode. The term \( e^{-\delta \kappa^-} \) includes a possible decay of \( \sigma_u^+ \) due to a previous overload in compression via the decay parameter \( \delta \). The shape of the post-yield function is solely controlled by the driving accumulated plastic strain \( \kappa^\pm \). For perfect plasticity, \( g = 1 \), while for bone in tension \( g^+ \) exhibits an exponential hardening behaviour with a slope parameter \( K^+ \). The softening post-yield behaviour found for bone in compression is modelled with a more complicated bi-
exponential function relying on the constants $A, K^-, \kappa^-_{\text{min}}, \kappa^-_{\text{max}}, g_{\text{min}}$

\[
g^-(\kappa^-) = e^{-\frac{(\kappa^- - \kappa^-_{\text{max}})^2}{\kappa^-_{\text{max}}^2 + A\kappa^-_{\text{min}}^2}} + g^-_{\text{min}} \left( 1 - e^{-\frac{(\kappa^- - \kappa^-_{\text{max}})^2}{\kappa^-_{\text{max}}^2 + A\kappa^-_{\text{min}}^2}} \right) - e^{-\left(\Sigma + K^-\kappa^-\right)} \quad \text{if } (\varepsilon - \varepsilon^p) < 0
\]

\[
g^+(\kappa^+; \kappa^-) = 1 - e^{-K^+(\kappa^+ + \frac{\kappa^+ - \kappa^-}{\kappa^+ - \kappa^-})} \quad \text{if } (\varepsilon - \varepsilon^p) \geq 0
\]

with

\[
\Sigma = -\ln \left( e^{-\frac{(\kappa^- - \kappa^-_{\text{max}})^2}{\kappa^-_{\text{max}}^2 + A\kappa^-_{\text{min}}^2}} + g^-_{\text{min}} \left( 1 - e^{-\frac{(\kappa^- - \kappa^-_{\text{max}})^2}{\kappa^-_{\text{max}}^2 + A\kappa^-_{\text{min}}^2}} \right) \right).
\]

Figure 3: The figure shows a schematic representation of the proposed damaged elasto-plastic material law. The behaviour is asymmetric ($\sigma^+_{y} \neq \sigma^-_{y}$) with an initially equal stiffness $\varepsilon_0$ that suffers two different damage ($D^+, D^-$) mechanisms. Furthermore, the proposed model allows for distinct hardening in tension and compression.

The numerical implementation follows classic schemes presented for instance in Curnier (1994); Simo and Hughes (2000). Briefly, a trial state $\varepsilon, \varepsilon^p_0, \kappa^+_0$, and $\kappa^-_0$ with a corresponding trial stress is assumed at the beginning of an increment (subscript 0 denotes $t = 0$, no subscript the end of the increment). If this stress state validates the yield criterion $Y \leq 0$ the stress increment is purely elastic. If $Y > 0$ the stress increment is inelastic and a back-projection on the yield point has to be performed. Using a Newton-Raphson algorithm, we can compute the unknowns $\varepsilon^p, \kappa^+, \kappa^-$. To do so the stress is linearised around the solution using the yield surface (12). By enforcing the Kuhn-Tucker conditions
and linearisation, this equation can be solved for the unknowns in an iterative fashion. The iteration is stopped as soon as a given tolerance is reached and the improved step results are computed.

3. Materials and Methods

**Specimen preparation:** Four femoral mid-diaphyseal shafts from different mature cows approximately five years of age were obtained from a local butcher. Each femoral shaft was divided into three approximately 40 mm long diaphyseal pieces by a cut perpendicular to the shaft’s length axis using a band saw (Exakt Apparatebau, Norderstedt, Germany). The two cutting faces were roughly polished using silicon carbide paper to visually check the osteonality. The pieces were then divided along the longitudinal direction into parallelepipeds with a cross-section of approximately $6 \times 6 \text{ mm}^2$. The superior and inferior ends of these parallelepipeds were inspected under a light microscope (Leica MS5). Specimens that did not show two fully osteonal cross-sections were discarded (Figure 4). Before and after sawing, all specimens were kept in 0.9 % sodium chloride. Sawing was performed under running tap water. The raw cortical pieces were dried for 24 h at room temperature. Subsequently, they were glued in cylindrical aluminium tubes with an inside diameter of 6 mm and an outside diameter of 10 mm using an epoxy adhesive (UHU plus endfest 300, Germany). If necessary, the edges of the pre-specimens were ground using silicon carbide paper (320) so that they fitted the inner diameter of the aluminium tubes. The outer side of the bone specimens as well as the aluminium tubes were defatted using

