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Fracture-matrix interactions during immiscible three-phase flow

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Abstract

Naturally Fractured Reservoirs (NFR) contain a significant amount of remaining petroleum reserves and are now considered for Enhanced Oil Recovery (EOR) schemes that involve three-phase flow such as water-alternating-gas (WAG) injection. Reservoir simulation of three phase flow is challenging because a proper set of flow functions, i.e. relative permeability and capillary pressure functions, that describe the underlying physics of fluid displacement is vitally important to obtain reliable production forecasts but associated with high uncertainty. For NFR, another challenge is the upscaling of recovery processes, particularly fracture-matrix transfer during three-phase flow, to the reservoir scale using dual porosity or dual permeability models.

In this work we approach a solution to these challenges by analysing three-phase flow during WAG injection at various scales, starting at the pore scale and then move on to an intermediate scale which is comparable to the scale of a single reservoir simulation grid block. At this scale, we represent fractures and matrix using a fine-grid model that employs empirical and pore-network modelling derived three-phase flow functions to study the effect of capillary and gravity forces on fracture-matrix transfer. We also consider different matrix wettabilities and permeabilities, as well as matrix block size distributions. We then perform an upscaling step that is typical for field-scale recovery simulations and use the dual porosity model to represent fracture-matrix transfer processes that were observed at the grid-block scale. This enables us to analyse and improve the accuracy of dual porosity models for three-phase displacement processes inherent to WAG in NFR.

We find that different three-phase saturation profiles develop inside matrix blocks, which are strongly dependent on wettability of the matrix. These profiles have a profound impact on recovery during WAG injection. The classical dual porosity model fails to capture these saturation profiles and hence miscalculates recovery during early WAG cycles. We present a double block dual porosity model, i.e. a simple multiple continua model, which better matches the fine grid simulation results.

Keywords: fracture-matrix transfer, WAG injection, dual porosity, three-phase flow

1. Introduction

Fractures can occur naturally in carbonate and clastic formations. Naturally fractured reservoirs comprise complex heterogeneities because the fractures are typically highly conductive but have small storage. Vice-versa, the rock matrix has high storage but normally only a small contribution to

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flow. This renders the design of enhanced oil recovery (EOR) schemes difficult because of poor injection fluid sweep efficiency and early water and/or gas breakthrough. In petroleum reservoirs, the latter often renders overall hydrocarbon recovery very low in fractured reservoirs. This has been shown in numerous case studies (e.g. Davidson and Snowdon, 1978; Denoyelle et al., 1988; Panda et al., 2009).

Continuous water injection into petroleum reservoirs is a well-established secondary recovery method which aims primarily to displace the oil and maintain the reservoir pressure. Oil displacement from the rock matrix in fractured reservoirs by injected water is capillary dominated and hence strongly dependent on the wettability of the rock (e.g., Behbahani and Blunt, 2005; Fernø et al., 2011; Schmid and Geiger, 2013). For unfavourable, i.e. mixed- to oil-wet matrix wettability, water flooding can be ineffective. In such cases, secondary recovery plans can be changed from water to gas injection to increase recovery (O'Neill, 1988; van Dijkum and Walker, 1991). In particular, gas oil gravity drainage (GOGD) provides an important drive mechanism in such cases because it can increase recovery factors irrespective of the reservoir wettability (e.g., Hagoort, 1980). Fractures extend the exposure of the injected gas with oil in reservoir rock, which renders GOGD more effective compared to unfractured reservoirs. Hence gas injection has been applied in many NFR (e.g., O'Neill, 1988; van Dijkum and Walker, 1991; Jakobsson and Christian, 1994; Saidi, 1996). However, as the gas mobility is high compared to water and oil, so is the risk of by-passed oil and gravity override, which can lead to early gas breakthrough (e.g., Panda et al., 2009). This is particularly true for NFR. In addition, the lack of availability of gas may limit implementation of a recovery scheme that solely relies on gas injection.

Water-alternating-gas (WAG) injection, both at miscible and immiscible conditions, combines the merits of the two injection fluids described above on macroscopic and microscopic scales while stabilizing the injection front, delaying breakthroughs, and therefore leading to increased oil recovery compared to continuous water or gas injection. This has been demonstrated in micro-model experiments that mimic multi-phase flow in conventional (Sohrabi et al., 2004) and fractured porous media (Er et al., 2010; Dehghan et al., 2012). In almost all reported cases, WAG application on the field-scale was observed to improve recovery (Awan et al., 2008; Brodie et al., 2012; Christensen et al., 2001).

Gas (continuous or as part of WAG flooding) injection represents more than 80% of EOR projects in carbonate reservoirs in the United States (Manrique et al., 2007) where the majority of the world’s WAG injections are applied (Christensen et al., 2001). In the North Sea, WAG is the most widely used EOR method (48%) and is typically applied in clastic reservoirs. In terms of incremental recovery, WAG has been regarded as the most successful EOR method in the North Sea (Awan et al., 2008). This is excluding other successful forms of WAG EOR methods, such as the Simultaneous WAG (SWAG) and Foam Assisted WAG (FAWAG). Elsewhere, preparations are underway to apply WAG to carbonate reservoirs in the Middle East (Arayni et al., 2013; Kalam et al., 2011; Rawahi et al., 2012) as well as the pre-salt carbonate reservoirs offshore Brazil (Pizarro and Branco, 2012).

Reservoir simulation of WAG injection includes additional complexities because all three phases, oil, gas and water, are mobile in parts of the reservoir. Hence representative three-phase relative permeability and capillary pressure functions, hereafter termed “flow functions”, are required to characterise the corresponding three-phase displacement processes. Three-phase relative permeability and capillary pressure data are very difficult to measure experimentally and there are
an infinite number of saturation paths that can occur. To overcome these challenges, empirical models are typically employed to predict three-phase flow functions from two-phase experiments (cf. Blunt, 2000). Empirical models are continuously improved to account for more processes that occur when three phases coexist (e.g., Fayers and Matthews 1984; Larsen and Skauge 1998; Blunt 2000). Although producing more accurate results, these improvements cannot easily overcome the major deficiency of the empirical models: They are mainly based on interpolating the much simpler physics of two-phase displacements, expressed in two-phase flow functions. They hence often fail to predict experimentally derived three-phase flow functions accurately, particularly when the rock is mixed or oil-wet (Delshad and Pope, 1989; Oak et al., 1990; Petersen et al., 2008; van Spronsen, 1982; Egermann et al., 2014).

Since most oil in a NFR is contained in the rock matrix, capillary and gravity forces can be more important in NFR in WAG compared to unfractured reservoirs. For example, capillary forces may either enhance or reduce recovery from matrix blocks depending on wettability (e.g., Gilman and Kazemi, 1988; Gang and Kelkar, 2008). Moreover, under some conditions, WAG may lead to water and gas displacing each other, while leaving the oil phase located in the rock matrix in place. Hence the choice of three-phase capillary pressure and relative permeability functions, which encapsulate how oil- or water-wet the rock is, will have a major impact on how fracture-matrix fluid transfer is predicted during reservoir simulation of WAG injection in NFRs.

At the field scale, the exchange between fractures and matrix is commonly modelled using dual porosity or dual permeability models. Both approaches employ transfer functions that simplify the exchange of fluids between fractures and matrix and resemble a fundamental upscaling process (e.g. Ramirez et al., 2009; Al-Kobaisi et al., 2009). This upscaling also neglects that the matrix properties are often heterogeneous, particularly in carbonate reservoirs. For example, Lichaa et al. (1993) found that that wettability indices in a Middle Eastern carbonate reservoir cover the full range from strongly water- to strongly oil-wet while the permeability varied over several orders of magnitude. These changes were observed in sections from two wells over a total length of less than 20 meters, i.e. at a length that is typically at or below the scale of a common reservoir simulation grid block.