![Figure 4: Cortical parallelepipeds were cut from the proximal part of the diaphysis of bovine femurs. After roughly polishing the end faces, osteonality was visually controlled under a reflecting light microscope. Subsequently, the cortical pieces were glued into aluminium cylinders. After curing, dumbbell shaped specimens were lathed from these aluminium-bone cylinders. The remaining aluminium parts provided proper endcaps. Totally 93 specimens were prepared of which 68 could be successfully tested.](image-url)
acetone prior to gluing. From these aluminium-bone specimens, dumbbell shaped spec-
imens were produced on a CNC lathe (Schaublin 102-CNC) equipped with a computer
umeric control (Heidenhain MANUALplus 620). The gauge length of the specimens had
a diameter of 2.95 mm and a length of 6.5 mm with an inlet radius of 7 mm. A linear-
elastic FE simulation confirmed that this radius produces a quasi-homogeneous stress state
in the gauge length. Furthermore, this preparation procedure resulted in properly aligned
decapped specimens with a total length of 40 mm. In total, 93 specimens were produced
for three sets of experiments.

Specimens were re-hydrated in Hank’s buffered salt solution (HBSS) for 12 h at 4°C
and at least two hours at room temperature before testing. The specimens were wrapped
in tissue soaked in HBSS solution and continuously kept wet for the duration of the test.

**Three different sets of mechanical experiments** were performed using a biaxial
15 kN and 150 Nm servo-hydraulic testing system (MTS Mini-Bionix 858). The specimens
were loaded in displacement control along the cylindrical main axis. Axial strains were
measured using a custom made biaxial extensometer (Epsilon Tech 3550HT). Quasi-static
tests were performed at a strain rate of \( \approx 3 \cdot 10^{-4} \) s\(^{-1}\) in tension and compression (con-
stant stroke rate of \( 6.4 \cdot 10^{-3} \) mm s\(^{-1}\)). All tests were performed at room conditions with
approximately 23°C and 44 % relative humidity. Data was acquired at a sampling rate of
100 Hz. The recordings included time, stroke, force, and axial strain.

**In experiment I, monotonic tests** were performed in tension and compression
on 10 specimens per load case to establish a baseline for the two following experiments.
The specimens were preconditioned with three load cycles up to 0.05 % strain. This low
pre-conditioning allowed to check whether the extensometer was attached, the loading
loop closed properly, and to rescue the specimen in case of early extensometer slippage.
Subsequently, they were loaded monotonically to failure (Table ??).

Nominal stress \( \sigma \) in compression and tension was calculated as force divided by the
initial cross-sectional area of the sample. Small strain \( \varepsilon \) in tension and compression was
directly captured by the extensometer. Yield stresses and yield strains were determined
from the monotonic stress-strain curves using a 0.05 % and a 0.2 % offset criterion. Please
note, we regard the 0.05 % criterion as onset of yield and the 0.2 % criterion as an upper
bound that allows us to compare our results to the literature. The initial elastic modulus
\( (\varepsilon_0) \) was determined using a moving regression with a box width of 0.2 % strain to identify
the stiffest section of the loading part. Ultimate stress and strain was measured as the
maximum stress and its corresponding strain.