In addition to the heterogeneities in matrix wettability and permeability, there are further heterogeneities to consider related to the scales at which natural fractures occur. Often the distribution of the matrix block sizes in a reservoir simulation grid block does not follow Warren and Root’s (1963) classical assumption that the matrix can be represented as uniform sugar-cube blocks. Instead, even at the scale below a single reservoir simulation grid block, individual blocks of the rock matrix often have multiple shapes and aspect ratios, which give rise to a distribution of fracture-matrix transfer rates (e.g., Haggerty et al., 2001; Di Donato et al., 2007; Geiger et al., 2013; Maier and Geiger, 2013). Since the classical dual porosity model assumes uniform matrix permeability, wettability, and uniform block sizes in each simulation grid-block, it is likely that some important recovery processes are misrepresented in the dual porosity upscaling process during WAG.

For two-phase flow processes in NFR, it has been demonstrated that lumping of capillary pressures can be used to account for heterogeneities in matrix permeability and wettability. Capillary pressure lumping involves the use of fine-grid simulations that represent matrix heterogeneities explicitly. The transfer function is then tuned to match the fine-grid results by readjusting the capillary pressure curve (Fung, 1993). To investigate fracture-matrix transfer during three-phase flow when WAG is applied to a NFR with a heterogeneous rock matrix, we follow a similar approach and use
fine-grid simulations to analyse the complex three-phase flow displacement processes. We then use the results from the fine-grid simulations to test how the dual porosity model could be adapted to capture three-phase fracture-matrix fluid transfer more accurately.

![Pore Scale to Grid-block scale to Reservoir Scale](image)

**Figure 1.** Step-wise upscaling procedure of recovery processes in naturally fractured reservoirs (NFR). Colours represent different phases (red = gas, green = oil, blue = water). Three-phase capillary pressure and relative permeability functions, so-called flow-functions, are derived from pore-network models to analyse the impact of wettability on three-phase displacement processes during water-alternating gas (WAG) injection in NFR. The three-phase flow-functions are then used to populate fine-grid simulation models for WAG where matrix heterogeneities and fractures are represented explicitly. Results from these fine-grid simulations are then compared to predictions from classical transfer functions and a new dual porosity model is formulated that allows us to capture the field-scale recovery processes during WAG in NFR more accurately.

Previous work has already investigated how the choice of hysteresis models impacts the predicted oil recovery during WAG in unfractured reservoirs (Spiteri and Juanes, 2006). Additionally, the impact of empirical and pore-network modelling derived three-phase flow functions on predicting recovery from a clastic reservoir during gas flooding after a prolonged waterflood was studied recently (Al-Dhahli et al., 2014). Here, we advance this research in that we compare three-phase relative permeability and capillary pressure curves derived from pore-network simulations with those from empirical models for predicting oil recovery from fractured reservoirs during WAG while considering different wettability states and other matrix heterogeneities.

Our work has three key objectives: First, we use a pore-network model to obtain physically consistent three-phase flow functions for immiscible displacements in realistic 3D pore geometries to estimate oil mobility at low oil saturations at different wettabilities. We then compare predicted recovery from a matrix block during WAG for the resulting flow functions and for empirical three-phase flow functions that interpolate two-phase relative permeability curves. Second, we use the three-phase flow functions derived from pore-network modelling in fine-grid simulations of WAG to analyse the emergent three-phase flow displacement processes and study the impact of various matrix heterogeneities. Last, we compare the resulting recovery curves to predictions from classical transfer functions and develop an approach that allows us to capture three-phase recovery during WAG more adequately in dual porosity models. Overall, this leads to a step-wise upscaling procedure (Fig. 1) where we analyse and attempt to preserve the displacement processes across various scales.

The paper is organized as follows: In the next sections, we introduce the pore-network model and show the resulting three-phase flow functions. Then we describe the fine-grid simulations at the intermediate scale and discuss the results from a sensitivity analysis where we show how different types of matrix heterogeneity impact recovery during WAG. Finally, we compare these results to predictions from classical transfer functions and develop an improved dual porosity model.
2. Pore-scale modelling of three-phase flow functions

We use a state-of-the-art pore-network model to compute the three-phase flow functions for immiscible displacement (Al-Dhahli et al., 2013a, 2013b). The model encompasses a number of features that enable us to capture a wide range of pore scale physics. Firstly, the model uses pore-network geometries from realistic 3D and multi-scale digital rocks as input. This is based on the work of Jiang et al. (2013a; 2013b). Secondly, the model comprises an improved pore shape characterization, which allows representation of complex pore throats and hence the computation of more accurate capillary entry pressures and hydraulic conductivities (Ryazanov et al., 2009). Thirdly, a novel thermodynamic criterion for oil layer formation and collapse (van Dijke and Sorbie, 2006) has been incorporated that allows for more accurate calculation of flow functions at low oil saturations in oil-wet pore systems. Finally, the model also accounts for multiple displacement chains occurring during three-phase flow that has been observed to cause low oil saturations during gas injection (Sohrabi et al., 2004). The model has been benchmarked against published three-phase relative permeability data for water-wet sandstone and oil-wet micro-model experiments (Al-Dhahli et al., 2013a, 2013b).

The pore-network we consider in this work is extracted using a process-based reconstruction of Berea sandstone (Øren and Bakke, 2003). It comprises 12348 nodes and 26146 bonds, resulting in permeability of 2673 mD and porosity of 18.3%. Further details of the pore-network are discussed in Al-Dhahli et al (2013a). We have selected the Berea sandstone because the three-phase flow functions obtained from pore-network modelling can be readily compared to experimental data for water-wet rocks (Oak et al., 1990; Blunt, 2000). Also, recovery processes in unfractured formations using three-phase flow functions from Berea sandstone have been discussed in the literature (Al-Dhali et al., 2014). However, the methods discussed here can be readily extended to more complex pore-networks extracted from carbonate rocks, as shown in Ahmed Elfeel et al. (2013) and Al-Dhahli et al. (2014).

Note that we only use the two- and three-phase flow functions from the pore-network model as input for the subsequent fine-grid simulations, not the permeability or porosity as especially the former would lead to a unrealistically high matrix permeability. We compute three-phase flow functions at various degrees of wettability by varying the contact angles. This allows us to quickly analyse how subtle changes in wettability impact recovery processes during three-phase flow. In this work, we consider two wettability cases: a strongly water-wet and a strongly oil-wet rock where we alter the contact angles after primary drainage.

For the strongly water-wet case, the oil/water contact angles are fixed (from 0 to 70°) and we generate relative permeability and capillary pressure data as follows: The network is initially saturated with water and oil flooding is simulated to model primary drainage. Then a series of water flooding (imbibition) simulations are carried out until the water saturation in the network reaches certain predefined values. Each water injection simulation is followed by a simulation of gas flooding. This leads to a series of saturation paths for water and gas injections. Hence, the effect of hysteresis due to a decrease in water and oil saturation during gas injection is automatically accounted for in the three-phase flow functions. However, hysteresis is not accounted for during subsequent water injections.
We carry out four two-phase displacements simulations to compute flow functions that provide the input for empirical three-phase models. These two-phase simulations model drainage and imbibition during water-oil flow as well as drainage and imbibition during gas-oil flow. The resulting flow functions match the experimental results of Oak et al. (1990) well, as discussed in Al-Dhahli et al. (2013a). The two-phase flow functions are a special case of three-phase flow functions and can be readily computed from the three-phase pore-network model. We use the resulting two-phase flow functions to initialise three different empirical models: Stone I and II (Stone 1970, 1973) and the saturation weighted interpolation (SWI) method of Baker (1988). Hysteresis was implemented in the empirical models using Carlson’s method for the two-phase flow functions (Carlson, 1981).

For the strongly oil-wet case, three-phase flow functions were computed following the same flooding sequence as above, but the oil-water contact angles are changed after the primary drainage to values ranging from 140 to 160°. This mimics ageing of the reservoir rock. Three-phase flow functions are represented by two-dimensional tables where relative permeability and capillary pressure of a given phase (e.g. oil) are functions of the other two saturations (e.g. water and gas) (Figs. 2 and 3 and Appendix A). The resulting relative permeability and capillary pressure data are smoothed to ensure monotonicity and enhance numerical convergence in the subsequent fine-grid reservoir simulations.

Figure 2. Three-phase relative permeability functions for water-wet (top) and oil-wet (bottom) Berea sandstone for the oil (kro), water (krw) and gas (krg) phases, respectively. Note the different colour scale for the water relative permeability. Coloured surfaces are fitted to pore-network modelling results (represented by the red dots) and have been smoothed to generate input tables for reservoir simulation.
3. Fine-grid simulations

A fine-grid simulation model comprising 64,000 cells was constructed at the approximate scale of a single reservoir simulation grid block. The fine-grid simulation model enables us to investigate how the three-phase flow functions generated above impact fracture-matrix transfer. For simplicity, the size of the grid block is uniform at 50x50x50 ft (15x15x15 m) with 27 (3x3x3) matrix blocks of 12x12x12 ft each. This resembles the classical, but highly idealized, sugar cube array of Warren and Root’s (1963) original dual porosity model (Fig. 4). The aim of this grid-block scale model is to incorporate the pore scale three-phase displacement processes, in the form of relative permeability and capillary pressure tables, at the continuum scale where fractures and matrix are present. We tested the effect of numerical dispersion by running simulations on a grid that was refined by a factor three but only observed negligible differences in the resulting recovery curves.

To simulate oil recovery from the matrix by water and gas injection, the fractures are initially filled with water. The initial saturations are the same in all models. For subsequent WAG cycles, the saturations of all three phases in the matrix are taken from the previous WAG cycle. However, the fracture saturation is changed instantly from water to gas and back to water during each WAG cycle. Matrix blocks were assigned two- and three-phase flow functions discussed in the previous section. The fractures were represented as high permeability features (1000 mD) in which we use linear relative permeability curves and assume zero capillary pressure. Further rock and fluid properties are summarized in Table 1. We note that the WAG cycle length in all simulations is two years to allow water and/or gas fronts to advance deep into the matrix blocks. We use the commercial reservoir simulator ECLIPSE in this analysis (Schlumberger, 2012).
Figure 4. Fine-grid model used to simulate fracture-matrix multiphase transfer (left). Cross-sectional view of the model showing the distribution of the phases (green = oil, blue = water) in the fractures (shown in dark blue as they are filled with water) and the matrix after a water injection cycle in an oil-wet matrix (right).

Table 1. Rock and fluid properties of the fine-grid model (note that matrix relative permeability and capillary pressure curves are derived using pore-network models or empirical models)

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matrix porosity</td>
<td>0.2 (Fraction)</td>
</tr>
<tr>
<td>Matrix permeability</td>
<td>1 mD</td>
</tr>
<tr>
<td>Matrix rock compressibility</td>
<td>3.60E-6 psi⁻¹</td>
</tr>
<tr>
<td>Fracture permeability</td>
<td>1000 mD</td>
</tr>
<tr>
<td>Viscosity, oil</td>
<td>0.523 cP</td>
</tr>
<tr>
<td>Viscosity, water</td>
<td>0.523 cP</td>
</tr>
<tr>
<td>Viscosity, gas</td>
<td>0.017 cP</td>
</tr>
<tr>
<td>Formation volume factor, oil</td>
<td>1.00 RB/STB</td>
</tr>
<tr>
<td>Formation volume factor, water</td>
<td>1.00 RB/STB</td>
</tr>
<tr>
<td>Formation volume factor, gas</td>
<td>0.650 RB/MSCF</td>
</tr>
<tr>
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<tr>
<td>Surface density, water</td>
<td>62.4 lb/ft³</td>
</tr>
<tr>
<td>Surface density, gas</td>
<td>0.042 lb/ft³</td>
</tr>
</tbody>
</table>

4. Results

In this section we show the results from the fine-grid simulations to compare predicted recovery from matrix blocks during WAG injection using pore-network derived and empirical three-phase flow functions. We then study the sensitivity of oil recovery considering heterogeneities in matrix permeability, wettability and block size distributions.

4.1. Water-wet case

The resulting oil recovery from the matrix during WAG for a water-wet case (see Figure 2 and 3 for the corresponding flow functions) is shown in Figure 5. Empirical models without hysteresis yield lower recoveries than the pore-network derived flow functions. When hysteresis is incorporated in the empirical models, the predicted recovery during the second WAG cycle (gas injection) is comparable to the results that employ pore-network derived flow functions. However, the oil
recovery continues to increase in subsequent WAG cycles. Ultimately, this causes up to 10% difference in recovery depending on which empirical model is chosen.

When hysteresis is not accounted for, trapping of the water and gas phases during gas and water injection, respectively, is not estimated correctly. This leads to inaccurate recovery predictions as water and gas displace each other, rather than oil, which leaves significant volumes of oil behind. We note that the flow functions from the pore-network model account for the hysteresis effect of a generic flow path; in this case, when gas saturations increase after water injection. Recovery predictions that employ pore-network model derived flow functions show hardly any increase during the third and later WAG cycles. This is because the pore-network model derived flow functions do not account for hysteresis and phase trapping during higher-order WAG cycles when the gas saturation decreases. Simulating hysteresis during higher-order WAG cycles with pore-network models could be achieved, in principle, by computing two-dimensional relative permeability tables for increasing and decreasing gas saturations and interpolating between the tables, but this remains subject to further research.

When hysteresis is accounted for in any of the empirical three-phase flow models using Carlson’s model, the observed recovery depends strongly on the choice of empirical model; up to 10% absolute difference in recovery have been observed during higher-order WAG cycles. This reiterates that the choice of three-phase relative permeability models is a key uncertainty in predicting oil recovery from fractured reservoirs during WAG.

Figure 5. Comparison of oil recovery from water-wet matrix blocks in the intermediate scale model predicted using three-phase flow functions from different empirical models, with and without hysteresis, and the pore-network model. Vertical grey lines show the boundaries between individual WAG cycles with 2 years length each. (G = gas, W = water). “no hyst” refers to three-phase flow functions without hysteresis.

The reason for the high uncertainty in oil recovery when considering hysteresis effects can be explained as follows: Hysteresis allows for trapping of the water and gas phases, which renders them less mobile and increases sweep efficiency and reduces the oil saturation to low values. A comparison between the three empirical models and pore-network results is given in Figure 6 at high and low oil saturations. There are significant differences of oil relative permeability values.
estimated by empirical models when the oil saturation is low, i.e. the saturation region that is targeted during WAG flooding.

As noted above, flow functions that were estimated from the pore-network model only account for hysteresis during increasing gas saturations, not decreasing ones which are encountered when water is reinjected. Since the recovery predictions using empirical three-phase models also vary greatly during later WAG cycles, we only consider the first two WAG cycles in the following analyses.

4.2. Oil-wet case

Figure 7 shows the resulting recovery profiles for the oil-wet case (see Figure 2 and 3 for the corresponding flow functions). It is well known that the wettability of the rock matrix impacts the recovery of oil from matrix blocks, particularly during capillary-driven imbibition and our simulations show the same: only 10% of the oil phase is recovered in the oil-wet case compared to 40% in the water-wet case during the first WAG cycle, i.e. during imbibition of water (compare Fig. 7 with Fig. 5).

In general, for capillary-dominated recovery, an increase in oil wetness decreases, both, the rate of recovery from the matrix and the recovery factor (Bebahani and Blunt, 2005, Haugen et al., 2008). This is because water-oil capillary pressure values are negative in oil-wet pores and hence capillary forces prevent water from entering the matrix blocks. Subsequently, the water phase can only enter the matrix to displace the oil phase when the gravity potential is higher than capillary forces.