**In the second experiment, damage interrogation tests** were performed on 24
samples for two overloading cases. In these experiments, overloading cycles in one loading
mode were followed by three conditioning cycles between the overloading stress and zero
load. Subsequently, three elastic interrogation cycles in the other loading mode were
performed. For instance, an overloading in tension was followed by three conditioning
cycles and three elastic cycles in compression. After the interrogation, the next overloading cycle was performed. The first overloading cycle was situated before reaching yield. The following three cycles were then in the post-yield regime. The limits for the overloading cycles were defined based on the results of the monotonic tests (Table ??). The strain limits for the elastic interrogation cycles were chosen to be below the 0.05 % yield strain for each loading mode. We accounted for different yield points in tension and compression. We fixed these limits to 0.35 % strain in compression and 0.25 % strain in tension. A sketch of the loading protocol can be found in the appendix (Figure B.11). Please see Figure 5 for a set of exemplary stress-strain curves.

Stress and strain curves were obtained from the force-displacement curves similar to the monotonic tests. Damaged elastic moduli in the overloading cycles were calculated by a linear regression through the full last cycle of the conditioning step after the overload. Elastic moduli in the interrogation cycles were obtained from a linear regression through the full last cycle of the interrogation step. Damage was determined as decay in modulus

\[ D_i = 1 - \epsilon_i / \epsilon_0 \] during the load steps \( \epsilon_i \). Plastic strains were obtained from the unloading part of the third cycle in the conditioning step per load step at zero stress (Figures 5).

To identify the impact of damage in one loading mode on the other loading modes, we relied on the split of the accumulated plastic strain (2) and used the proposed damage functions (4). The interaction parameters \( \beta^+ \), \( \beta^- \), \( \beta^{++} \), and \( \beta^{--} \) are identified by using both damage functions. Suppose an overloading in tension. Only \( \kappa^+ \) accumulates so that \( \kappa^+ \neq 0 \) and \( \kappa^- = 0 \). Hence, \( \beta^{++} \) can be identified by using \( D^+ \) and \( \beta^+ \) by using \( D^- \).

It is possible to identify the initial hardening functions as proposed in (13) from the interrogation experiments. First, we correlated plastic strains with the imposed strain levels. We assumed \( \kappa = m(\varepsilon - \varepsilon_{0.05}) + n \) (with \( m, n \) slope and intercept) which was fitted to the measured plastic strains of the third conditioning cycle and the corresponding strain limits \( \varepsilon \) from load step 2 to 4. We enforced \( n = 0 \) and used \( \varepsilon_{0.05} \) as monotonic yield strain based on the 0.05 % yield criterion in each loading mode. Plastic strains of the initial step of the interrogation tests were in the order of \( 10^{-4} \) and in the order of \( 10^{-3} \) for both load cases. As yield strains are located between load step 1 and load step 2, we neglected the plastic strains in step 1 based on the fact that plastic strains in steps 2, 3, and 4 were at least an order of magnitude larger. Using these relations we could convert the monotonic stress – strain curves into stress – accumulated plastic strain curves which were normalised to the corresponding strength. These curves were then used to identify the hardening behaviour for each loading mode using a non-linear least square fit. In case of compression, the plastic strain at maximum stress \( \kappa^+_{\text{max}} \), plastic strain at the last stress point \( \kappa^-_{\text{min}} \), and \( g_{\text{min}} \) as the ratio of the last stress to ultimate stress can be identified from these curves. After normalisation of the stress-plastic strain curves to the ultimate stress a non-linear least squares fit was performed to identify the curvature parameters \( K^+, K^- \), and \( A \).
In the third experiment, hardening tests were performed based on the monotonic and the interrogation experiments on twelve samples per loading – reloading case (Table ??). For each loading – reloading combination three different overloading limits (four samples each) were specified. In contrast to experiment II, we overloaded to one plastic step. Similar to experiments II, we performed three conditioning cycles after the overloading and an elastic interrogation. Instead of a subsequent plastic overloading along the previous load direction we monotonically overloaded in the other loading mode. For instance, an overloading in tension was followed by three conditioning cycles, three elastic interrogation cycles in compression to identify the damage interaction and a monotonic reloading in compression. Please note, we also performed the elastic step 1 similar to experiment II for the sake of comparison (Table ??). Stress – strain curves and all mechanical variables were identified in a similar fashion as in experiment I and II. A set of exemplary stress-strain curves illustrating the parameter identification is shown in Figure 7.