Figure 7 also shows that gas injection is more effective in the oil-wet case compared to the water-wet case. The incremental recovery during the second WAG cycle was 27% in the oil-wet matrix (Fig. 7) compared to 13% in the water-wet matrix (Fig. 5), but we note that these values will vary for other rock types. The reason is twofold: First, the initial oil saturation is higher when the matrix is oil-wet as a result of poor recovery after the water injection cycle. Secondly, because the recovery is gravity dominated when the matrix is oil-wet, water accumulates at the bottom of matrix blocks (Fig. 8) while gas displaces oil starting from the top. This leads to a smaller three-phase region. In contrast, three-phase regions are larger in the water-wet case because gas interacts with the previously injected water that has imibed the matrix from all directions (Fig. 8). This is discussed in more detail in Section 5.
Note that unlike in the water-wet case, the impact of hysteresis in the oil-wet case is less and recovery predictions for pore-network model derived and empirical flow functions generally agree. As mentioned in the previous paragraph, the injected water and gas displace oil at different locations in the matrix, gas at the top and water at the bottom. This minimises the areas where three phases coexist and hence reduces the impact of hysteresis.

**Figure 7.** Comparison of oil recovery from oil-wet matrix blocks in the intermediate scale model predicted using three-phase flow functions from different empirical models and a pore-network model. The vertical grey line separates the two WAG injection cycles (approximately 2 years length each). “no hyst” refers to three-phase flow functions without hysteresis.

**Figure 8.** Cross-sectional view of the distribution of water saturation ($S_w$) after the water injection cycle, i.e. the first WAG cycle, in a water-wet (left) and an oil-wet (right) rock matrix. Gas that is injected in subsequent WAG cycles has to displace the previously injected water before it can displace the oil in the water-wet case. Hence hysteresis has a stronger impact in the water-wet case compared to the oil-wet case.
4.3. Effect of variations in matrix block size

As mentioned above, it is likely that matrix blocks of different sizes and shapes exist at a scale less than that of a single grid-block. An outcrop image and conceptual simulation models of this behaviour are shown in Figure 9. We simulate recovery for models with various matrix block sizes and shapes to analyse how four different geometrical heterogeneities in matrix shape (Fig. 9) impact recovery. In all simulations we assume that the rock is uniformly water-wet and the rock matrix permeability is uniform as well.

![Image of fractured carbonate outcrop showing matrix blocks of various sizes at a scale below the size of a single reservoir simulation grid block (Left) (Image courtesy of H. Boro). Side views of an idealized fracture-matrix arrangement with different matrix block sizes, showing the saturation distribution after a water injection (top right) and subsequent gas injection (bottom right). Matrix block geometries are (a) a sugar cube distribution, (b) non-uniform matrix block sizes, (c) match sticks, and (d) single block matrix shapes. Colours indicate fluid phases (green = oil, blue = water, red = gas).](image)

Recovery factors vary between 20% (single block model) and 49% (sugar cube model) during water injection. The incremental recovery after the gas injection cycle varies between 16% (single-block model) and 10% (match-stick model) (Fig. 10). While there is a clear positive correlation between fracture-matrix interface area and recovery factor during water injection, the incremental recoveries due to gas injection is influenced by the oil saturation after water injection and differs from the correlation observed for continuous gas injection without prior water flooding (Fig. 11).
Figure 10. Comparison of oil recovery profiles for models with variable matrix block sizes and shapes (see Fig. 9). The numbers give the fracture-matrix interface area for each model (Left). The correlation between recovery after the water injection cycle and fracture-matrix interface areas (Right).

![Comparison of oil recovery profiles](image)

Figure 11. Comparison of oil recovery due to continuous gas injection from models of various matrix block shapes and sizes (see Fig. 9). The numbers give the average matrix block vertical height for each model (Left). The correlation between recovery after continuous gas injection and average matrix block vertical height (Right).

![Comparison of oil recovery due to continuous gas injection](image)

4.4 Effect of variations in matrix permeability and wettability

Figure 12 shows the recovery profiles for the WAG simulations for uniform and heterogeneous permeability and wettability. First, we consider three different matrix permeabilities, all assuming a water-wet rock. In the first case, the matrix permeability is homogeneous. The two other cases have heterogeneous matrix permeability. The average permeability is the same in all cases but in the heterogeneous cases, the matrix permeability varies over three orders of magnitude and is either randomly distributed (“random permeability”) or the matrix permeability increases continuously upwards (“ordered permeability”). The heterogeneous cases have a Dykstra-Parson permeability variation coefficient of 0.86. Second, we consider three different matrix wettabilities, all using the ordered permeability model. In the first case, the matrix remains uniformly water-wet. The two other cases have a random distribution of water- and oil-wet rock (“random wettability”) or consider a scenario where the upper third of the matrix (with high permeability) is water-wet and the lower two thirds of the rock matrix (with low permeability) are oil wet (“ordered wettability”).

There are distinct differences in the recovery profiles depending on the arrangement of matrix permeability and wettability (Fig. 12). Recovery in the heterogeneous permeability case with uniform wettability is most sensitive to the low permeability regions. Fluid displacement by
spontaneous imbibition and gravity drainage occurs at slower rates in the low-permeability regions of the heterogeneous matrix. Hence the speed of recovery is faster in the homogeneous case compared to both heterogeneous permeability cases. The randomly distributed permeability yields higher recovery factors compared to the ordered matrix permeability at the end of the first WAG cycle. This is due to the high correlation length of the low permeability regions in the ordered permeability model whereas in the random permeability distribution, the low permeability regions are small and surrounded by high-permeability regions. During water imbibition, the displacement becomes increasingly slower in the ordered heterogeneous case. During gas injection, the vertical permeability arrangement in the ordered permeability model (highest permeability at the top) facilitated a faster recovery compared to the other two cases. Hence, the incremental recovery was higher in the ordered heterogeneous model and the cumulative recovery after two WAG cycles is similar for the two heterogeneous models.

Figure 12. Comparison of recovery profiles in three different permeability distributions (left) and three different matrix wettability scenarios (right). Ordered refers to a model where permeability or wettability is layered, random refers to a model where permeability and wettability are randomly distributed. Refer to the text for further discussion. Note that J-scaling of the capillary pressure curve does not help to reduce the difference in recovery predictions.

A water-wet rock matrix always yields the highest recovery during the first WAG cycle due to the rapid spontaneous imbibition. Although the high-permeability region in the ordered wettability model is oil-wet and water does not spontaneously imbibe into this region, gas oil gravity drainage during the second WAG cycle is accelerated in this region. The low-permeability region is water-wet and oil can be produced from this region during the first WAG cycle. The combination of capillary forces acting in the low permeability region and gravity forces acting in the high permeability regions leads to higher recoveries in the ordered wettability case compared to the random wettability case. Here, the low permeability regions can be oil-wet and vice-versa the high permeability region can be water-wet, which both slow down overall recovery.

5. Discussion
In this section, we analyse the results shown in the previous section and their impact on WAG injection recovery in NFR. We also discuss how different types of matrix heterogeneity and the resulting displacement processes during three-phase fracture-matrix transfer could be captured accurately in dual porosity models.
5.1. Three-phase displacement processes

Figure 13 schematically shows how oil, gas and water displace each other when they enter the matrix block at water- or oil-wet conditions. In the water-wet case and during the water injection cycle, oil recovery follows two distinct behaviours. The first occurs over short time-scales and is characterized by fast recovery as water imbibes into the matrix. The second occurs over longer time-scales and corresponds to slow recovery after water injection. These two distinct behaviours can be observed in recovery profiles in Figures 5 and 10 where 50% of the recoverable oil during the first water injection cycle was produced during the first few days. A ring-like region emerges which surrounds the oil in the matrix centre. This region has low oil saturation and low oil mobility. When gas is subsequently injected, it interacts first with the water in the ring-like region developing a three-phase region at the top of the matrix block (Figure 13).

The gas water interaction causes a delay in the incremental recovery during the gas injection cycle. The delay can be seen in recovery profiles of the second WAG cycles in Figures 10 and 12. This implies that when gas is injected after a water flood, gas can first displace the water that was injected in the previous WAG cycle, leading to higher water-cut but not necessarily higher oil cut. Similarly, when water is injected after gas injection, some of the water will displace gas first.

The above analysis can potentially explain why secondary gas injection might not significantly affect water and oil production in fractured reservoirs (e.g. Jackobsson et al., 1994). It further implies that if WAG injection cycles are short compared to the size of the matrix block, water and gas are likely to displace each other instead of recovering additional oil as the centre of the matrix blocks are not reached by the water and gas fronts.