Statistical analyses were performed with Gnu R (R Development Core Team, 2008). Due to linear quantile–quantile plots and a positive post-hoc Shapiro-Wilk normality test (Shapiro and Wilk, 1965), normal distributions were assumed and t-tests were performed at a significance level of $p < 0.025$. Damage and hardening functions were fitted using R’s nonlinear and linear least squares function. Log-space conversion was used in the latter case. Please note, we combined the data from the three experiments where appropriate to increase the statistical power. For instance, all initial moduli were pooled after a t-test showed no significant differences among the groups. Furthermore, the damage identified in the overloading step in experiment III was pooled with the corresponding damage functions of experiment II.

4. Results

In the monotonic tests in experiment I, all ten samples in each load case could be tested to failure. The known features of cortical bone for tension and compression (Reilly and Burstein, 1975; Fondrk et al., 1988; Martin et al., 1998; Currey, 2002; Garcia, 2006; Garcia et al., 2010) were reproduced (Figure 5, Table 1). Briefly, the initial stiffness in tension and compression is the same ($p = 0.78$). Yield and ultimate stresses and yield strains are significantly different between tension and compression ($p < 0.001$). Ultimate strain in tension and compression is not significantly different ($p = 0.95$). With this, a valid baseline for experiment II and III is provided.

In the interrogation tests of experiment II, ten samples survived all four overloading steps in tension and 6 in compression. The slopes of the linear correlation between plastic strain and stresses that were used to identify the hardening function are calculated as $m^+ = 0.60$ ($SE = 0.03$, $R^2 = 0.90$) and $m^- = 0.46$ ($SE = 0.04$, $R^2 = 0.78$).
Figure 5: The figure shows typical stress-strain curves for the experiment I and II. Top, nominal stress-strain plot for monotonic loading in uniaxial tension (UT, red) and uniaxial compression (UC, blue). Inflection points of experiment II (solid circles) and experiment III (solid triangles) are superimposed on the monotonic results. Monotonic stress strain results of the finally identified model (Section 2) are superimposed as black, solid line for each load case. Bottom left, experiment II overloading in tension and elastic interrogation in compression. Bottom right, experiment II overloading in compression and elastic interrogation in tension. The dotted lines and the black circles illustrate how stiffness and plastic strains, respectively were identified from the experiments.

Compressive and tensile moduli of step 0 and step 1 are the same for all overloading modes. The initial moduli are the same when compared to the monotonic results.

Tensile overloading (Figure 6, left) results in a significant reduction of moduli for all plastic load steps 2, 3, and 4. Interrogating in compression shows that moduli are signif-
Table 1: The table lists the mechanical properties determined in uniaxial tension (UT) and uniaxial compression (UC) in the monotonic tests of experiment I. Elastic moduli were pooled over all three experiments in the end. Yield and ultimate points are determined for Experiment I only since the samples in experiment I witnessed the interrogation cycles so that the monotonic situation was not exactly reproduced.

<table>
<thead>
<tr>
<th>Mechanical Property</th>
<th>Unit</th>
<th>Load Case</th>
<th>UT</th>
<th>UC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulus</td>
<td>$\epsilon_0$</td>
<td>GPa</td>
<td>21.63 ± 5.23</td>
<td>21.76 ± 5.09</td>
</tr>
<tr>
<td>Yield Stress 0.05 %</td>
<td>$\sigma_{0.05}$</td>
<td>MPa</td>
<td>76.22 ± 14.79</td>
<td>113.48 ± 24.27</td>
</tr>
<tr>
<td>Yield Strain 0.05 %</td>
<td>$\varepsilon_{0.05}$</td>
<td>mm/mm</td>
<td>0.0041 ± 0.0008</td>
<td>0.0058 ± 0.0011</td>
</tr>
<tr>
<td>Yield Stress 0.2 %</td>
<td>$\sigma_{0.2}$</td>
<td>MPa</td>
<td>88.78 ± 11.60</td>
<td>166.91 ± 25.49</td>
</tr>
<tr>
<td>Yield Strain 0.2 %</td>
<td>$\varepsilon_{0.2}$</td>
<td>mm/mm</td>
<td>0.0062 ± 0.0007</td>
<td>0.0098 ± 0.0014</td>
</tr>
<tr>
<td>Ultimate Stress</td>
<td>$\sigma_u$</td>
<td>MPa</td>
<td>102.54 ± 10.74</td>
<td>189.24 ± 23.44</td>
</tr>
<tr>
<td>Ultimate Strain</td>
<td>$\varepsilon_u$</td>
<td>mm/mm</td>
<td>0.015 ± 0.005</td>
<td>0.015 ± 0.004</td>
</tr>
</tbody>
</table>

Figure 6: Damage interaction as identified in experiment II. We compare the impact of an overloading in either uniaxial tension (UT, red) or uniaxial compression (UC, blue) on the other loading mode via elastic interrogation tests. The moduli are normed to the initial elasticity of load step 0 so that $(1 - D)$ is shown on a log-scale. The tables show slope, intercept, and Pearson’s correlation coefficient for the decay of moduli versus the accumulated plastic strain of the overloading step. Please note, in the left image $m = \{\beta^{++}, \beta^{+-}\}$ and in the right image $m = \{\beta^{+-}, \beta^{--}\}$ for the red and the blue curve respectively and compare Table 2.
Compressive moduli remain over 20 GPa in all load steps.

Compressive overloading (Figure 6, right) results in significant modulus reduction in tension and compression in all overloading steps. The slopes of the decay regressions are similar and the moduli in tension and compression are not significantly different for all overloading steps 2 to 4. This indicates that damage in compression is fully coupled to tension.

Identifying the damage functions (4) provides a set of significant parameters that explain the evolution of damage as a function of accumulated plastic strain. The interaction parameters reflect the impact of a previously encountered damage in another loading mode. They are comparable to the slopes of the decay regressions in Figure 6. They represent the impact of an irreversible load history with a stronger impact of compressive than tensile damage. The standard error (SE) for all fits is smaller than 13% (Table 2).

The fits of the hardening functions and (13) were significant with an $SE < 5\%$ (Table 2). These hardening functions constitute the initial unaffected hardening behaviour per loading mode.

<table>
<thead>
<tr>
<th>Damage Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D^+$ $\beta^{++}$ $D^-\beta^{+-}$ $\beta^{+\ +}$ $\beta^{--}$</td>
</tr>
<tr>
<td>186.21 143.76 55.38 135.46</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Hardening Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>load case hardening  $K$ in mm $\kappa_{max}$ in mm $\kappa_{min}$ in mm $g_{min}$ in MPa $A$ in mm $\delta$ in mm</td>
</tr>
<tr>
<td>UT exponential 336.89 - - - - 203.25</td>
</tr>
<tr>
<td>UC softening 608.20 0.0028 0.02 0.58 0.1 -</td>
</tr>
</tbody>
</table>

In the loading – reloading tests of experiment III, no impact of the tensile overloading step on the ultimate stress in the compressive reloading mode could be identified (Figures 8 and 9). It seems that both, the value of the compressive strength as well as the shape of the post-yield curve do not change. Ultimate stress of the reloading curve was not significantly different from the ultimate stress in the monotonic loading.

A compressive overload and a tensile reloading shows a significant reduction of the tensile yield point (Figures 8 and 9). Ultimate stress of the reloading curve was significantly lower than the ultimate stress in the monotonic experiment-I. Furthermore, the fit of the
decay parameter $\delta$ in (12) was significant (Table 2).

Taken together, this rebuts the existence of Bauschinger’s effect which would have been a strong indicator for a kinematic hardening mechanism.

With respect to ultimate strains, tensile overloading seems to have no impact. No significant difference was found between the ultimate strain of the compressive reloading (Figure 9) and the ultimate strain of a monotonic compression (Figure 5). An initial compressive overload, however, significantly reduces the ultimate strain in a tensile reloading. Thus, a compressive overload reduces the ductility on the tensile side while a tensile overload seems not to reduce the ductility on the compressive side.

Using the final set of parameters, the model developed in Section 2 can be identified. The identified model is able to reflect the major features of the piece-wise material behaviour (Figure 10).