![Figure 13. Idealised sketch showing the fluid interactions during WAG in a rock matrix with different wettability states.](image)

In the oil-wet case, water accumulates at the bottom and gas at the top of the matrix due to gravity forces. Hence a three-phase interface region develops only when the two injected fluids meet inside the matrix blocks. Before they meet, standard two-phase relative permeability give reasonable estimates of recovery because hysteresis effects has minimal effect on oil recovery (Fig. 7).

In the three-phase regions where gas and water are mobile and oil is close to or at residual saturation, it becomes increasingly difficult to quantify oil mobility and residual oil saturation as the oil saturation decreases because empirical three-phase relative permeability models estimate these properties differently (Fig. 6). The oil iso-perm corresponding to a relative permeability of zero in
Figure 6.b is of particular importance in this context because it defines the residual oil saturation, which varies significantly based on the selected three-phase relative permeability model. Of the three empirical relative permeability models considered in this work, only the Stone 1 model can be modified so that its three-phase residual oil saturation matches that computed using pore-network modelling. We reiterate that pore-network derived relative permeabilities at low oil saturations are physically consistent and agree well with experimental measurements (Al-Dhahli et al., 2013a; 2013b). Therefore, the choice of empirical three-phase relative permeability model impacts the estimation of three-phase residual oil saturation values, which impacts fracture-matrix fluid exchange and ultimately affects the field-scale forecasting of oil production.

5.2. Upscaling to dual porosity models
The detailed fine-grid simulations shown in the previous sections are not possible at the field scale. Hence recovery processes in NFR are typically upscaled using appropriate transfer functions in dual porosity and dual permeability models (e.g. Kazemi et al., 1976; Thomas et al., 1983; Ramirez et al., 2009; Al-Kobaisi, 2009). The transfer function employs parameters that can grouped into three categories, (1) fluid properties, (2) three-phase flow functions, and (3) shape factors. While the fluid properties are normally well known, the choice of three-phase flow function will have a great impact on recovery, as discussed above.

The shape factors are the imbibition shape factor ($\sigma$), gravity drainage shape factor ($\sigma_2$) and the matrix vertical block height ($L_2$). They impact the ratio of capillary to gravity forces during the individual WAG cycles. A frequent assumption is that imbibition and gravity drainage shape factors are the same. However, this assumption can lead to inaccurate results during WAG injection in NFR because gravitational and capillary forces interact differently with each other depending on the size and shape of the matrix blocks. For example the match sticks and single block cases in Figures 10 and 11 have similar recovery results during gas injection but very different recovery results during water injection.

Saturation profiles in the matrix blocks that were obtained from the fine-grid simulations show that there is a significant period of transient flow during water injection that is not captured by a classical dual porosity model that assumes a steady-state exchange of fluids between fracture and matrix (Fig. 14). This leads to significant differences in the predicted recoveries at early and late time (Zimmermann et al., 1995). It is possible to capture this transient behaviour in a dual porosity model by adjusting the shape factors so that the dual porosity model matches the fine-grid simulation results (Ueda et al., 1989; Thomas et al., 1983). However, a change in shape factor only increases or decreases the rate of recovery, but does not change the shape of the recovery curve (Abushiaka and Gosselin, 2008). Hence the match between the fine-grid simulations and dual porosity model predictions remains poor (Fig.15). Additionally, shape factors values should reflect matrix block sizes which are very well defined in the fine-grid model. As shown in Ahmed Elfeel et al. (2013), the dual porosity model overpredicts recovery during three-phase flow for both, structurally simple pore-networks such as those corresponding to a Berea sandstone and more complex pore-networks corresponding to carbonate rocks. The general features during WAG injection were observed to be similar for the sandstone and carbonate samples in that wettability affects recovery profiles and the mutual displacement of gas and water leaves the oil phase largely untouched.
Another alternative to match results from fine-grid simulations in a dual-porosity model are time-dependent shape factors (e.g., Penuela et al., 2002; Sarma and Aziz, 2006; Rangel-German and Kovscek, 2006; Geiger et al., 2013). However time-dependent shape factors usually involve solving an analytical equation that models the (non-linear) diffusion of water into the matrix blocks, which imposes certain simplifying assumptions. Most notably, gravity forces are often assumed to be negligible which is inappropriate for WAG injection. It is also possible to lump (or “pseudoise”) capillary pressure curves by treating them as a free parameter that can be adjusted to match the recovery curves predicted from a dual porosity model to fine-grid simulation results (e.g., Dean and Lo, 1988; Rossen and Shen, 1989; Fung, 1993; Gurpinar and Kossack, 2000). Lumping of capillary pressures is an iterative process that is often unphysical and impractical. For WAG injection, it is also not clear if and how the three-phase regions of low oil-mobility that control recovery behaviour (Fig. 12) can be represented adequately through capillary pressure lumping.

As capillary pressure lumping and transient shape factors may be inadequate for modelling WAG injection in NFR, we apply the multiple interacting continua (MINC) model (Pruess and Narasimhan, 1985; Wu and Pruess, 1988; Gilman, 1986; Rubin, 2007; Karimi-Fard et al., 2006; Gong et al., 2008, Tatomir et al., 2011) instead. In the MINC model, the matrix is discretised into multiple sub-regions such that each sub-region can contain its own saturation values and saturation gradients in the matrix can be modelled. The MINC model hence can represent the low oil mobility region and therefore approximate recovery during three-phase flow more accurately.

The original MINC model was developed for capillary dominated multi-phase flow, assuming that each matrix sub-region comprises concentric grid cells. The mass flux between these matrix sub-domains is calculated using their volumes, interface areas and nodal distances. These geometrical
parameters are computed once for a given matrix block size. Changes in matrix block sizes, e.g. as part of reservoir history matching, can be implemented by changing matrix permeability (Wu and Pruess, 1988). For gravity-dominated displacement, the MINC model can be recast such that the matrix sub-regions comprise horizontal layers that preserve vertical gradients in saturation (e.g., Gilman, 1986; Fung, 1993; Rubin, 2007).

To accommodate fast updates of the matrix block sizes in a MINC model, we present a simple but efficient double-block model (Fig. 15). The full mathematical development is given in Appendix B. This double-block model is a classical dual porosity formulation applied to two interacting matrix regions. The first region is the outer matrix block \( M_1 \) which enables fast imbibition from the fractures \( F \) into \( M_1 \). The second region is an inner matrix block \( M_2 \), for which flow only comes from the outer block \( M_1 \). Capillary and gravity driven exchange between \( M_1 \) and \( M_2 \) is governed by the classical dual porosity model, i.e. utilises the same transfer functions that govern fluid exchange between \( F \) and \( M_1 \). A new parameter \( \Delta \) is introduced that quantifies the size of \( M_1 \). This parameter is used to calculate the bulk volume of \( M_1 \) and \( M_2 \) and defines the geometry of each matrix block, honouring the actual fracture spacing. For \( \Delta = 0 \) or \( \Delta = L_{xyz} \), where \( L_{xyz} \) is the fracture spacing in \( x \), \( y \) or \( z \) direction, we obtain \( M_2 = 0 \) or \( M_1 = 0 \), respectively, and the classical dual porosity model is recovered.

What distinguishes the double-block model from the MINC model is that the size of \( M_1 \) is determined by the size of the three-phase region after the first WAG cycle where oil saturation and oil mobility are low. Hence the size of the \( M_2 \) is determined by the size of the matrix region where oil is still at high saturation. Recovery from \( M_2 \) is controlled by the mobility and size of \( M_1 \). The actual sizes of \( M_1 \) and \( M_2 \) are defined through a shape factor that quantifies their interfacial area. This has two advantages: First, we can model fluid exchange between due to gravity drainage and capillary imbibition between \( M_1 \) and \( M_2 \) by using the appropriate shape factors for each process. Fine grid simulations and dimensionless analysis of three-phase displacement in NFR can be used to guide the size of \( M_1 \) and to reflect the changes of the sub-matrix blocks as a function of the WAG cycle length (e.g. Pirker and Heinemann, 2008). Second, using shape factors to define \( M_1 \) and \( M_2 \) allows us to stay within the classical dual porosity concept and implement this formulation quickly into commercial and research grade reservoir simulators (e.g. Lie et al., 2012) as part of standard reservoir characterisation workflows that employ Discrete Fracture Network models to compute the effective properties of the fracture network (Dershowitz et al., 2000).