5. Discussion

This study aimed at investigating how damage is coupled between tension and compression in cortical bone. Furthermore, we have been interested in how a damage state accumulated under one loading mode influences the post-yield regime in a subsequent reloading in another mode.

Figure 7: Typical stress-strain curves for the hardening tests of experiment III are shown. The overloading steps are followed by an elastic interrogation and a reloading step in another mode. Similar to step 1 in experiment II an elastic step is introduced for the sake of comparison. Left, an overloading in uniaxial tension (UT, red) and reloading in uniaxial compression (UC, blue). Right, overloading in compression and reloading in tension. The dotted lines and the black circles illustrate how stiffness and plastic strains were determined. The coloured circles show how yield and ultimate points were identified from the data.
Microdamage accumulated under uniaxial tension has a small effect on the compressive stiffness (37.5%). Even though there is a significant difference when comparing the modulus reduction for the tensile and compressive side when overloading in compression the decay of the interrogation stiffness cannot be neglected. However, the slope of the interrogation regression line was less than half of the slope of the overloading regression. Both decay regressions in Figure 5 are significant for all interrogation modes. This indicates that we cannot take on hypothesis i) since a tensile overload has some effect on the compressive side. However, the impact of a tensile overload is much less than vice versa.

Microdamage accumulated under compression has a strong effect on the elastic properties in a subsequent loading in tension. Judging from the decay regression, the impact on the interrogation mode is as high as on the overloading mode. Taking the overloading results in tension and compression together indicates the existence of a unilateral effect known from concrete (Horii and Nemat-Nasser, 1983; Mazars et al., 1990; Chaboche, 1992; Welemane and Cormery, 2003; Challamel et al., 2005; Cormery and Welemane, 2010; Goidescu et al., 2013). This means we can take on hypothesis ii) in the sense that the tensile properties are affected by a previous compressive overloading. This impact is almost

![Figure 8: Overloading in uniaxial tension (UT) seems to have only a minor impact on the strength in uniaxial compression (UC). Overloading in compression leads to a significant decrease of the strength in tension. Stresses are normalised to the initial stiffness. The bold medians and whiskers indicate the overloading mode. The thin medians and whiskers indicate the strength in the reloading mode. These stresses are compared to the strength from the monotonic loading results of experiment I (EXP-I). The solid circles indicate the mean stress at the respective strain level from the monotonic experiment I for the sake of comparison. The black triangles indicate the reloading part of the model. The black solid line illustrates the significant reduction of the tensile strength due to a compressive overload (12). The asterisk indicates a significantly lower stress in comparison to the monotonic result.](image-url)
Figure 9: The reloading part of experiment III is compared to the monotonic loading of experiment I. Left, strength $\sigma$ versus strain at strength in tension or compression. Strengths from monotonic tests in uniaxial tension (UT, solid red square) and uniaxial compression (UC, solid blue circle) performed in experiment I are compared to the reloading strengths of experiment III, e.g. overloading in compression followed by reloading in tension (UCUT) or vice versa. Right, reloading curves of experiment III show a decreased ductility when compared to the monotonic loading shown in Figure 5. Comparing the monotonic loading curve of the fitted model (solid black lines) indicates that an overload in one loading mode has little impact on the yield point when reloading in the other loading mode. Dashed black lines show the interaction hardening model after an initial overload. Stresses are normalised to the initial stiffness.

We identified an initial post-yield behaviour that is comparable to the literature (Reilly and Burstein, 1975; Fondrk et al., 1988; Martin et al., 1998; Jepsen et al., 1999; Garcia, 2006; Garcia et al., 2010). Judging from Figure 8 there is no effect of a previous plastic overloading in tension on the plastic behaviour of a subsequent loading in compression. However, Figure 8 indicates a drop of tensile properties due to a previous overloading in compression.