For completeness, we note that the fracture-matrix transfer could also be estimated utilising recent extensions of the MINC model. Gong et al. (2008), Karimi-Fard et al. (2006), and Tatomir et al. (2011) have demonstrated that the shape of each MINC sub-region can be estimated accurately for more complex fracture geometries than those discussed in our work. The sub-region shapes can be computed using local unstructured-grid simulations that explicitly account for both, fractures and matrix, for each reservoir simulation grid block. This MINC extension also leads to very accurate full-field predictions but comes at the extra computational cost of the computing the shape of the sub-regions locally for each reservoir simulation grid block before the reservoir simulation commences. In contrast, our proposed double block model requires less computational effort and uses the same input parameters as the standard dual porosity model, i.e. fracture spacing, in addition to one geometrical factor.
Figure 15. Conceptual image for the double-block model, i.e. the simple-most form of the MINC model. A matrix region is surrounded by fractures F which communicate with the outer matrix region M1 that is coupled to the inner matrix region M2. The size of M1 is related to the size of the three-phase region after the first WAG cycle where oil saturation and oil mobility are low (Fig. 5). Oil recovery from M2 where the oil saturation is high is hence controlled by the oil mobility in M1.

Figure 16 compares the recovery from the first and second WAG cycles of the water-wet case fine-grid simulations (Fig. 5) with the recovery estimated by a classical dual porosity model and the double-block model. Note that the early-time recovery is matched well in the double block model because water can imbibe quickly into M1. M1 also controls the exchange with M2 and hence honours the low oil mobility region, which leads to a better recovery predictions at late time. The two effects combined cause a markedly improved match of the fine grid simulation.
5.3. Effect of sub-cell heterogeneity

So far, our analysis of recovery processes and related upscaling to dual porosity models only considered uniform matrix properties in terms of wettability and block size distribution. However, we showed that oil recovery is sensitive to these heterogeneities (Figs. 10).

The non-uniform matrix block sizes model (Fig. 9b) provides an illustrative example to investigate the best way to estimate recovery from a heterogeneous rock matrix by averaging the matrix block size distribution. The model contains one large matrix block (32x32x32 ft) and 19 smaller matrix blocks (16x16x16 ft). The volume weighted average for the height and width of the matrix block is 20.74 ft. The average block dimensions are used to calculate the imbibition and gravity drainage shape factors for classical dual porosity models (Equation B.22).

![Graph comparing oil recovery using fine grid simulation, single block and the double block model.](image)

Figure 17 Comparison of oil recovery using fine grid simulation, single block and the double block model. The matrix block sizes are variable as shown in Figure 9b.

One alternative to capture the different transfer rates, for example due to differently shaped matrix blocks or permeability variations, in a single reservoir simulation grid block is through multi-rate dual porosity models. Multi-rate dual porosity models comprise a distribution of transfer rates within each reservoir simulation grid block to account for the fact that a single reservoir simulation grid block is likely to contain different matrix blocks with different shapes, permeabilities, and wettabilities. Each of the matrix blocks has its own transfer rate. The multi-rate dual porosity model hence requires less averaging of the matrix properties (Di Donato et al., 2007; Maier et al., 2013; Maier and Geiger, 2013). The double block model that we introduce here can be integrated readily in such a multi-rate dual porosity approach, so as to approximate spatial variations in saturation for each individual matrix block that is modelled with its own unique transfer function (Appendix B).

Figure 16 compares results from the fine grid simulation with a classical single block, a double block, and a multi-rate double block dual porosity model. The classical single block dual porosity model uses the averaged shape factor for the matrix blocks and fails to match both, early and late time recovery. The double block model predicts low recovery compared to the fine grid during the gas injection cycle but still yields a much improved match compared to the classical single block dual porosity model. The best result is obtained with the multi-rate double block dual porosity model.
However, this model did not completely match fine grid simulation results during gravity drainage because the outer matrix blocks geometry as discussed previously. Nevertheless, the multi-rate dual double block model is capable to match recovery significantly more accurately compared to the other dual porosity models.

The reason why the single-rate double block model matches the recovery during the second WAG cycle less well is the averaging of fracture spacing. The size and geometry of the matrix block controls the speed of recovery (Figs. 10 and 11). Hence the variably sized blocks will have different phase saturations and the sizes of $M1$ and $M2$ vary depending on the overall dimensions of the matrix blocks. If only one average matrix block size is considered, $M1$ and $M2$ will also be averaged and the size of the three-phase region, which controls the release of oil from the centre of the matrix block, will be estimated inaccurately.

The question that arises then is whether we can actually characterize a multi-rate dual porosity medium from the data that are typically collected for a fractured reservoir. Clustering analysis and spatial organization of wellbore intersecting fractures can be used in approximating matrix block size distribution. Outcrop analogues provide information on fracture spacing and matrix block sizes. This information can be used in combination with discrete fracture network (DFN) modelling to obtain different likelihoods of block sizes and shape factor distributions (e.g., Sarda et al., 1997; Dershowitz et al., 2000). Classical geomodelling of the rock matrix already provides information on permeability and wettability distributions in the rock matrix.

### 6. Conclusion

We examined capillary and gravity dominated fluid exchange between matrix blocks and fractures during three-phase flow using models at multiple scales. Three-phase relative permeability and capillary pressure data were derived from a state-of-the-art pore-network model and compared with flow functions computed with empirical models (Stone I, Stone II, Saturation weighted interpolation and Carlson’s hysteresis model). A high-resolution fine grid simulation model enabled us to investigate the first-order parameters that control three-phase fracture-matrix transfer processes.

Our work shows that, like for unfractured reservoirs, hysteresis is key parameter that influences the prediction of oil recovery during WAG in fractured reservoirs. The choice of three-phase model and hysteresis model, can cause up to 10 % difference in predicted oil recovery from a water-wet rock matrix. The difference in recovery is smaller for an oil-wet rock matrix because of a smaller size of the three-phase region and less impact of hysteresis.

A sensitivity study showed that matrix heterogeneities in terms of wettability, block geometry and permeability affect fracture-matrix transfer rates significantly. The different transfer rates arising from these heterogeneities cannot be averaged using classical dual porosity models. We found that the classic dual porosity model provides incorrect estimates of recovery because the assumption that saturations are uniformly distributed in the rock matrix leads to wrong estimates of the transfer rates. This effect is well-known for two-phase flow and exacerbated for three-phase flow because regions where three phases coexist and oil mobility is low or hysteresis is important cannot be modelled properly.

We therefore presented a new form of the Multiple Interacting Continua (MINC) model, a double block model comprising an inner and outer matrix block. This new model already provides
significantly better estimates of fracture-matrix transfer rates during three-phase flow at negligible computational cost. The double block uses shape factors estimated from the geometries of the sub-domain grid blocks and the fracture spacing and can be extended easily to a multi-rate dual porosity model. The multi-rate model incorporates different transfer rates arising from matrix heterogeneities (e.g. different block size distributions), provides the best estimates for fracture-matrix transfer rates as it captures these sub-cell heterogeneities more adequately than a classical single- or the new double block model. Multi-rate dual porosity models can be combined readily with the double block dual porosity approach to account for both, multiple transfer rates due to matrix heterogeneity and heterogeneous saturation distributions in each rock matrix block.

7. References


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Appendix A

In three-phase flow functions, relative permeability and capillary pressure are tabulated as functions of two saturations (enough for three-phases as we have the closing relationship $S_3=1.0-S_1-S_2$). For example, oil relative permeability, $k_{ro}$, can be treated as a function of water and gas saturations, $S_w$ and $S_g$, as shown in Fig. A.1.