Furthermore, a plastic process in compression seems to decrease the ductility of a subsequent tensile reloading (see also (13)). This was not found in case of a compressive reloading. This reflects a similar coupling between the two loading modes as in case of damage. This hardening behaviour is more in line with our expectations that a tensile overload has no impact on the compressive properties but vice versa. This rather clear result makes it somewhat difficult to explain the relatively high impact of a tensile overload on compressive damage. One reason why this was not visible in the hardening experiment could have been the small number of specimens per load step. Nevertheless, we can take on hypothesis iii) since an overloading in tension does not affect the strength in compression but an overload in compression affects the strength in tension. The shape of the post-yield
Figure 10: The identified model is well able to represent the elasto-plastic behaviour found in experiment II and III. This can be seen from the well captured differing damage behaviour in uniaxial tension (UT, red) and uniaxial compression (UC, blue) after a tensile overload. Please note, that the non-linearity present when changing the loading modes (compare Figure 5) was removed from these plots since it was not a feature of the model. Due to the same reason, the interrogation part was only a fifth in case of loading in compression and reloading in tension. This was necessary since the impact on the yield stress triggered plastic flow on the other side earlier than encountered in the experiment due to missing non-linear transition.

The post-yield curve does not change and the tensile ductility is decreased after a compressive overload. However, the impact of a tensile overload on the compressive properties was different than in case of the damage variables.

We expected that the impact of a previous plastic process on the post-yield surface can be captured by the same set of interaction parameters as in the damage function (4).
However, we found that the relative importance of an overloading is not properly described by $\beta_{kl} (k,l = \{+, -\})$ and an own parameter $\delta$ is necessary. Figure 9 indicates that reloading is controlled only by the newly accumulated plastic strain. This contrasts our findings in case of the damage interaction. However, introducing an exponential damper in the tensile yield function (12) captures the decreasing yield stress after a compressive overload.

Regarding Bauschinger’s effect (Bauschinger, 1886), Figures 9 and 8 show that it seems not to be present in cortical bone. This effect would have translated the yield points in tension and compression on the expense of one another due to an overloading process. Instead we see a significant impact of a compressive overload on the yield properties in a tensile reloading but not vice versa. Therefore, we can take on hypothesis iv) and rule out kinematic hardening as a plastic process.

We have chosen an elasto-plastic model to illustrate the effect of different damage accumulations in tension and compression. Clearly, Figure 10 shows hysteresis and non-linearity that are due to viscous effects. However, the piece-wise behaviour encountered in bone is primarily due to the opening and closing of microcracks and a viscous part could be added to the rheologic model for both the elastic spring as well as for the plastic slider. However, for illustrating the hardening behaviour the viscous part is not essential as our identification results show (Figures 10 and 5). Nevertheless, comparing the model to the experimental results is somewhat tedious since the nonlinear transition zone when changing the loading modes will lead to a higher strain before the yield point is reached. The elasto-plastic model would underestimate this strain as the non-linearity is not included.

The damage interaction was modelled by a weighted additive impact of different accumulated plastic strains which contradicts other models in the literature were the interaction was modelled by an additive impact of damage variables (Mazars et al., 1990; Chaboche, 1992; Halm and Dragon, 1996; Welemane and Cormery, 2003; Challamel et al., 2005; Cormery and Welemane, 2010; Goidescu et al., 2013; Saï and Cailletaud, 2007; Saï et al., 2011; Saï, 2011). This strong coupling of plasticity and damage may be in contrast to earlier CDM models (Horii and Nemat-Nasser, 1983; Bažant and Oh, 1983; Lemaitre, 1984; Bažant and Oh, 1985; Bažant and Prat, 1988a,b; Chaboche, 1992; Lemaitre and Chaboche, 2000; Lemaitre et al., 2000; Lemaitre and Desmorat, 2005; Chaboche, 2008). However, we found that this allows to very efficiently capture the different mechanisms in tension and compression. Especially, when comparing the new model to an earlier effort to capture this interaction (Garcia et al., 2010) it turns out that the new model complies with the experimental findings.

The prepared samples turned out to be robust. The maximum shear stress in the glue layer was calculated to be 10.6 MPa assuming a maximal axial load of 2000 N on an interface area of minimally $\pi \cdot 6 \cdot 10$ mm$^2$. This was three times lower than the critical shear stress of the glue. The gluing interface failed only in five cases. Most likely due to the fact...
that the acetone defatting failed. We observed detrimental effects due to swelling of the
419 glue after keeping the specimens submerged in HBSS for days. The 12 hours rehydration
prior to testing, however, should have been harmless. With respect to the tissue it is
420 known that drying and re-wetting has only a small impact on the mechanical properties
421 of cortical bone (Currey, 1988).