Figure A1. Oil relative permeability values as a function of two saturations (All in percentage). The red dots represent the pore-network model results. For input to numerical simulators, a surface is fitted and smoothed and represented as two-dimensional table (Fig. A2).

Two-phase flow functions represent one row in the 2D tables. For example, the three-phase oil relative permeability table shown in Figure A2 includes the two-phase oil-water displacement flow functions (at $S_g = 0$).

Appendix B

The classic dual porosity model assumes a uniform saturation profile inside the matrix. Fine grid simulations showed that the saturation in the matrix is not uniform and that a low oil mobility region traps oil at the centre of the matrix blocks. One of the proposed solutions is to provide more discretisation inside the matrix domain to capture the saturation gradient. We consider here a simple MINC model to allow for modelling of the low oil mobility region by having an inner and outer matrix region, whose sizes are defined by the fracture spacing and the size of the three-phase region (Fig. 15).

In our fine grid simulations, we assume the fractures are instantaneously filled with water or gas. Hence, we only solve for saturation inside the matrix blocks. Thomas et al. (1983) developed a three-
phase model for simulating flow of water, oil and gas in a NFR using the dual porosity system. We
describe the fracture matrix transfer based on their equations.

For the fractures, we assume zero capillary pressure values. Therefore,

\[ p_{wf} = p_{of} = p_{gf} = p_f \] \hspace{1cm} \text{B. 1}

The matrix/fracture flow equations for M1 (Thomas et al., 1983):

\[ -\lambda_{w1}(p_{wml} - p_f + G_{wgl} + G_{wol}) = \frac{V_{bl}}{\Delta t} \bar{S}(\phi b_w S_{w1}), \] \hspace{1cm} \text{B. 2}

\[ -\lambda_{o1}(p_{oml} - p_f + G_{ogl} + G_{owl}) = \frac{V_{bl}}{\Delta t} \bar{S}(\phi b_o S_{o1}), \] \hspace{1cm} \text{B. 3}

and

\[ -\lambda_{g1}(p_{gml} - p_f + G_{gol} + G_{gwl}) = \frac{V_{bl}}{\Delta t} \bar{S}(\phi b_g S_{g1}). \] \hspace{1cm} \text{B. 4}

The phase mobilities are defined as follows (Thomas et al., 1983)

\[ \lambda_{\alpha 1} = \sigma_{M1} k_{rad1} \left( \frac{k V_{bl}}{B_\alpha \mu_\alpha} \right), \] \hspace{1cm} \text{B. 5}

where the subscript \( \alpha \) denotes the phase, relative permeability is based on the upstream value.

The multiphase gravity terms are pseudo capillary pressure terms that are defined as follows:

\[ G_{\alpha \beta} = \frac{\sigma_{\alpha M1}}{\sigma_{M1}} (\rho_\alpha - \rho_\beta) \frac{g L_{mol}}{2} (h_{mlol} - h_{gl}). \] \hspace{1cm} \text{B. 6}

Where the subscripts \( \alpha \) and \( \beta \) denote two different phases and \( h_\alpha \) is a dimensionless variable that depends on the saturation of \( \alpha \), i.e.

\[ h_\alpha = \frac{S_\alpha - \min(S_\alpha)}{1 - \min(S_\alpha) - S_{or\alpha}}. \] \hspace{1cm} \text{B. 7}

Equations B. 2 to B. 4 are three equations to solve for six unknowns: \( p_{wml}, p_{oml}, p_{gml}, S_{wml}, S_{oml} \) and \( S_{gml} \). The closing equations are:

\[ S_{wml} + S_{gml} + S_{oml} = 1, \] \hspace{1cm} \text{B. 8}

\[ p_{wml} = p_{oml} - p_{f wml} \] \hspace{1cm} \text{B. 9}

and

\[ p_{gml} = p_{oml} + p_{g methyl}. \] \hspace{1cm} \text{B. 10}
We introduce the following operators to solve the flow equations in a coupled manner:

\[ \alpha_o = \frac{b_o}{b_w} \text{ for Equation B.3} \]

and

\[ \alpha_g = \frac{b_w}{b_g} \text{ for Equation B.4.} \]

Summing up the flow equations after multiplying by the operators gives:

\[
-\lambda_w \left( p_{oml} - p_f + G_{wg1} + G_{wol} \right) - \alpha_o \lambda_o \left( p_{oml} - p_f + G_{og1} + G_{owl} \right) - \alpha_g \lambda_g \left( p_{oml} - p_f + p_{cgml} + G_{gw1} + G_{gol} \right) = \frac{V_{b1}}{\Delta t} \phi w \Delta \left( S_{wml} + S_{oml} + S_{gml} \right).
\]

B. 11

The right hand side collapses because \((S_{wml} + S_{oml} + S_{gml})\) is always constant. We introduce \(p_{diff}\) such that:

\[ p_{diff} = p_{oml} - p_f, \]

B. 12

and solve for it in Equation B. 11,

\[
\frac{\lambda_w}{\phi w} \left( -p_{cmml} + G_{wg} + G_{wo} \right) + \alpha_o \lambda_o \left( G_{og} + G_{ow} \right) + \alpha_g \lambda_g \left( p_{og} + G_{go} + G_{gw} \right) = -\left( \frac{\lambda_w}{\phi w} + \alpha_o \lambda_o + \alpha_g \lambda_g \right).\]

B. 13

To find \(S_{wml}\) and \(S_{gml}\) we substitute the \(p_{diff}\) value back in Equation B. 2 and B. 4, respectively.

\[
S_{wml}^{n+1} = S_{wml}^n - \frac{\Delta t \lambda_{w1}}{\phi w} \left( p_{diff1} - p_{cmml} + G_{wg1} + G_{wol} \right),
\]

B. 14

\[
S_{gml}^{n+1} = S_{gml}^n - \frac{\Delta t \lambda_{g1}}{\phi g} \left( p_{diff1} + p_{cgml} + G_{gw1} + G_{gol} \right).
\]

B. 15

The flow equations for M2:

\[
-\lambda_{w2} \left( p_{um2} - p_{um1} + G_{wg2} + G_{wo2} \right) = \frac{V_{b2}}{\Delta t} \phi w \Delta (\phi w S_{w2}),
\]

B. 16

\[
-\lambda_{o2} \left( p_{om2} - p_f + G_{og2} + G_{ow2} \right) = \frac{V_{b2}}{\Delta t} \phi o \Delta (\phi o S_{o2}).
\]

B. 17

and

\[
-\lambda_{g2} \left( p_{gm2} - p_f + G_{go2} + G_{gw2} \right) = \frac{V_{b2}}{\Delta t} \phi g \Delta (\phi g S_{g2}).
\]

B. 18
The solution follows the same definition as with M1. Potential differences are taken between the two matrix blocks. For example, phase gravities are defined as:

\[
G_{\alpha\beta} = \frac{\sigma_{M2}}{\sigma_{M2}} (\rho_\alpha - \rho_\beta) \frac{gL_{z2}}{2} (H_{m2} - H_{m1}),
\]

And that \( p_f \) is replaced by \( p_{oml} \). Consequently,

\[
S_{wm2}^{n+1} = S_{wm2}^n - \frac{\Delta t \lambda_{w2}}{\phi_w V_{b2}} (p_{diff2} + p_{cmwl} - p_{cwmm2} + G_{wg2} + G_{wo2}) \quad \text{B. 20}
\]

\[
S_{gm2}^{n+1} = S_{gm2}^n - \frac{\Delta t \lambda_{g2}}{\phi_g V_{b2}} (p_{diff2} + p_{cgml} - p_{cgmm2} + G_{gw2} + G_{go2}) \quad \text{B. 21}
\]

To determine the shape factors, the double block model takes the average fracture spacing in three directions \( L_x, L_y, \) and \( L_z \) in addition to the outer block thickness \( \Delta/2 \) as shown in Figure B1. This additional parameter can be determined empirically from fine grid simulation as it depends on matrix permeability and fluid viscosities and dimensionless analysis.