More critical for the testing was the attachment of the extensometer. Twenty samples
422 failed due to slippage of its tips or detachment during plastic loading. A biaxial exten-
423 someter was used which is a coupled mechanism. To avoid misalignment errors, a custom
424 made mounting system was designed that clamped the extensometer tips on top of each
425 other between two prism brackets in exactly the gauge length distance of 6 mm. After
426 attachment of the rubbers the clamping brackets opened symmetrically due to a parallelo-
427 gram mechanism. If excessive misalignment was noticed during the initial preconditioning,
428 the test was stopped and the extensometer re-attached.

We have chosen displacement control with strain limits in the experiment. The reason
433 for this were mainly safety issues for the system. The attachment of the biaxial extensome-
434 ter is a quite intricate operation and slippage was easily possible. Running the experiments
435 under true strain control would have threatened both sample and equipment severely.

6. Conclusion

We performed three kinds of experiments in this study. First, monotonic experiments
438 were performed to identify the necessary strain limits and to establish a baseline compara-
439 ble to the literature. Subsequently, stress interaction experiments were performed to reveal
440 the impact of an overloading in one loading mode on the elastic behaviour in another load-
441 ing mode. These results directly help to understand whether a previous overloading in one
442 loading mode increases the risk of fracture in another loading mode. Such a situation is for
443 instance encountered when comparing stance and fall situations. Tensile overloading and
444 subsequent loading in compression seems not to be a problem since microcracks are closed.
An overloading in compression, however, seems to have a huge impact on the mechanical
446 resistance of the structure since its impact is fully coupled into a tensile reloading. The
447 proposed model illustrates how we envision a possible implementation for a 3D model.
Such a model would be a beneficial tool to include loading mode dependent damage and
449 plasticity into fracture risk analyses of whole bones and bone implant structures.

Appendix A. Basic Fuchsin Protocol

We coloured the specimen with a toluidine-fuchsin staining after standard polymethyl-
452 methacrylate infiltration and microscopic polishing.

1. Submerge polished sections in 40% alcohol and clean in ultrasound bath for 5 min.
2. Discard alcohol.
3. Apply formic acid 0.1% for 5 min.
4. Rinse each cut under running tab water for approximately 10 s.
5. Stain with toluidine blue for 5 min. Apply staining directly onto the cut using a pipette. Pay attention to not scratch the samples and to avoid air bubbles.
6. Discard toluidine blue and collect poisonous solution.
7. Rinse with distilled water.
8. Stain 1-2 min. with basic fuchsin 0.05% dissolved in water by applying it directly to the polished sections.
9. Discard basic fuchsin solution and rinse with distilled water.

Appendix B. Loading Protocols

Figure B.11: Strain-time curves for experiment II. Top: overloading in uniaxial tension (UT) and elastic testing in uniaxial compression (UC). Bottom: overloading in compression and elastic testing in tension. Please note, the loading protocols used in experiment III are adapted versions of the loading protocols in experiment II and indicated with the thin dashed arrows.
Conflict of interest statement

The authors have no potential conflict of interest regarding this article.

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References


Bauschinger, J., 1886. Über die Veränderung der Elastizitätsgrenze und die Festigkeit des Eisens und Stahls durch Strecken und Quetschen, durch Erwärmen und Abkühlen und durch oftmals wiederholte Beanspruchungen,. Mitteilungen aus dem Mechanisch Technischen Labor der Königlich Technischen Hochschule München.


Gupta, H. S., Zioupos, P., 2008. Fracture of bone tissue: The 'hows' and the 'whys'. Medical Engineering & Physics 30 (10), 1209–1226.


osteons as barriers to crack growth in compact bone. International Journal of Fatigue 29 (6), 1051 – 1056.


for bone tissue. Biomechanics and Modeling in Mechanobiology 12 (2), 201–213.