![The geometrical arrangement in the double block model. All matrix sub-domains have the same pressure and saturation values at a given time during fracture-matrix transfer.](image)

The imbibition shape factor can be calculated using matrix block geometry based on the following equation (Kazemi et al., 1992)

\[
\sigma = \frac{1}{V} \sum_{j=1}^{j} \frac{A_j}{d_j},
\]
where \( j \) denotes a matrix block in grid block with a total number of matrix blocks \( J \). \( A_j \) is the total surface area, \( d_j \) is the distance the imbibition front can move from a side of the matrix block and \( V \) is the volume of the matrix block.

The surface area over which spontaneous imbibition occurs at the outer matrix block is the sum of the areas of the matrix block sides, \( 2L_xL_y + 2L_xL_z + 2L_yL_z \). The potential distance of the imbibition front is the same as the outer block thickness \( \Delta/2 \) and the volume is given by \((L_x, L_y, L_z) - ((L_x - \Delta)(L_y - \Delta)(L_z - \Delta))\). Due to the large area and a relatively short front movement distance, the shape factor for M1 is high compared to that of M2 (see below) and is given by:

\[
\sigma_{M1} = \frac{\frac{2L_xL_y + 2L_xL_z + 2L_yL_z}{2}}{\left[(L_xL_yL_z) - ((L_x - \Delta)(L_y - \Delta)(L_z - \Delta))\right]}.
\]

Similarly, the area and volume are computed geometrically for the inner matrix block, M2. The imbibition front can advance from the matrix faces to the centre and is approximated by \(\frac{1}{2}(L_x - \Delta)(L_y - \Delta)(L_z - \Delta)/2 \). The imbibition shape factor for M2 is calculated as,

\[
\sigma_{M2} = \frac{\frac{2L_xL_y - 2L_xL_z + 2L_yL_z}{2}}{\left[(L_xL_yL_z) - ((L_x - \Delta)(L_y - \Delta)(L_z - \Delta))\right]}.
\]

Unlike the spontaneous imbibition, gravity drainage displacement is unidirectional. Here, we start with M2 as it is straightforward. The acting cross-sectional area for such a displacement for M2 is approximated by \((L_x - \Delta)(L_y - \Delta) + L_z - \Delta)(L_x - \Delta) + 2(L_y - \Delta)(L_z - \Delta)\) and the potential distance of gravity drainage front is equal to M2’s vertical height \((L_z - \Delta)\). Consequently, the gravity drainage shape factor for M2 is

\[
\sigma_{z:M2} = \frac{(L_x - \Delta)(L_y - \Delta) + 2(L_x - \Delta)(L_z - \Delta) + 2(L_y - \Delta)(L_z - \Delta)}{(L_z - \Delta)(L_x - \Delta)(L_y - \Delta)}.
\]

The gravity drainage cross-sectional area for M1 is computed through weighted averaging,

\[
A_{z,M1} = \frac{\Delta}{L_z} (L_xL_y + L_x - \Delta) + \frac{\Delta}{L_x} (L_yL_z - L_y - \Delta) + \frac{\Delta}{L_y} (L_xL_z - L_z - \Delta).
\]

B.26

Similarly, for the front vertical displacement distance,

\[
d_{z:M1} = \frac{\Delta}{\sqrt{L_xL_y}} (L_z) + \frac{\Delta}{\sqrt{L_yL_z}} (L_x).
\]

B.27

Hence, the gravity drainage shape factor for M1 is given by
\[
\sigma_{m1} = \left( \frac{1}{L_x} \right) \left( \frac{1}{L_y} \right) \left( \frac{1}{L_z} \right) \left( \frac{\Delta (l, L_x)}{L_x} + \frac{\Delta (l, L_y)}{L_y} + \frac{\Delta (l, L_z)}{L_z} (l, L_x, -\Delta) + \frac{\Delta (l, L_z)}{L_z} (l, L_y, -\Delta) \right)
\]

The vertical height for the matrix sub-domains is given by:

\[h_1 = L_z, \quad \text{while } h_2 = L_z - \Delta.\]

The bulk volume of the matrix sub-domains is calculated based on the volume of a simulation grid block as follows:

\[V_{b1} = \frac{V_m}{L_x L_y L_z} \left( L_x L_y L_z - (L_x - \Delta)(L_y - \Delta)(L_z - \Delta) \right),\]

and

\[V_{b2} = \frac{V_m}{L_x L_y L_z} (L_x - \Delta)(L_y - \Delta)(L_z - \Delta);\]

where \(V_m\) is the total matrix volume in the simulation grid block. Note that if \(\Delta = 0\), then \(V_{b1} = 0\); while if \(\Delta = L\), then \(V_{b2} = 0\). In both previous cases, the double block model behaves as the classic dual porosity model.

**Implementation and validation of the double-block model**

Equations B.14, B.15, B.20 and B.21 were incorporated into a C++ code (available from the first author upon request) to calculate recovery from the matrix using the double block model. The code reads in two-dimensional table that represent three-phase flow functions derived from the pore-network model. The phase saturations are calculated explicitly as the mobility terms, \(\lambda_{\alpha}\), are evaluated at the saturations of the previous time-step. However, very small time-steps were selected to stabilize the solution. To validate this approach, a comparison was made with results from the fully implicit implementation of a commercial Black-Oil simulator (Fig. B2).
Figure B2. Comparison of matrix-fracture transfer calculation in a commercial Black-Oil simulator and the double-block model with the parameter Δ = 0. The geometrical constants used were 0.0469 ft^-2 and 0.0195 ft^-2 for σ and σ_z, respectively.

However, the main validation step was the comparison against fine grid simulation results. Figs. 16 and 17 show that the double block model is capable of matching fracture-matrix transfer with various shape factors representing different matrix block sizes. The results are also qualitatively comparable to the original MINC model results. Wu and Pruess (1988) showed that the classical dual porosity model may result in large errors, particularly, when the matrix permeability is low or when the matrix block sizes are large. This is because the transient effects take longer times in these two cases. Generally, they showed that recovery is overestimated when the matrix domain is not divided into sub-domains and a constant shape factor is used. Our double block model captures this transient behaviour as the quickly changing saturation in the outer matrix block diminishes the capillary diffusion process and leads to accurate matrix recovery.

Extension to Multi-rate models

Based on the flow equation (B2), the fracture-matrix transfer rate for water in a two phase oil-water case can be written as:

$$
\tau_w = \sigma V_b \frac{k_{rw}}{\mu_w} \left( p_{wf} - p_{wm} \right) - \frac{\sigma_z}{\sigma} \left( \rho_w - \rho_o \right) \frac{g L_{cl} \delta}{2} \left( h_{wf} - h_{wm} \right)
$$

Eq. B.32

Multi-rate dual porosity models (MRDP) accommodate a distribution of matrix block rates due to heterogeneity (Di Donato et al., 2007; Maier et al., 2013; Maier and Geiger, 2013). The heterogeneity can be due to geometrical heterogeneity or porosity and permeability heterogeneity. If the permeability of the matrix blocks is assumed to be constant a multi-rate transfer rate due to geometrical heterogeneity can be written as follows:

$$
\tau_w = \sum_{i=1}^{N} \sigma_i V_b \frac{k_{rw}}{\mu_{wi}} \left( p_{wfi} - p_{wm} \right) - \frac{\sigma_z}{\sigma_i} \left( \rho_{wi} - \rho_o \right) \frac{g L_{cl} \delta}{2} \left( h_{wfi} - h_{wm} \right)
$$

Eq. B.33

where $N$ is the number of rates. The characterisation factors needed for each rate are $\sigma$, $\sigma_z$, and $V_b$. Saturation dependent and pressure dependent parameters such as $k_{rw}$ and $\mu_w$ vary because each sub matrix component holds its phase saturations and pressures.

Highlights

- We use three-phase pore-network model results to study matrix-fracture transfer
- We perform a careful sensitivity on the effect of matrix heterogeneity
- We present a simple MINC model that better match fine grid simulation results